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Tracking Algorithms for Optimal Trajectories Based on Augmented Pursuit Guidance

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Abstract

This paper proposes an Augmented Pursuit Guidance (APG)-based guidance law that faithfully tracks optimal trajectories by reproducing the optimal lead angle history with respect to the target. While optimal guidance laws offer theoretical performance advantages, their real-time applicability is often limited due to computational complexity. To address this gap, the proposed APG method provides a practical and implementable alternative that closely approximates the optimal solution using precomputed or polynomial-approximated data. Simulation study shows that APG is a simple and efficient method for trajectory tracking during the mid-course guidance phase of a long-range air-to-air missile equipped with a dual-pulse rocket.

1. Introduction

Midcourse guidance of long-range flights is a difficult problem since simple guidance laws such as proportional navigation (PN), which is very useful for terminal homing guidance, are not applicable. Due to large variations in speed and altitude during the midcourse phase, the optimal trajectory is only available through numerical trajectory optimization. Real-time computational guidance based on convex programming, which has been extensively investigated recently, has a high potential of practical application. Nonetheless, how to track the optimal trajectory is a separate issue. Even if the optimal control input, for example, the optimal normal acceleration or angle of attack, is accurately applied to the autopilot, there is no guarantee that the missile will precisely follow the optimal trajectory. Due to this problem, the optimal trajectory should be updated frequently during the flight. Path following (or trajectory tracking) is a better option in this situation since the missile will arrive at the intended terminal position. Various carrot chasing algorithms could provide satisfactory tracking performance but the performance of the optimal trajectory may not be achieved, depending on the algorithm details.

APG has been studied for midcourse guidance of long-range air-to-air missiles due to its ability to accurately follow polynomial trajectories under terminal position constraints. [1,2,3] For the pitch channel, APG is expressed as

$$a_{com} = NV(\lambda_{ref} - \lambda) \tag{1}$$

where a_{com}, N, V, λ_{ref}, and λ represent the normal acceleration command, guidance gain, missile speed, reference trajectory's lead angle, and missile's lead angle, respectively. The lead angle is defined as $\lambda = \gamma - \sigma$ where γ is the flight path angle and σ is the line-of-sight angle from the missile to the target position, as illustrated in Figure 1.

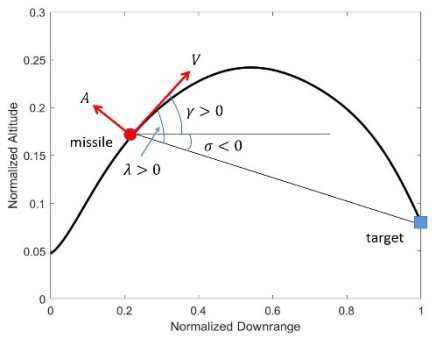


Figure 1: Definitions of symbols

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In this work, the reference trajectory is defined as the trajectory to be tracked. The altitude of the reference trajectory can be specified by a polynomial function of the downrange. Or a data table of [downrange, altitude, flight path angle] is required. The pseudo-spectral method, widely used for the parameterization of the trajectory optimization problem, is based on the Lagrange polynomial collocation. Hence, APG can be directly applied to the tracking of the optimal trajectory.

2. Tracking errors of APG

To simplify the analysis, APG is revised as follows:

$$a_{com} = N\ V_m(\lambda_{ref} - \lambda) + V_m\dot{\sigma}\ + g_{ref} \tag{2}$$

where $\dot{\sigma}$ is the line-of-sight of the target relative to the missile and g_{com} is the gravity compnsation term. Note that the second term of the right-hand side of (2) is PN guidance of unity gain. We also assume that the autopilot time lag is small enough to be neglected. Then,

$$\dot{\gamma}\ = \frac{a_{com} - g\ cos\ \gamma}{V_m} = N\ (\lambda_{ref} - \lambda) + \dot{\sigma} + \frac{g_{com} - g\ cos\ \gamma}{V_m} \tag{3}$$

By definition, $\lambda = \gamma - \sigma$. Hence, if the gravity compensation is chosen as $g_{com} = g\ cos\ \gamma$, (3) is simplified as

$$\dot{\lambda}\ = N\ (\lambda_{ref} - \lambda) \tag{4}$$

Since (4) is a simple first-order dynamics with a time constant of $1/N$, we observe that λ tracks λ_{ref} with a time lag of $1/N$.

Now we need to show that the true trajectory successfully follows the reference trajectory if $\lambda \approx \lambda_{ref}$. Suppose that $\lambda = \lambda_{ref}$ but the true trajectory is lower than the reference trajectory by $\Delta h;\ h_{ref} - h = \Delta h$. Then we see that

$$\sigma_{ref} - \sigma \approx (h_T - h_{ref})/R - (h_T - h)/R = -\Delta h/R \tag{5}$$

where h_T is the altitude of the target and R is the range to go. We also observe that

$$\frac{d(\Delta h)}{dt} = V(\sin\gamma_{ref} - \sin\gamma) \approx V\cos\gamma_{ref}(\gamma_{ref} - \gamma) \tag{6}$$

By definition of λ, we see $\gamma_{ref} - \gamma = \sigma_{ref} - \sigma$ when $\lambda = \lambda_{ref}$. Hence, from (5) and (6) we obtain a differential equation of Δh written as

$$\frac{d(\Delta h)}{dt} = -\frac{V\cos\gamma_{ref}}{R}\Delta h \tag{7}$$

This equation shows that Δh always converges to 0 when $\lambda = \lambda_{ref}$. If $\Delta\lambda\ = \lambda_{ref} - \lambda \neq 0$ but finite, then

$$\gamma_{ref} - \gamma = \sigma_{ref} - \sigma\ + \Delta\lambda \tag{8}$$

and (7) should be modified as

$$\frac{d(\Delta h)}{dt} = \frac{V cos\ \gamma_{ref}}{R}(-\Delta h\ + R\ \Delta\lambda\ ) \tag{9}$$

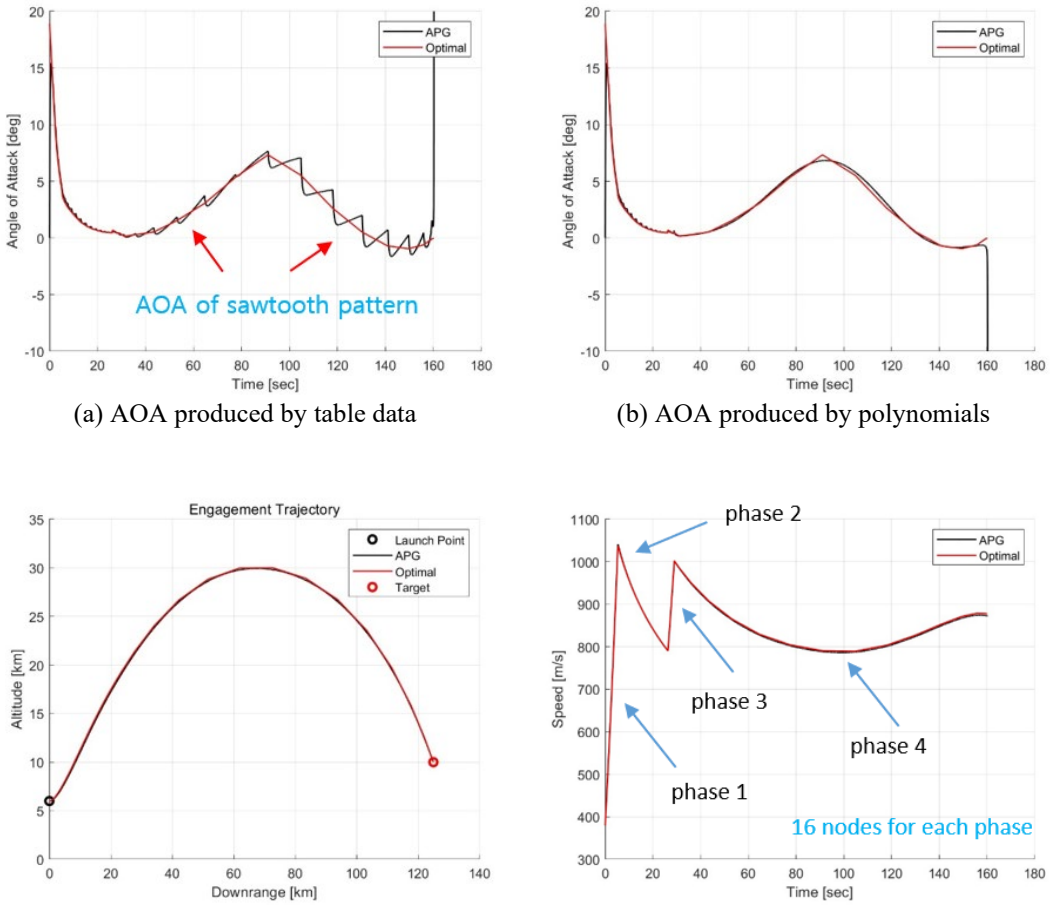
From (9), we observe that the trajectory tracking error Δh tracks $R\ \Delta\lambda$ if $\Delta\lambda$ is constant. Since R goes to 0 as the missile approaches to the target, (9) implies that Δh will not diverge but decreases to 0 even if $\Delta\lambda \neq 0$.

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3. Polynomial approximation of the reference trajectory

In this study, the optimal trajectory is assumed to be obtained by using the pseudo-spectral method. In such case, the optimal trajectory is typically expressed as a high-order polynomial in either time or downrange. This study adopts downrange as the independent variable. The rationale behind this choice is that discrepancies in the time profile of speed, caused by model errors, can result in a trajectory that deviates from the optimal path, resulting in a failure to reach the target at the terminal time even if the time-altitude profile is followed precisely. In contrast, using the downrange-altitude profile of the optimal trajectory enables the generation of a trajectory that remains close to the optimal trajectory and arrives at the target precisely even when the aerodynamic and propulsion models are different from the true ones.

The reference trajectory considered here is the optimal trajectory of an air-to-air missile equipped with a dual-pulse rocket. The target position is (125, 10) km and the missile is launched horizontally from (0, 6) km with a speed of 380 m/s. The optimization problem, solved by the pseudo-spectral method, aims to maximize the terminal speed. The resulting optimal terminal speed and flight time are 877.5 m/s and 160.18 s, respectively. The optimal solutions of the states and control input (angle of attack) are given at 61 nodes distributed over four phases. Phase 1 is the interval of the first pulse burn, Phase 2 is a brief unpowered flight, Phase 3 is for the second pulse, and Phase 4 is the controlled glide flight to reach the target. In the pseudo-spectral formulation, each phase is discretized into 15 intervals, leading to a total of 61 collocation nodes across the complete midcourse phase. Since each phase exhibits dynamic characteristics that differ from those of adjacent phases, a single-phase formulation is not adequate. The performance index of the optimal trajectory is to maximize the terminal speed.



(a) AOA produced by table data

(b) AOA produced by polynomials

(c) Altitude

(d) Speed

Figure 2: Comparison of APG and the optimal trajectory

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The optimal trajectory is calculated by using the pseudo-spectral method and a commercial SOCP(second-order cone programming) solver. Figure 2 (a) and (b) show the histories of the angle of attack produced by APG of (1). In Figure 2 (a), which is produced by using the table data for all phases, a sawtooth pattern of the angle of attack is observed. This phenomenon occurs because the lead angle of the reference trajectory generated by the table data is not smooth; it is continuous over the entire interval but its derivative is not continuous at the nodes. Since the sawtooth pattern is significant when the node spacing is large, the table data of phase 4 is replaced by a 6th-order approximation polynomial to obtain a smooth angle-of-attack response of Figure 2 (b). The effect of the sawtooth pattern of the angle of attack is not noticeable in the altitude and speed histories of Figures 2 (c) and (d). However, the zig-zag pattern of the angle of attack may not be desirable for obtaining smooth autopilot responses.

Let ξ and h denote the downrange to go and altitude of the missile normalized by the initial range of the target, X_T;

$$\xi = 1 - X/X_T, \qquad \eta = H/X_T \tag{10}$$

Then, the approximation polynomial $\eta(\xi)$ of order n can be found from the normalized table data $[X_k, H_k], k = 0, \cdots m$, by the least-square method. First, let $\eta(\xi) = a_0 + a_1\xi + a_2\xi^2 + \cdots + a_n\xi^n$ and $p = [a_1 \;\; a_2 \cdots a_n]^T$. Note $a_0 = H_T/X_T$. Then, construct m simultaneous equations given as

$$[\, \xi_k \quad \xi_k^2 \quad \xi_k^3 \cdots \quad \xi_k^n \,]\; p \;= \eta_k - a_0\,, \qquad k = 0, \cdots (m-1) \tag{11}$$

which can be written as a matrix equation $A\,p \;=\; B$ where A is a $m \times n$ matrix and B is a $m \times 1$ vector. Normally, we choose n to be smaller than m so that the least-square solution p_{LS} is calculated as

$$p_{LS} = (A^T A)^{-1} A^T B \tag{12}$$

For Phase 4 of the numerical example of this study, we have $m = 16$ and $n = 6$. Note that the line-of-sight angle of the target and the flight path angle of the reference trajectory are simply calculated from

$$\tan\sigma = (\eta_T - \eta)\,/\,\xi\,, \qquad \tan\gamma \;= -\,\mathrm{d}\eta/d\xi \tag{13}$$

to generate the lead-angle command $\lambda_{ref} = \gamma_{ref} - \sigma_{ref}$.

4. Simulation study

For the numerical experiments, a typical air-to-air missile model with a maximum range of 125 km is employed. We are concerned with a three-dimensional engagement scenario but only the longitudinal motion in the vertical plane is considered to simplify the presentation of the simulation results. The APG algorithm for the lateral guidance is simpler than the longitudinal guidance since the gravity compensation is not necessary. The logitudinal equations of motion are given as follows:

$$\dot{X} = V\cos\gamma \tag{14}$$

$$\dot{Z} = -V\sin\gamma \tag{15}$$

$$\dot{V} = \frac{T\cos\alpha - D}{m} - g\sin\gamma \tag{16}$$

$$\dot{\gamma} = \frac{T\sin\alpha + L}{m\,V} - \frac{g\sin\gamma}{V} \tag{17}$$

$$\dot{m} = -\frac{T}{I_{sp}\,g} \tag{18}$$

where (X, Z) are the position coordinates of the missile in the reference NED frame, T is the thrust, D is the drag force, L is the lift force, m is the mass, and I_{sp} is the specific impulse of the rocket. The X-axis of the reference system is horizontal and points in the direction of the target and the Z-axis points downwards. For trajectory optimization, the control variable is α bounded by 20 degrees. The drag and lift models are typical ones for long-range air-to-air missiles. A simple model is also included for the induce drag, which is the additional drag produced by the lift. The propulsion

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system is a dual-pulse solid-propellant rocket motor. The thrust, burn time, and specific impulse of the first pulse are 23120 N, 5.33 s, and 244 s, respectively, and those of the second pulse are 11493 N, 2.67 s, and 237 s. In this example, the target of the midcourse guidance is the prediced intercept point against an aerial intruder. The initial positions of the missile and the target are (0, 6) km and (125, 10) km, respecively. The missile is fired horizontally with a speed of Mach 1.2, and the objective of the trajectory optimization is to maximize the terminal speed. The optimal duration of Phase 2, which is the unpowered phase between the first pulse and the second pulse, is determined as 21.0 sec by trajectory optimization for this scenario.

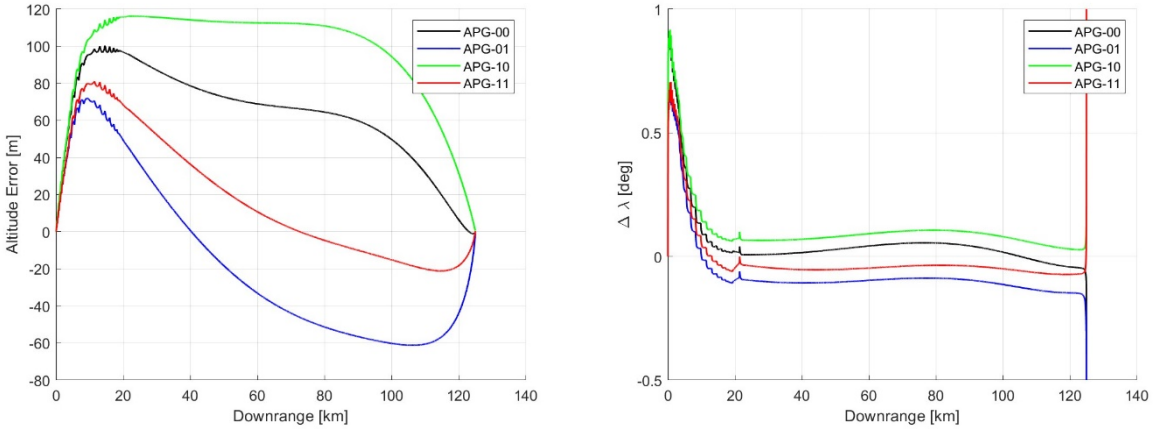
Four variants of APG listed in Table 1 are compared in this simulation study. The terminal state values of these four APG algorithms are compared with the optimal values as shown in Table 2. Note that there is no significant differences in the terminal values among the four APG algorithms. It is also observed that the terminal altitude and speed errors are very small for all APG variants. However, the histories of the altitude errors and lead angle errors are quite different to each other as shown in Figure 3. The missile's initial trajectory is lower than the optimal trajectory since the optimal trajectory requires a quick pull-up manuever but the missile is launched horizontally. Note that Δh is defined as $\Delta h = h_{ref} - h$ so that Δh increases quickly in Phase 1, as shown in Figure 3 (a). In terms of tracking accuracy, APG-11 outperforms other variants as observed in this figure. The second best is APG-00, which provides good performance in tracking the lead angle history of the optimal trajectory as shown in Figure 3 (b). Figure 4 illustrates the error histories of the flight path angle and the LOS angle. These angle errors increase as the missile approaches to the target but their magnitudes are limited to the order of 1 deg.

Table 1: APG algorithms tested in this study

APG-00	$a_{com} = N\ V_m(\lambda_{ref} - \lambda)$	equation (1)
APG-01	$a_{com} = N\ V_m(\lambda_{ref} - \lambda) + g_{ref}$	
APG-10	$a_{com} = N\ V_m(\lambda_{ref} - \lambda) + V_m\dot{\sigma}$	
APG-11	$a_{com} = N\ V_m(\lambda_{ref} - \lambda) + V_m\dot{\sigma} + g_{ref}$	equation (2)

Table 2: Terminal State Errors of APG algorithms

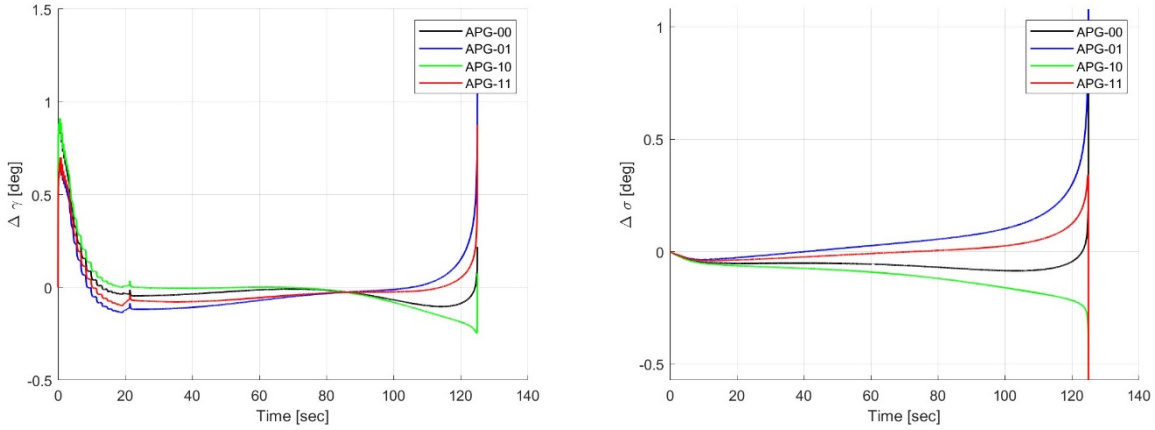
Terminal States	Optimal	APG-00	APG-01	APG-10	APG-11
Time (s)	160.320	160.290	160.403	160.204	160.314
Altitude (km)	10.0000	10.0010	10.0015	10.0002	10.0002
Speed (m/s)	877.54	875.68	876.08	874.34	874.97
Flight Path Angle (deg)	-42.16	-42.64	-42.50	-42.49	-43.29



(a) Altitude errors (b) Lead angle errors

Figure 3: Altitude and lead angle errors of APG algorithms

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(a) Path angle errors (b) LOS angle errors

Figure 4: Path angle and LOS angle errors of APG algorithms

5. Conclusions

This paper presents a novel trajectory-following method for long-range air-to-air missiles based on Augmented Pursuit Guidance (APG). The tracking accuracy characteristics were analyzed to evaluate the performance of APG. Additionally, realistic simulations were performed to demonstrate its ability to follow the optimal trajectory with high precision and minimal loss of terminal speed. The proposed method can be applied to the tracking of any feasible midcourse trajectory, allowing for easy adaptation to changes in the predicted intercept point by utilizing the normalized downrange. This feature is particularly valuable in situations where in-flight updates of the optimal trajectory are not possible. The extension of APG to three-dimensional engagements is straightforward, as the lateral APG algorithm mirrors the longitudinal one, except for gravity compensation. Future work will explore the application of APG to other long-range guidance problems.

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