# Mathematical model for LES of evolution of far vortex wake behind an airplane

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## Abstract

Evolution and decay of far vortex wake behind airplane is modeled. The boundary problem for space filtered Navier-Stokes equations is described. It involves special setup for initial and boundary conditions. Two different cases are considered: freestream and inside the ground boundary layer. For freestream conditions the model is verified for high levels of background turbulence. In the ground layer the leeward tip vortex pops up and decays fast, while the windward hovers for long time with slow lateral movement.

## 1. Introduction

For the past time the problem of vortex wake behind airplane was a challenging in sense of understanding the physics and correct modeling. Interaction of wake vortices with turbulent atmosphere involves different types of interactions such as instabilities and non-linear decay. Some processes can be described solely with analytic expressions. The others require computations.

Large eddy simulation (LES) is a computational approach that provides good result for reasonable computational cost. It is not as accurate as DNS. But DNS requires sufficient resources, yielding order of Re<sup>2.25</sup> elements. The size of computational domain for vortex wake can extend by hundreds of meters, therefore the usage of DNS becomes unfeasible by the moment. To the contrary, the usage of RANS seems to be simple and cheap way. The specific turbulence models<sup>11</sup> had been already developed. However, in case of far vortex wake the first hand analysis show that time averaging blurs the vortex wake structure.

The process of vortex wake evolution and decay is complex. It can be characterized with intensity, scale and processes leading to wake decay. These features are primarily defined by leading aircraft properties and the state of background atmosphere. The interaction of wake vortices with background turbulence, along with vortex self induction, results in wake instability. There are different types of instabilities that can be observed in vortex wake.<sup>8</sup> Eventually, these instabilities lead to the formation of vortex rings and wake decay. Another sufficient mechanism is "circulation loss", which is intensive turbulent transfer of vorticity out from wake. All described mechanisms can be modeled with LES.

The LES approach is being used for vortex wake simulations for long time. One of the first works uses coarse meshes and high order accurate schemes for simulation.<sup>9</sup> With increasing computational resources available the solved tasks become more complex. Recent papers show full vortex wake modeling from its generation till full decay.<sup>7</sup> Finally, it becomes possible to move from qualitative analysis to quantitative. Consequently, an accurate task setup is required.

#### 2. Mathematical formulation of the problem

Considered is the full Navier-Stokes equation system for the space-filtered variables.<sup>14</sup> Non-stationary equation system for viscid compressible gas flow without heat generation can be represented as follows (vector sign will be omitted):

$$\frac{\partial U}{\partial t} + \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z} = 0$$
(1)

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The equation (1) is written in Cartesian coordinate system. The vector of conservative variables has following form:  $U = [\rho; \rho v_x; \rho v_y; \rho v_z; \rho e]^T$ . Flux vectors F can be written as a sum of convection and diffusion contributions:  $F_x = F_x^{conv} + F_x^{diff}$ ,  $F_y = F_y^{conv} + F_y^{diff}$ ,  $F_z = F_z^{conv} + F_z^{diff}$ . The vectors of convective fluxes along the *x*, *y* and *z* axes are written as follows:

$$\mathbf{F}_{x}^{\mathrm{conv}} = \begin{bmatrix} \rho v_{x} \\ \rho v_{x}^{2} + p \\ \rho v_{x} v_{y} \\ \rho v_{x} v_{z} \\ (\rho e + p) v_{x} \end{bmatrix}, \quad \mathbf{F}_{y}^{\mathrm{conv}} = \begin{bmatrix} \rho v_{y} \\ \rho v_{x} v_{y} \\ \rho v_{y} v_{y} \\ (\rho e + p) v_{y} \end{bmatrix}, \quad \mathbf{F}_{z}^{\mathrm{conv}} = \begin{bmatrix} \rho v_{z} \\ \rho v_{x} v_{z} \\ \rho v_{y} v_{z} \\ \rho w^{2} + p \\ (\rho e + p) v_{y} \end{bmatrix}.$$

The vectors of diffusive fluxes along x, y and z axes contain the second derivatives and have the following form:

$$F_{x}^{\text{diff}} = \begin{bmatrix} 0 \\ \tau_{xx} \\ \tau_{yx} \\ \tau_{zx} \\ \tau_{xx} v_{x} + \tau_{yx} v_{y} + \tau_{zx} v_{z} + \sigma_{x} \end{bmatrix}, F_{y}^{\text{diff}} = \begin{bmatrix} 0 \\ \tau_{yy} \\ \tau_{zy} \\ \tau_{xy} v_{x} + \tau_{yy} v_{y} + \tau_{zy} v_{z} + \sigma_{y} \end{bmatrix},$$
$$F_{z}^{\text{diff}} = \begin{bmatrix} 0 \\ \tau_{xz} \\ \tau_{yz} \\ \tau_{zz} \\ \tau_{zz} \\ \tau_{zz} \\ \tau_{zz} v_{x} + \tau_{yz} v_{y} + \tau_{zz} v_{z} + \sigma_{z} \end{bmatrix},$$

where

$$\begin{aligned} \tau_{xx} &= -\left(\mu + \mu_{SGS}\right) \left( 2\frac{\partial v_x}{\partial x} - \frac{2}{3} \operatorname{div} \mathbf{V} \right), \ \tau_{yy} &= -\left(\mu + \mu_{SGS}\right) \left( 2\frac{\partial v_y}{\partial y} - \frac{2}{3} \operatorname{div} \mathbf{V} \right), \\ \tau_{zz} &= -\left(\mu + \mu_{SGS}\right) \left( 2\frac{\partial v_z}{\partial z} - \frac{2}{3} \operatorname{div} \mathbf{V} \right), \ \tau_{xy} &= \tau_{yx} = -\left(\mu + \mu_{SGS}\right) \left( \frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x} \right), \\ \tau_{xz} &= \tau_{zx} = -\left(\mu + \mu_{SGS}\right) \left( \frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x} \right), \ \tau_{yz} &= \tau_{zy} = -\left(\mu + \mu_{SGS}\right) \left( \frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y} \right), \\ \operatorname{div} \mathbf{V} &= \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}. \end{aligned}$$

The closure equation for the system above is ideal gas relation  $p = \rho RT$ , total energy of gas unit is  $e = (v_x^2 + v_y^2 + v_z^2)/2 + RT/(\gamma - 1)$ , where  $R = 287.07 \text{ J} / (\text{kg} \times \text{K})$ ,  $\gamma = 1.4$ . The dynamic coefficient of molecular viscosity is determined with Sutherland's formula:<sup>12</sup>

$$\mu(T) = \mu(T_0) \times \left(\frac{T}{T_0}\right)^{3/2} \frac{T_0 + C}{T + C},$$

where C = 110 K,  $\mu(T_0) = 1,72 \times 10^{-5} \text{ kg/(m} \times \text{s})$  is air viscosity at  $T_0 = 273 \text{ K}$ . To determine the dynamic coefficient of turbulent (subgrid) viscosity, Smagorinsky model<sup>10</sup> is used:  $\mu_{SGS} = \rho(C_S \Delta)^2 [2S_{ij}S_{ij}]^{1/2}$  with  $S_{ij} = 0.5(\partial v_i/\partial x_j + \partial v_j/\partial x_i)$  for velocity deformation tensor,  $\Delta = [V_{\Omega}]^{1/3}$  ( $V_{\Omega}$  being a characteristic volume, e.g. mesh cell size) and  $C_S = 0.1$  for Smagorinsky constant.

A number of different boundary conditions is used. Some of them are standard and used both for solving the problems and for the formulation of special boundary conditions characteristic to the studied cases. A "periodic condition" for front boundaries should be noted as a special. It is used for a pair of opposite boundaries of the computational domain. In present case the boundaries are located along the main axes of a Cartesian coordinate system and represent planes. If we take a point  $\mathbf{x}_a$  in a plane x = a then it will match the point  $\mathbf{x}_b$  at the boundary of the section x = b. The periodic boundary condition is formulated so as all the elements of the vector  $\mathbf{Q} = [\rho; v_x; v_y; v_z; p]^T$  in point  $\mathbf{x}_b$  are assigned to the corresponding values in the vector at  $\mathbf{x}_a$ :  $\mathbf{Q}(\mathbf{x}_b) = \mathbf{Q}(\mathbf{b}, \mathbf{y}_0, \mathbf{z}_0) = \mathbf{Q}(\mathbf{a}, \mathbf{y}_0, \mathbf{z}_0) = \mathbf{Q}(\mathbf{x}_a)$ .

### 2.1 Calculation method

In the current study, a finite-volume numerical method of the second approximation order both in space and in time is used (ANSYS CFX 15.0). LES calculation is performed for incompressible fluid. Smagorinsky LES model with the constant  $C_S = 0.1$  is used for turbulence modeling. Unsteady computations are performed with dual time-stepping approach with maximum Courant number  $CFL_{max} = 2$ .

## 3. Model of vortex wake

The model of vortex wake is used to setup consistent initial conditions. It describes the velocity field of vortices in the wake at the distance of 7-8 wingspans behind an airplane (see Figure 1, a). The wake is characterized there as rolled up, consisting of a number of strong counter rotating vortices. The number of vortex pair varies depending on the flight regime (cruise, landing or take-off).



Figure 1: Schemes: a) Initial condition location. Color shows different rotation of vortices; b) Flow with 4 vortices

Considered in this paper is the wake consisting of 4 vortices that corresponds to approach or landing/take-off regime of flight (See Figure 1, b). Such topology is adopted after experimental study of Huenecke.<sup>6</sup> The pair of vortices above comes from the wing-tip. The lower pair of vortices with the rotation in counter direction results from vorticity generated by fuselage and stabilizer (*additional* vortices). The position of additional vortices  $(y_1, z_1)$  is determined from preliminary computations of aircraft vortex wakes.

The vortices in upper pair are described with Proktor's model.<sup>9</sup> This model provides velocity distribution in each vortex, the distance between them being calculated as for the wing with elliptic circulation distribution. The circumferential velocity  $V_{\tau}$  in a single vortex complies the following relations:

$$V_{\tau} = 1.4 \frac{\Gamma}{2\pi r} \left[ 1 - \exp\left(-10(r_c/l)^{0.75}\right) \right] \left[ 1 - \exp\left(-1.2527(r/r_c)^2\right) \right], \ r < r_c,$$
(2)

$$V_{\tau} = \frac{\Gamma}{2\pi r} \left[ 1 - \exp\left(-10(r_c/l)^{0.75}\right) \right], \ r > r_c, \tag{3}$$

$$\Gamma = \frac{4G}{\pi l \rho V_{\rm flt}}, \ b = \frac{\pi l}{4} \tag{4}$$

where b is the vortex span, l is the wingspan, G is the aircraft weight,  $\rho$  is air density,  $V_{\rm flt}$  is aircraft flight velocity,  $r_c$  is the radius of vortex core. In the paper<sup>9</sup> the value of  $r_c$  isn't determined. However, it is shown<sup>13</sup> that the core radius can be taken at the landing regime as  $r_c = 0.05l$ .

Velocity distribution in the lower pair of vortices is described with Lamb model:

$$v_{\tau} = \frac{\Gamma_1}{2\pi r} \left[ 1 - \exp\left(-1.25 \frac{r^2}{r_{c1}^2}\right) \right].$$
 (5)

The circulation of an additional vortex is taken to be  $\Gamma_1 = -0.15\Gamma$  and radii of vortex cores are taken as  $r_{c1} = 0.07l$ .

### 4. Models of turbulent atmosphere

The evolution and decay of vortex wakes is very dependent on background conditions. These conditions are defined by the state of turbulent atmosphere and vicinity of ground, the last being of key importance. Therefore, two different setups could be suggested: freestream and near-ground.

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#### LES FOR FAR VORTEX WAKE

The atmospheric background consists of two main components that are turbulent velocity fluctuations and mean wind. Velocity fluctuations form turbulence, taken as uniform and isotropic for freestream conditions. Turbulence properties are different in ground layer. However, the method for getting the field of wind gusts is the same for these two cases.

The field of background turbulence is achieved with a known algorithm. The components of velocity pulsations  $v'_i$  at a point with radius vector **r** are represented as a sum of harmonic oscillations:

$$v_{i}'(\mathbf{r}) = \sum_{n=1}^{N} \hat{v}_{i}'(\mathbf{k}_{n}) \cos\left(k_{nx}x/L_{x} + k_{ny}y/L_{y} + k_{nz}z/L_{z} + \varphi_{n}\right).$$
(6)

The random variable  $\hat{v}'_i$  for phase vector  $\mathbf{k}_n$  is generated in the way to have a preconditioned covariance matrix  $c_{ij}$ . The value  $\varphi_n$  is a phase, which is uniformly distributed random value in  $[0, 2\pi)$  range. The covariance matrix is expressed through correlation tensor of isotropic and uniform turbulence and the volume of phase space element:

$$c_{ij} = \langle \hat{v}'_i \hat{v}'_j \rangle = \frac{E(k)}{4\pi k^2} \left( \delta_{ij} - \frac{k_i k_j}{k^2} \right) \Delta^3 k$$

Karman energetic spectrum is used in the form suitable for modeling the atmospheric turbulence.<sup>5</sup> For unit level and scale of turbulence it has a following form:

$$E(k) = \frac{55}{27\pi} q^2 L_A \frac{(\alpha_k L_A k/(2\pi))^4}{\left[1 + (\alpha_k L_A k/(2\pi))^2\right]^{17/6}},\tag{7}$$

where  $\alpha \approx 1.339$ . The random vector  $\hat{v}'_i$  is achieved by multiplying a vector consisting of random variables (normally distributed with zero average and unit dispersion) by the result of the Choletsky decomposition for covariance matrix.<sup>1</sup>

The turbulence outside the ground layer (vortex wake in freestream conditions) is taken as uniform and isotropic. The scales  $L_x$ ,  $L_y$ ,  $L_z$ , as well as dispersion  $s_{xx}$ ,  $s_{yy}$ ,  $s_{zz}$ , are then taken equal for each direction:

$$s_{xx} = s_{yy} = s_{zz},$$

$$L_x = L_y = L_z = \begin{cases} y, & y < 760 \text{ m}; \\ 760, & y \ge 760 \text{ m}, \end{cases}$$

where *y* is a distance to ground.

The relations differ essentially in case of vortex wake within ground boundary layer. It is assumed that there is a wind that blows along the ground in the direction perpendicular to the flight trajectory. The state of atmosphere is assumed to be neutral. The correspondent velocity profile of boundary layer and anisotropic turbulence parameters are the following:<sup>2</sup>

$$v_z = 0.19 v_{10} \ln\left(\frac{y + y_0}{y_0}\right),\tag{8}$$

$$s_{xx} = s_{zz} = 4s_{yy}, \ s_{yy}^{1/2} = 0.09v_{10},$$
(9)

$$L_y = y, \ L_x = L_z = 200 \,\mathrm{m} \tag{10}$$

Here  $v_{10}$  is wind velocity module at the height y = 10 m,  $y_0 = 0.05$  m is the roughness of the underlying surface.

## 5. Vortex wake at approach stage

Considered is the vortex wake behind the airplane flying at approach stage. The domain for solving the vortex wake problem has the shape of rectangular box with dimensions of 0 < x < 700 m, 0 < y < 300 m, -100 < z < 100 m. The axes of wake vortices lay along the longest side of box. The chosen dimensions are determined by: 1) along x – by Crow wavelength;<sup>4</sup> 2) along y – by the distance that the vortex pair passes in vertical direction during its life; 3) along z – by vortex span. Periodical boundary condition is used at the fronts of computational domain (along x axis). For other boundaries a constant (in time) distribution of turbulent velocity fluctuations is utilized. Such boundary condition provides lower damping of atmospheric turbulence within then computational domain during the computation, compared to other "standard" boundary conditions.

Gas parameters are taken for air of constant density, having temperature 298 K and 1 atm pressure. Background turbulence is assumed to be isotropic and uniform. Three levels of background turbulence are considered:  $q_A =$ 

0.25, 0.5, 1.0 m/s with the scale  $L_A = 200$  m. These parameters provide boundary condition with frozen velocity field to be consistent. The validity is proved by estimate of turbulence characteristic time  $T = L_A/q_A = (200 \div 800)$  s, being greater than vortex wake lifetime  $T_L = (50 \div 100)$  s. The parameters that define vortex wake intensity and vortices location are taken for long-range heavy commercial aircraft.

The computation results were postprocessed and are shown on Figure 2. There are the fields of axial vorticity at the plane x = 500 m for  $q_A = 1$  m/s. Vorticity is shown by color, the color scale having been chosen to depict flow evolution. Initially there are two pairs of tip and additional vortices. Additional vortices undergo deformation and destruction in the field of strong tip vortices (t = 8, 16 s on the figure 2). Therefore the tip vortices appear to be surrounded by highly turbulent field with prevailing vorticity of opposite sign (t = 24 s). Under such conditions the tip vortices undergo fracturing that intensifies transfer of vorticity out from vortex wake. This process is called as "loss" of circulation.



Figure 2: Vorticity in the wake,  $q_A = 1$  m/s, section at x = 500 m

The intensity of vortex wake is measured in control circle 5-15 m surrounding the vortex in yz plane during the vortex lifetime. The resulting plots showed on figure 3 depict circulation decrease for cuts at x = 200 m and x = 500 m. The decrease pace is compared to the one predicted by TsAGI's vortex wake model.<sup>2</sup> The results of comparison show that for high levels of turbulence q = 0.5, 1.0 m/s the circulation decrease rate is in agreement with the model, whereas for low level of turbulence q = 0.5 m/s the differences are observed.

The same picture was obtained with vortex wake after aircraft of another type.<sup>3</sup> This means that for high levels of atmospheric turbulence the presented LES model can be suggested as quite reliable.

## 6. Vortex wake in ground boundary layer

The wake evolution behind the large aircraft in ground layer of turbulent atmosphere is solved in the region 700 < x < 0 m, 100 < y < 0 m, 300 < z < 100 m. The region sizes are determined as: 1) along x – by Crow wavelength;<sup>4</sup> 2) along y – by the height, where the vortex pair has been generated (h = 30 m); 3) along z – by side wind velocity ( $v_{10} = 2 \text{ m/s}$ ) and vortex span.

Within the ground layer the model with adiabatic stratification of atmosphere (neutral atmospheric state, section 4) is adopted. The equations are solved with assumption of incompressible fluid and Boussinesq approximation. On the basis of tests<sup>3</sup> meshing of the domain has been made with the cells of following size:  $\Delta y = \Delta z = 0.6 \text{ m } \Delta x = 1.2 \text{ m}$ . The resulting grid contains 110 million cells. The ratio of cells is equal to 2 so the mesh is consistent for LES.

At the fronts of computational domain along x axis, a periodical boundary condition is used. The boundary condition at the bottom of the domain is no-slip wall. The velocity distribution of wind in ground layer with turbulent fluctuations is assigned for remaining boundaries. The wind field with turbulent fluctuations, along with the velocity field of four vortices, is set up as initial condition.

The results of computation are shown as axial vorticity distribution on figure 4. The fields are taken for the section x = 500 m for different times: t = 0, 24, 48, 96 s. The side wind on the images is blowing along z direction, from left to the right. Initially two pairs of tip and additional vortices can be seen, as well as vorticity in the ground layer. The system of vortices is not able to move downwards infinitely and begins the lateral movement under screen effect and influence of side wind.

The interaction with ground boundary layer plays a key role in flow evolution. The right negative vortex produces additional downwash on the surface, therefore vorticity in the boundary layer increases. The right additional, positive, vortex interacts with this fat boundary layer and gets enforced forming a new vortex pair. The resulting secondary



Figure 3: Decrease of circulation in wake vortices, approach stage



Figure 4: Vorticity in the wake in ground layer, cut at x = 500 m

vortex pair eventually rises up to a height of  $h = 20 \div 40$  m and decays fast. The trajectories of vortices are shown at figure 5. After the pop-up this vortex system decays quickly.

The left positive tip vortex imposes the downwash on the surface that makes the boundary layer weaker. The additional left negative vortex, having the sign of vorticity opposite to the sign of the boundary layer, is destroyed once interacting with it. The resulting windward vortex system consists of one tip vortex and its imaginary counterpart below surface. This system would move to the left if no wind. However the wind is present and it blows to the right and, therefore, the vortex lateral movement is compensated.

On the contrary to the leeward vortex, the windward exists for very long time. This can be seen on figure 6, where



Figure 5: Tip vortex trajectories at cuts x = 200 and x = 500 m. The left vortex hovers, the right pops up

vortex circulations are plotted. However, that vortex is still subject to decay and two mechanisms for this process can be sorted out. The first is circulation "loss", due to the turbulent field produced by additional vortex after destruction in the ground layer. The other is long-wave instability in the pair of vortex and its image below the screen. Finally, it should be emphasized that, depending on the background conditions the left vortex can hover for long time at the same place. This vortex can be a serious hazard for the following aircraft.



Figure 6: Decrease of vortex circulations at cuts x = 200, x = 500 m, landing stage

## 7. Concluding remarks

As a result of the study a reliable LES model for simulation of the vortex wake destruction behind the aircraft at the approach and landing regimes have been built. The key features of the model are proper initial and boundary conditions. The model allows to simulate key processes of vortex wake and decay.

The two different scenarios of vortex wake destruction are described. At approach stage on case of high level of background turbulence the wake decay is the result of circulation "loss", which is splitting the vortices into fractions and consequent turbulent transfer of tip vorticity outside the vortex pair.

The tip vortices show different behavior in earth proximity. It is very much affected by the presence of a side wind and ground boundary layer. The leeward vortex "pops" to a height equal to wingspan and looses its structure and circulation. The windward vortex "hovers", decaying much slower. This vortex can be very dangerous for the following aircraft.

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