Region of Attraction Estimation Using Multiple Shape Functions in Sum-of-Squares Optimization for Aerospace System

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Abstract

Stability analysis is of particular importance in the aerospace system. One commonly used approach for stabilizing a system is the linearization of a nonlinear system and the cancellation of nonlinearities. However, a more capable system can be achieved if the peculiarities of a nonlinear system are not cancelled out but utilized. Most nonlinear systems are stable only in a specific region around an equilibrium point (EP). This invariant set, the so-called region of attraction (ROA) of the relevant EP, is an important metric for system stability and robustness and specifies the extent to which the initial states can be disturbed away from the expected steady states. Knowledge of the stable region is essential in aerospace since it delivers a safe region for robust and safe operation. In addition, the flight control system requires an intensive clearance process for certification purposes.

Finding or estimating the exact ROA for a nonlinear system, both numerically and analytically, can be a laborious task. The problem is complicated by the additional use of nonlinear filters, adaptive control, or parameter estimation strategies. There are several promising novel numerical approaches, including Sum-of-Squares (SOS) optimization-based ones that estimate ROA by finding a Lyapunov function (LF).

The SOS optimization is based on an SOS polynomial. An SOS polynomial is a non-negative polynomial that can be expressed as a sum of squares of other polynomials. An LF can be expressed as an SOS polynomial since it is also a non-negative polynomial. The shape function (SF), which specifies a region of variable size, is a very essential part of SOS optimization. The existing method uses an origin-cantered SF in SOS optimization to estimate the ROA. The SF is a positive definite function, and its encircled region is surrounded by the estimated LF. It forces the newly generated LF to capture a more stable area by increasing its size in every iteration. However, the existing SOS technique is very conservative for real aerospace systems, especially in the case of non-symmetrical or unbounded ROAs. To overcome the limitations of the SOS technique, we propose a new method based on multiple SFs in SOS optimization, which estimates a very large subset of ROA even if the actual ROA is irregular.

The effectiveness of the proposed method is demonstrated via ROA estimation of trim points for generic transport aircraft and fighter aircraft. It is shown that our method significantly over performs state-of-the-art methods.