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Modeling and validation of propulsion system for small battery powered electric aircraft

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Abstract

Many rules and disciplines need to be taken into account for the preliminary design of the small battery powered electric aircraft, such as mass and power characteristics of the motor, aerodynamic characteristics of the propeller and parameters of the aircraft itself. In this paper, the model of propulsion system for battery powered aircraft is developed. Comparison with the experimental data has proved the model to be reliable. Then the propulsion model is used for range and endurance estimation to evaluate the performance of the designed aircraft and the impact of battery capacity is investigated.

1 Introduction

Small electric unmanned aerial vehicles (UAVs) have been increasingly used in both civilization and military applications such as surveillance and aerial photo. The accurate mathematical model of propulsion system including motor, electric speed controller (ESC), propeller and battery is generally hard to obtain due to the nonlinearity of motor and ESC and the complex aerodynamic property of propeller. However, the simple model developed in this paper fits well with the experimental results which can provide simulation data for the evaluation of aircraft performance under a range of flight conditions.

In the current investigation, the battery, motor and propeller are modeled individually while the impact of ESC is ignored. The battery model is obtained from the manufacturer's discharge curve and validated with the experimental data. The model of motor-propeller system is described with several classical equations and its simulation results is compared with the static thrust experiments, which has validated the model's effectiveness.

2 Theoretical model

2.1 Battery model

The battery model used in this paper is described by a controlled voltage source E in series with an internal resistance R as it's shown in figure 1, and the model's parameters can be extracted from the battery manufacturer's discharge curve [1], which can provide data for the model's validation as well.

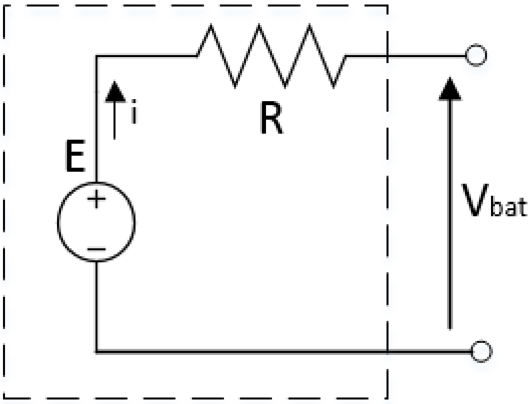

Figure 1: Battery model

The controlled voltage source is described as the function of current consumption:

$$\mathrm{E} = \mathrm{E}_0 - K\frac{Q}{Q - \int idt} + \mathrm{A}\exp(-\mathrm{B}\cdot\int idt) \tag{1}$$

Then the battery voltage can be expressed as:

$$\mathrm{V}_{\mathrm{bat}} = \mathrm{E} - R\cdot i \tag{2}$$

Where E is the controlled voltage source (V), $\int idt$ represents the actual battery consumption (Ah), E_0 is the battery constant voltage (V), Q is the battery capacity (Ah), R is the battery internal resistance(Ω), K, A and B are constant parameters that can be obtained from the battery discharge curve provided by the manufacturers.

To use the proposed battery model, it is assumed that the internal resistance and battery temperature are constant and the Peukert effect [2] is ignored which means the battery capacity doesn't change with the discharge currents.

In the current research, Dualsky Xpower ECO-S series lithium polymer batteries [3] are studied and the lines in figure 2 and figure 3 show the manufacturer's experimental discharge curves at 10C and 20C for the capacity of 2200mAh and 2700mAh.

In the reference [1], the discharge curve of battery is separated into several sections and the model's parameters (E_0, K, A and B) are extracted from the curve directly. The approach is not quite applicable for the lithium polymer battery as the battery voltage declines rapidly with the discharge time.

According to the equation (1) and (2), the battery voltage is the function of discharge time when the discharge current is already known. A nonlinear curve fitting is carried out based on equation (1) and (2) to fit the 10C discharge curve and the fitted voltage is drawn with solid lines in both figure 2 and figure 3. This method is simple but practical and the model's parameters are obtained as presented in the following table.

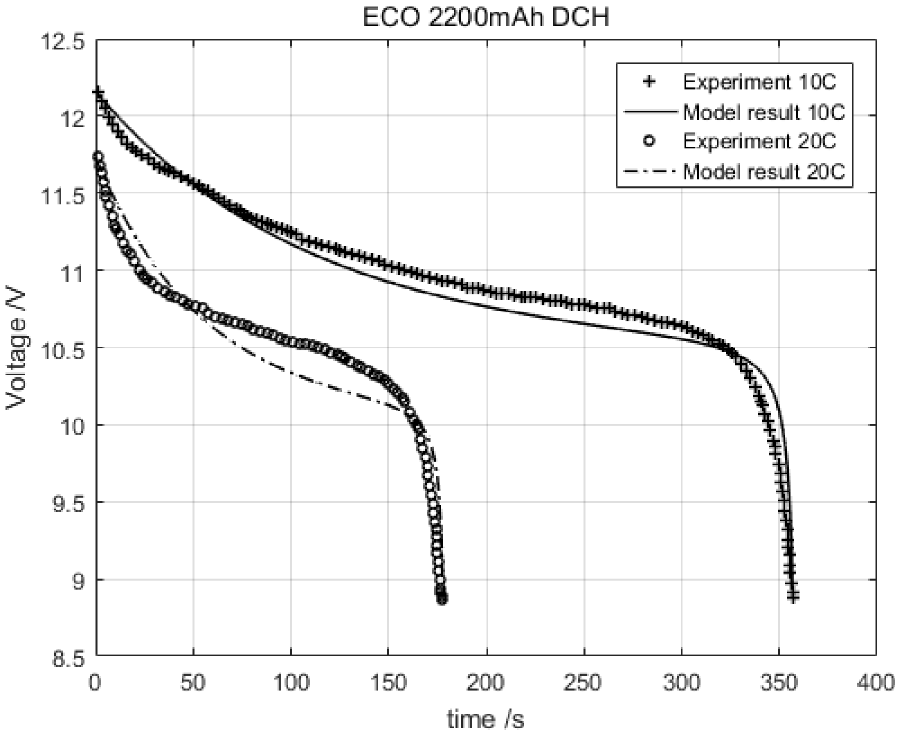

Figure 2 Battery discharge curve of ECO-S 2200mAh

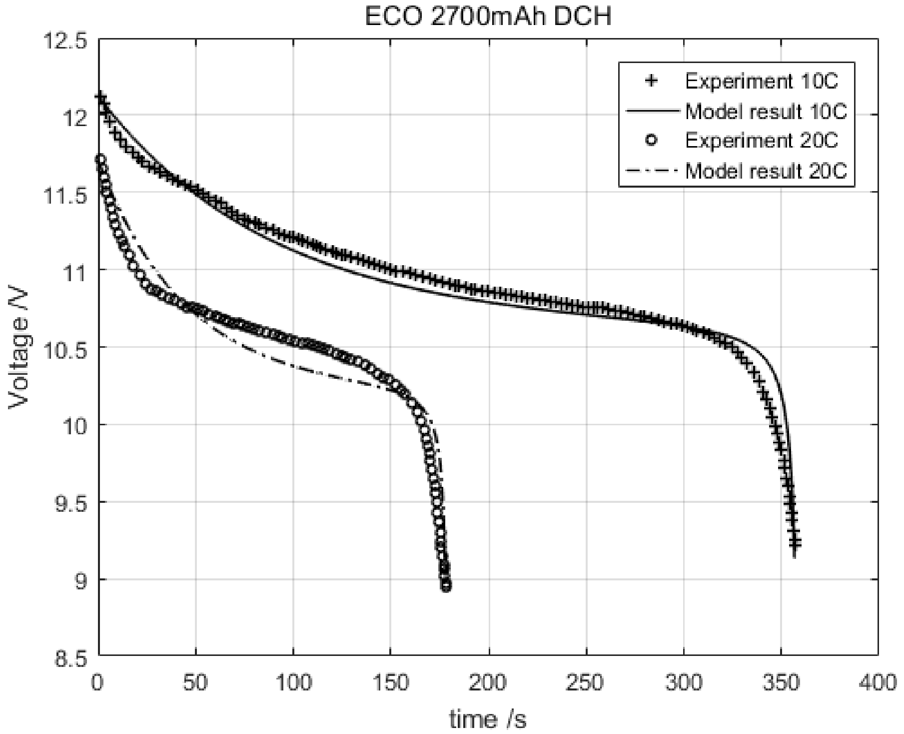

Figure 3 Battery discharge curve of ECO-S 2700mAh

Table 1 : Battery model parameters

Battery type	Q (Ah)	R (Ω)	E_0 (V)	A (V)	B (Ah^{-1})	K (V)
ECO-S 2200mAh	2.2	0.0195	10.963	1.640	1.50	0.0144
ECO-S 2700mAh	2.7	0.0153	11.078	1.473	1.50	0.0140

The validation of the proposed model is conducted by comparing the experimental 20C discharge curve with the model's results at the same discharge current. As it's shown in figure 2 and 3, the model results drawn with dash-dotted lines are in good agreement with the experimental data, which demonstrates that the model's results can represent the behavior of the battery correctly. Then the voltage of the battery can be calculated in any currents consumption by the proposed model.

2.2 Motor model

The traditional brushless direct current motor (BLDC) model is generally described with classical motor constants as shown in figure 4. There are three crucial parameters for each motor: the speed constant $\mathrm{K_v}$ (rpm/V), the no-load current $\mathrm{I_0}$ (A), and the motor internal resistance $\mathrm{R_m}(\Omega)$, which can be got from the motor manufacturers. The parameters of Sunny Sky BLDC motor studied in this paper are given in table 2 and the test data with different propellers can be found in the website [4].

Table 2: Motor constants

Motor type	K_v (rpm/V)	Weight (g)	$R_m(\Omega)$	I_0(A)	Outside Diameter(mm)	Length (mm)	Maximum Amps (A)
SunnySky X2814	1000	110	0.091	1.7	35	34.3	30
SunnySky X2216	1390	72	0.125	1.6	27.8	34.2	25

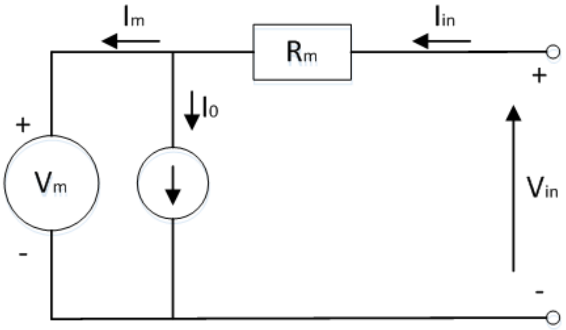

Figure 4 : BLDC Motor model

The power losses [5] due to the current required to overcome mechanical friction, magnetic hysteresis and eddy current losses during the motor running are characterized by the no-load current and internal resistance. Based on figure 4, two fundamental equations are derived to calculate the output power and rpm (revolutions per minute) of the BLDC motor:

$$P_{out} = I_m \cdot V_m = (I_{in} - I_0)(V_{in} - I_{in}R_m) \tag{3}$$

$$n = K_v \cdot V_m = K_v(V_{in} - I_{in}R_m) \tag{4}$$

And the motor efficiency is:

$$\eta = P_{out}/P_{in} = (I_{in} - I_0)(V_{in} - I_{in}R_m)/I_{in}V_{in} \tag{5}$$

According to the equations, for a determined input voltage there exists a maximum efficiency with the specific current value, which is decided by the motor load (the torque needed to run the propeller).

Generally, the motor resistance R_m is considered as a constant independent of motor voltage. However, the experimental data [4] indicates that R_m is approximately linear to the motor input voltage V_{in}. To make it more realistic, the motor model used in the following research takes R_m as a linear function of input voltage, which can be expressed as:

$$R_m = m_a + m_b \cdot V_{in} \tag{6}$$

Where the function parameters are shown in table 3

Table 3: Parameters for function of motor resistance

Motor type	m_a (Ω)	m_b (Ω/V)
SunnySky X2814	0.0050	0.0516
SunnySky X2216	0.0066	0.0649

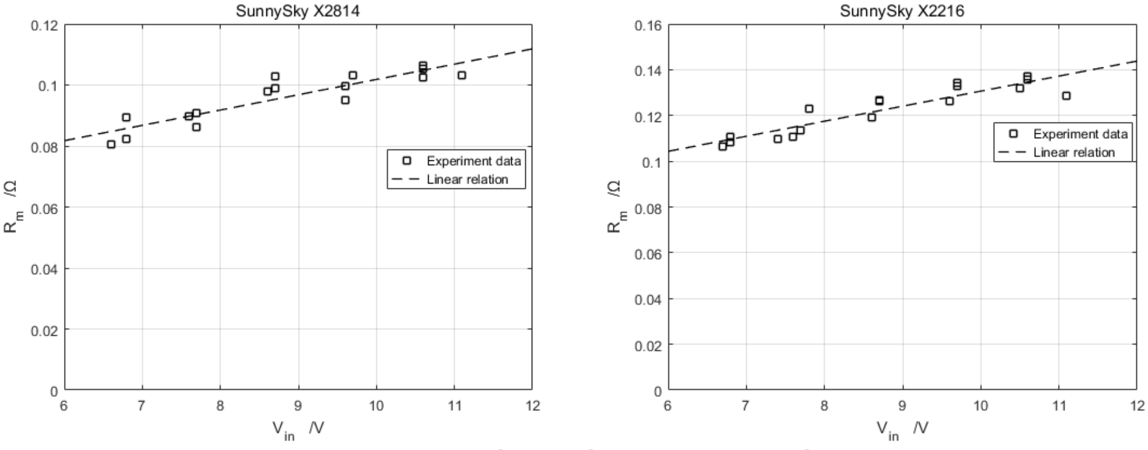

Figure 5: Input voltage and motor resistance relations

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Equations (3) to (6) indicate that motor output power P_{out} and rpm are determined by the input voltage and operating current. When the propeller is directly driven by the motor, the power absorbed by propeller should be equal to the motor output power P_{out} and they share the same rotational speed as well. In the practical flight, the thrust required to drive the aircraft forward is based on the flight conditions and it determines the rotational speed of the propeller. With the power needed to drive the propeller, input voltage and operating current can be calculated according equations (3) to (6). The motor model will be validated with the propeller model by comparing with thrust test results, which will be discussed later.

2.3 Propeller model

The propeller model is based on the wind tunnel measurements in the UIUC Propeller Database [6, 7]. The following equations give the relations between thrust T and the coefficient of thrust C_{T}, power P and the coefficient of power C_{P}.

$$T = C_T \rho n^2 D^4 \tag{7}$$

$$P = C_P \rho n^3 D^5 \tag{8}$$

Where $\rho(\mathrm{kg/m^3})$ is the density of air, n(revolutions per sec) is the rotational speed and D(m) is the diameter of propeller.

Figure 6 shows the ‘GWS DD 11x7’ propeller test results in different advance ratio J with four RPMs. The advance ratio J is calculated as

$$J = \frac{v}{nD} \tag{9}$$

Where v is the air speed vertical to the propeller disk. The advance ratio represents the relative value between the air speed and the propeller tip speed.

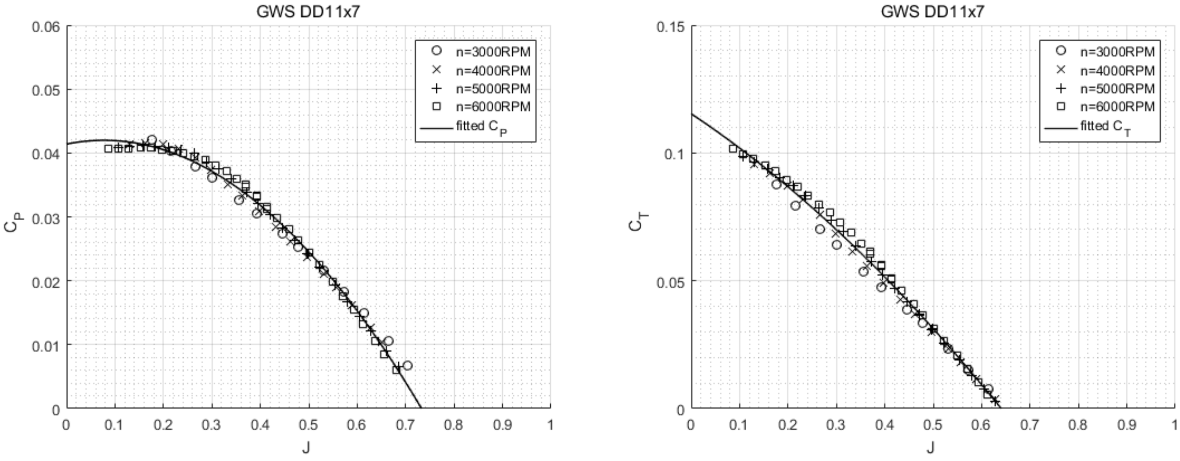

Figure 6 GWS DD11x7 propeller test resutls

As it’s shown in figure 6, different RPMS have little influence on the coefficient of thrust or power when the advance ratios are close, which suggests that the coefficients can be considered as the functions of advance ratio. Therefore, the experimental data is fitted according to the following polynomial equations and drawn in a solid line. The fitting parameters of propellers in different size are given in table 4.

$$Cp = p_1 \cdot J^2 + p_2 \cdot J + p_3 \tag{10}$$

$$C_T = t_1 \cdot J^2 + t_2 \cdot J + t_3 \tag{11}$$

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Table 4: Propeller model parameters

Propeller name	p_1	p_2	p_3	t_1	t_2	t_3
GWS DD 11x7	-0.0977	0.0151	0.0414	-0.0848	-0.1258	0.1153
GWS DD 10x6	-0.1035	0.0239	0.0361	-0.0941	-0.1085	0.1047
GWS DD 9x5	-0.0579	-0.0101	0.0419	-0.0455	-0.1478	0.1094

With the equations of C_T andC_P , thrust and power of the propeller can be calculated in any air speed and revolutions, with makes it possible to simulate the thrust required by the aircraft and the battery consumption in different flight conditions.

2.4 Comparison with experimental results

In order to validate the motor and propeller model, simulation results are compared with thrust test data [4]. Two different motors are tested with GWS propellers in different size. The parameters of motors and propellers are given in table 2 and table 4.

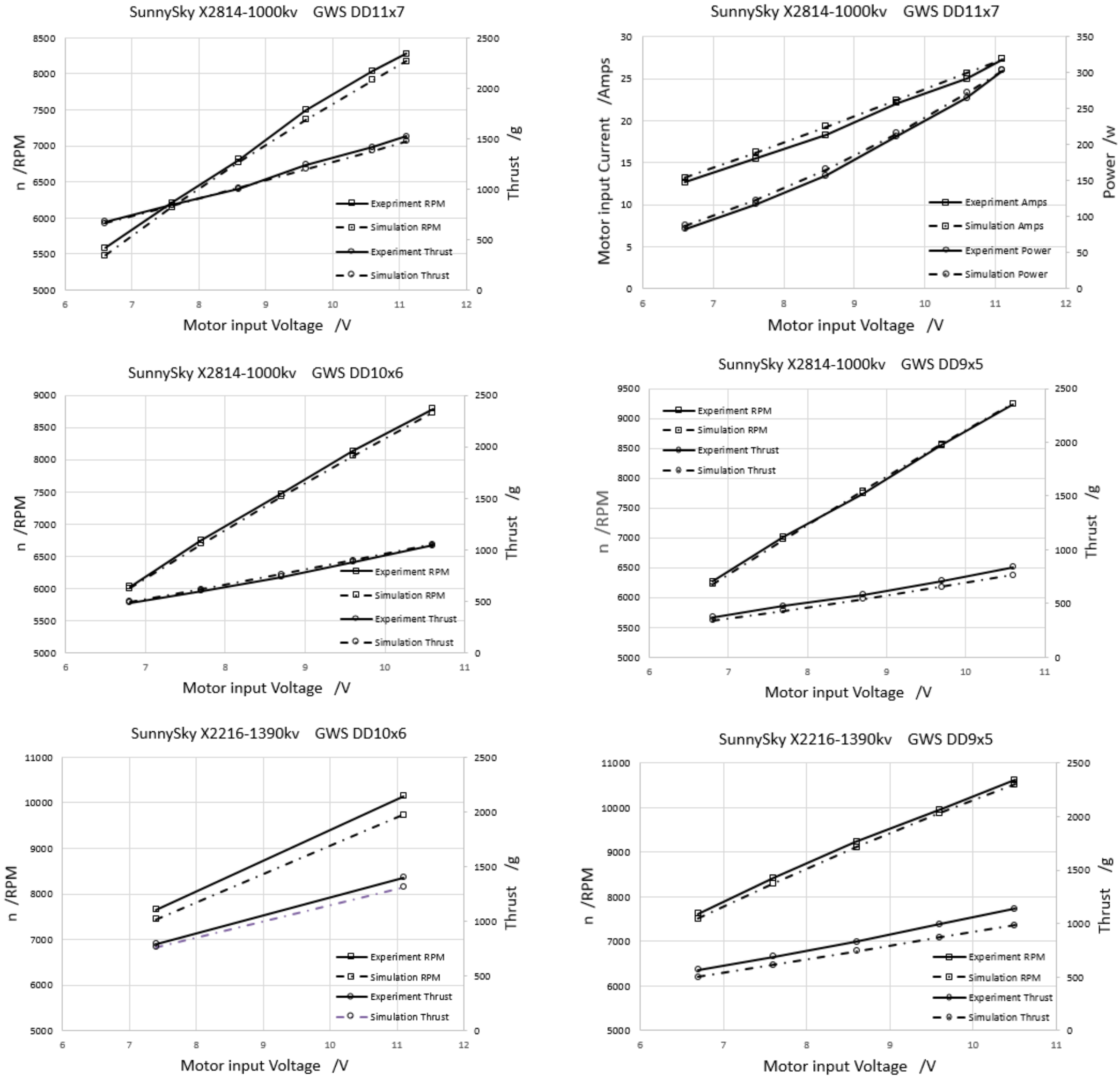

Figure 7 Simulation and experiments comparison

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The proposed motor model shows good agreement with the experimental results in power consumption as well as rotational speed, as it's shown in the first two charts of figure 7. Although the propeller model is obtained from wind tunnel experimental data and the polynomial equations of C_T and C_P are fitted with data in which the air speed is far above zero, the comparison shows it can be used for static thrust estimation. In most cases, the simulation results are slightly smaller than the experiments. One of the possible reasons is that the simulated C_T and C_P according to equations (10) and (11) are greater than the real values when the advance ratio is equal to zero, which leads the rotational speed relatively small values according equations (7) and (8). However, this error can be ignored during the estimation of range and endurance as the advance ratio is much more than zero in most conditions.

3 Range and endurance estimation

The individual models including battery, motor and propeller are combined as a propulsion system model for the electric aircraft which is able to simulate the aircraft flight mission and help to analyze the aircraft performance in the preliminary design.

3.1 Aircraft model

The small electric UAV designed in this paper is aimed to carry 1 kg payload while cruising below 500m. The mass characteristics is given in table 5, where the W_{eng} is the weight of engine system including motor, propeller, ESC and its cables, the W_{bat} is the weight of battery and $W_{payload}$ is the payload carried by the aircraft as well as other onboard devices such as flight controller and servos.

Table 5: Mass distribution of the aircraft

W_{eng} (kg)	W_{bat} (kg)	$W_{payload}$ (kg)
0.192	0.176	1.21

$$\mathrm{W_{to}} = \frac{W_{eng}+W_{bat}+W_{payload}}{1-Wf_{str}} \tag{12}$$

The total weight of the aircraft $\mathrm{W_{to}}$ is expressed as the equation (12) and Wf_{str} is the coefficient of structural weight. The empirical value of Wf_{str} is usually between 0.25 and 0.35 [8] and $Wf_{str} = 0.3$ is taken for the aircraft.
When the aircraft is in level flight, the thrust required to overcome the drag is produced by the motor-propeller drive system. Sunny Sky X2216 motor and GWS DD 10x6 propeller are used for the range and endurance estimation. The basic aerodynamic parameters are specified in table 6, where AR is the aspect ratio and S_w is the wing area. The coefficient of drag is calculated according to the equations (13) to (15) [9].

Table 6 Aerodynamic parameters

AR	Wing Span (m)	S_w (m^2)	S_{fuse} (m^2)	S_{tail}(m^2)	C_{fe}
8	1.55	0.3	0.3116	0.0415	0.0055

$$\mathrm{C_d} = C_{d0} + \frac{C_l^2}{\pi e AR} \tag{13}$$

$$C_{d0} = C_{fe} \cdot S_{wetted}/S_w \tag{14}$$

$$S_{wetted} = S_{fuse} + 2.1S_w + 2S_{tail} \tag{15}$$

The Oswald factor e is calculated by Shevell method [10] and $C_{fe}$is the friction coefficient of the aircraft skin, S_{fuse} and S_{tail} is the surface area of the fuselage and tail. Drag and lift can be obtained with the following equations [11].

$$D = 0.5\rho U^2 S_w C_d \tag{16}$$

$$L = W = 0.5\rho U^2 S_w C_l \tag{17}$$

3.2 Battery Capacity impact on aircraft performance.

The battery capacity has significant influence on aircraft range and endurance as it's the only power source. Increase in capacity can raise the flight time and bring larger total weight at the same time. Figure 8 illustrates the estimation of range and endurance with a range of battery capacities and the curves indicate that the increasing capacities benefits both range and endurance. According to the paper [11], for a given aircraft, there should be optimal flight speed for maximum range and endurance and the figure below prove the point. The optimal flight speed increase with larger capacities as it requires higher speed to achieve the best lift coefficient. For the designed aircraft above, the optimal flight speed is around $14\mathrm{m/s}$ for the maximum range when carrying the battery of 5200mAh while it's $10\mathrm{m/s}$ for the maximum endurance nearly 68 minutes.

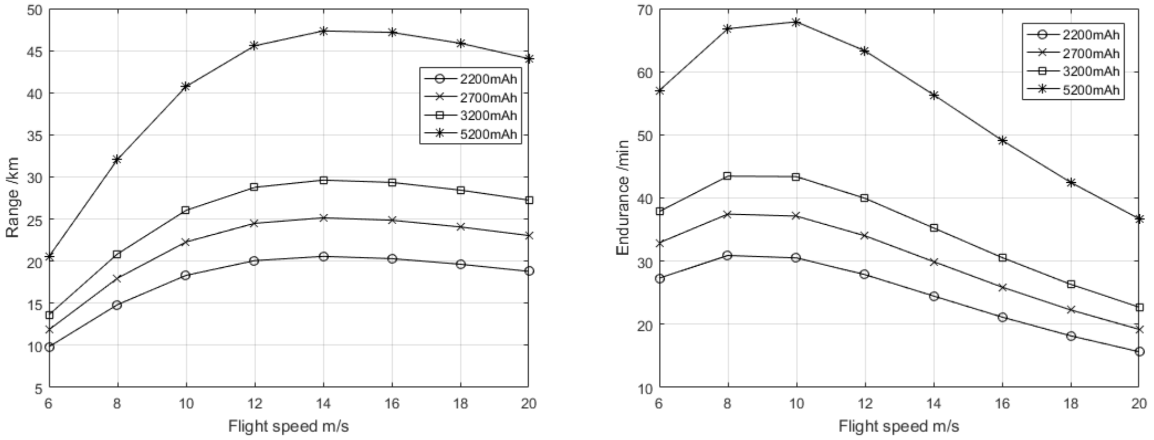

Figure 8 Range and endurance estimation for different battery capacity

4 Conclusion

The estimation of range and endurance is the basic evaluation of aircraft performance in the preliminary design. To obtain accurate estimation, the propulsion model including battery, motor and propeller is developed. Each component is modeled individually and validated with experimental data. The several sets of motor-propeller simulation results have proved the model's reliability. Then the propulsion model is used to estimate the range and endurance of the designed aircraft and impact of batter capacity is analyzed. The results show that the proposed model can be a good tool for the analysis of battery powered electric aircraft.

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