An arbitrary order H_∞ interval filter for spacecraft navigation units

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This paper presents a new H_∞ interval-based estimation filter for navigation units used in space applications. The key element of the proposed approach consists of the structure of the proposed estimator. Rather than an observer-based structure, the interval estimator is formulated as a generic state-space realization to design. Thus the problem turns out to the design of an arbitrary order state-space realization, in spite of observer gain matrices. In that sense, the proposed interval estimator offers more degrees of freedom (its order can differ from the system model), than current existing interval observers [1]-[4]. The H_∞ paradigm is used to enhance robustness against sensor misalignment errors, noises and other unknown inputs.
The application support is inspired by the Microscope satellite mission [5]. The satellite combines simultaneously a rotational motion ω_o around the Earth on a sun-synchronous, quasi-circular dawn-dusk orbit, while performing simultaneously a rotation around its y-axis ω_s. The tracking of the trajectory is ensured by a AACS (Attitude and Acceleration Control System), whose schematic structure is illustrated in Fig.1. The avionics is assumed to be composed of a μASC (micro Advanced Stellar Compass) and the SAGE (Space Accelerometer for Gravitation Experimentation) system, that provide the attitude $\Theta_m \in \mathbb{R}^3$ and both the angular acceleration $\dot{\omega}_m \in \mathbb{R}^3$ and linear acceleration $\Gamma_m \in \mathbb{R}^3$ (the index "m" denotes measured variables). The navigation unit is in charge of providing an estimate of the linear acceleration Γ (role of the hybridation filter), the lower and upper bounds $\underline{\Theta}, \overline{\Theta}, \underline{\omega}, \overline{\omega}$ of the true attitude Θ and angular rates ω (role of the interval estimator), and an estimate $\hat{\Theta}$ of the attitude, derived by means of a l_1-optimal fusion rule. The control is ensured by a 6DOF controller that delivers the 3-dimension force $F_c \in \mathbb{R}^3$ and torque $C_c \in \mathbb{R}^3$, that are then converted to thruster firing commands $\tau_c \in \mathbb{R}^8$.
The satellite mission is implemented in a functional engineering simulator (FES), which includes highly representative models of sensors (misalignment errors, non Gaussian noises...) and actuators (including dead-zone, Minimum Impulse Bit effect...), and Dynamics Kinematics and Environment models. The environment modules contain the spatial disturbances such as the magnetic field, the aerodynamic drag, the gravitational disturbances, the solar and the albedo radiations.

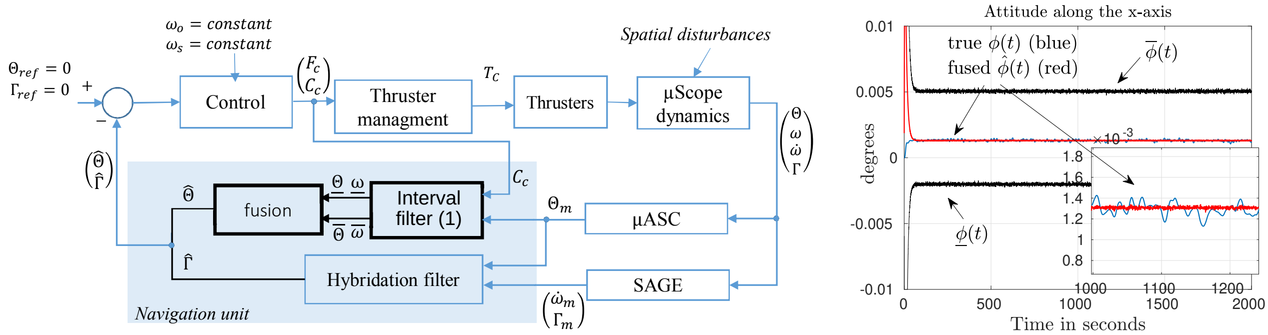

Fig. 1. Left: The AACS and avionics architecture - Right: attitude angle $\phi(t)$, its upper/lower bounds and the fused value

More precisely, the proposed H_∞ interval estimator admits the following structure (see Fig.1 to identify input/output signals):

$$\begin{aligned}
\dot{\overline{s}}_f &= A_F\overline{s}_f + (B_{F_1} - B_F)u_F + \overline{\epsilon}_1(\overline{z}_f, \underline{z}_f, \overline{w}, \underline{w}) \\
\dot{\underline{s}}_f &= A_F\underline{s}_f + (B_F - B_{F_2})u_F + \underline{\epsilon}_2(\overline{z}_f, \underline{z}_f, \overline{w}, \underline{w}) \\
\dot{x}_f &= A_F x_f + B_F u_F
\end{aligned}
\qquad
\begin{aligned}
\overline{x}_f &= \overline{s}_f + x_f \\
\underline{x}_f &= x_f - \underline{s}_f \\
\overline{z}_f &= C_{F_1}\overline{x}_f + D_F u_F + \overline{\psi}_1(\overline{x}_f, \underline{x}_f, \overline{w}, \underline{w}) \\
\underline{z}_f &= C_{F_2}\underline{x}_f + D_F u_F + \underline{\psi}_2(\overline{x}_f, \underline{x}_f, \overline{w}, \underline{w})
\end{aligned}
\tag{1}$$

$\overline{s}_f$, $\underline{s}_f \in \mathbb{R}^{n_f}$ refer to the filter states and $\overline{x}_f$, $\underline{x}_f \in \mathbb{R}^{n_f}$ denote the upper and lower bounds of the state x_f. $\overline{z}_f = [\overline{\Theta}^T\ \overline{\omega}^T]^T$ and $\underline{z}_f = [\underline{\Theta}^T\ \underline{\omega}^T]^T$ are the filter outputs and $u_F = [C_c^T\ \Theta_m^T]^T$ is its input. $w \in \mathbb{R}^{n_w}$ refer to all disturbances and unknown inputs, assumed to be bounded by known bounds $\underline{w}, \overline{w}$.
The objective is to determine the functions $\overline{\epsilon}_1, \underline{\epsilon}_2, \overline{\psi}_1, \underline{\psi}_2$ and the matrices $A_F, B_F, B_{F_i}, C_{F_i}, D_F$, $i = 1, 2$, so that it is guaranteed theoretically that the true vector $[\Theta^T\ \omega^T]^T$ belongs to the interval $[\underline{z}_f, \overline{z}_f]$, and so that the interval length $\overline{z}_f - \underline{z}_f$ is minimized, in the H_∞-criterion sense. The proposed solution exploits the cooperative property and the H_∞ paradigm. It is shown that the design of the interval filter can be formulated as an optimisation problem under linear and bilinear matrix inequalities (BMI) constraints, that can be solved using BMI solvers like PENLAB.
Results obtained from the FES demonstrate that, as expected, the true attitude and angular velocity belong to their associated intervals, i.e. $\underline{\Theta}(t) \leq \Theta(t) \leq, \overline{\Theta}(t)$ and $\underline{\omega}(t) \leq \omega(t) \leq \overline{\omega}(t)$, $\forall t \geq 0$, despite the presence of misalignment errors in the sensors, non Gaussian noises and other existing unknown inputs. Fig. 1 gives an illustration of the obtained results.

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