7TH EUROPEAN CONFERENCE FOR AERONAUTICS AND AEROSPACE SCIENCES (EUCASS)

Modeling of High Density Polyethylene Regression Rate in the Simulation of Hybrid Rocket Flowfields

D. Bianchi^{\star^{\dagger}}, G. Leccese^{\star} and F. Nasuti^{\star}

Sapienza University of Rome, Department of Mechanical and Aerospace Engineering Via Eudossiana 18, Rome, Italy 00184 daniele.bianchi@uniroma1.it · giuseppe.leccese@uniroma1.it · francesco.nasuti@uniroma1.it C. Carmicino *University of Naples "Federico II", Department of Industrial Engineering Piazzale Tecchio 80, Naples, Italy 80125 carmicin@unina.it †Corresponding author: daniele.bianchi@uniroma1.it

Abstract

Numerical simulations of the flowfield in a hybrid rocket engine are carried out with a Reynolds-averaged Navier-Stokes solver integrated with a customized gas-surface interaction wall boundary condition and loosely coupled with a radiation code based on the discrete transfer method. The fuel grain wall boundary condition is based on mass, species and energy conservation equations coupled with thermal radiation exchange and finite-rate Arrhenius kinetics for fuel pyrolysis modeling. Fuel pyrolysis is governed by the heat flux reaching the surface due to convection and radiation and by the energy required for the grain material to heat up and pyrolyze. Attention is focused here on a set of static firings performed with a lab-scale GOX/HDPE motor. A sensitivity analysis has been carried out on the literature pyrolysis models for HDPE, to evaluate the possible role of the uncertainty of such models on the actual prediction of regression rate. A reasonable agreement between measured and computed averaged regression rate and chamber pressure is obtained, with a noticeable improvement with respect to solutions without including radiative energy exchange.

1. Introduction

Hybrid Rocket Engines (HRE) are promising propulsion systems.^{1,29} Their performance is similar to that of storable or semi-cryo liquid rocket engines and they share appealing features of both solid rocket motors and liquid rocket engines. Moreover, they are cheaper and safer than liquid and solid rockets and they are more environmentally friendly than storable-liquid and solid rockets. Therefore, many research programs focus on HRE development for applications that range from earth-suborbital flights to space exploration. Despite the above-mentioned advantages, some issues hinder the success of HRE: low regression rates of commonly used polymeric fuels, reduced combustion efficiency, and uncertainty of regression-rate law and its scalability are the most challenging problems. Uncertainty in regression rate and its scalability remains one of the most challenging issue, since regression rate is a parameter that strongly drives the design process and heavily influences the performance of the entire propulsion system.

To date, research in developing hybrid propulsion system strongly depends on firing tests and experimental trials and errors, which are expensive and time-consuming. In particular, for what concerns the estimate of the fuel regression rate, empirical correlations based on linear interpolation in the logarithmic plane are mostly used as well as surface energy balances which include simplified boundary layer correlations based on bulk transfer coefficients to evaluate the convective heat flux to the solid fuel. Such approaches are useful during the preliminary design and analysis process, providing a faster and simpler estimation of the motor performance, but rely on a very simplified modeling of the complex physical and chemical phenomena involved. Therefore, they need to be accurately calibrated relying on the availability of existing experimental data specific for each analyzed motor. Consequently, such models are limited to providing qualitative understanding of the trends but are inadequate to provide the kind of quantitative data needed for motor design and optimization. Indeed the extension of such models to new motors which can be different in scale, geometry, etc. is hardly possible without the availability of experimental data for each motor. For this reason there is a renewed interest in the development of more advanced models^{4,7,11,15,16,19,21,28,30,36,39} based on

Computational Fluid Dynamics (CFD) and is capable of representing more accurately the physico/chemical phenomena involved. The numerical modeling of the combustion process and the fluid dynamics in the combustion chamber and nozzle of a hybrid propellant rocket is a challenging task as it involves strongly interacting multiphysics processes such as fluid dynamics, solid fuel pyrolysis,^{3,37} oxidizer atomization and vaporization, mixing and combustion in the gas phase,^{4,14,28,36} nozzle thermochemical erosion,^{8,11} particulate formation, and radiative characteristics of the flame. Commercial CFD tools are generally not optimized to this task, as they are typically less flexible for the treatment of fluid/solid boundary conditions which are prescribed as constant temperature or heat flux and zero mass transfer. To obtain a suitable tool for the analysis of hybrid rocket flowfield, CFD codes must take into account spatially varying surface temperature, heat flux, and fuel regression rate, realistic surface energy and multispecies mass balances, thermal soak into the fuel grain, radiative energy exchange, and finite-rate Arrhenius kinetics for fuel pyrolysis modeling. With a commercial software, then, one has to built proper user-defined functions²⁰ or, in the most general case, an in house code has to be used.

The objective of this study is the simulation of the flow inside the combustion chamber and nozzle of a hybrid rocket. The target configuration is a lab-scale hybrid motor in the 1 kN class tested at the University of Naples "*Federico II*" equipped with an axial subsonic nozzle to feed gaseous oxygen into the port of High-Density PolyEthylene (HDPE) grains. With this injector arrangement, regression rate is appreciably larger than expected from the literature⁴³ especially at very low mass fluxes (around 10 kg/m²s), at which a 2.4-fold increase can be achieved; it shows lower dependence on the mass flux and, at a given mass flux, an increase with the port diameter is measured. Furthermore, motor stability and combustion efficiency are both favored.

The numerical simulations presented in this paper are carried out by solving Reynolds-averaged Navier-Stokes equations for single-phase multicomponent turbulent reacting flows,^{5,6} including the required sub-models in order to describe the homogeneous combustion in the gas phase, the radiative energy exchange, and the fluid-surface interaction in the combustion chamber (solid fuel pyrolysis model) for HDPE grains. The in-house code used for the simulations and its gas-surface interaction modeling capability has been validated for re-entry flows,¹⁰ whereas the same gas-surface interaction model has been used for the study of Hydroxyl-Terminated PolyButadiene (HTPB) grains^{7,9,11} and of hybrid rocket nozzle thermal protection system behavior.^{8,11}

2. Theoretical and Numerical Model

The study of hybrid rocket motor flowfields requires a suitable modeling of the motor internal ballistics including both gas phase reactions (pyrolyzed fuel and oxidizer combustion) and gas-surface interaction (fuel pyrolysis). Concerning the latter aspect, a detailed gas-surface interaction model based on surface mass and energy balances, including thermal radiation exchange and finite-rate Arrhenius kinetics for fuel pyrolysis, is coupled with a three-dimensional chemically-reacting fluid dynamics (CFD) code and a two-dimensional axisymmetric radiation code.

The CFD software is a finite-volume solver for three-dimensional turbulent compressible multi-component reacting flows^{5,6} with variable thermodynamic and transport properties. The thermodynamic properties of individual species are approximated by seventh-order polynomials of temperature and the transport properties are approximated by fourth-order polynomials.²⁴ Mixture properties for conductivity and viscosity are derived from the Wilke's rule. The diffusion model is based on an effective diffusion coefficient obtained assuming a constant Schmidt number. The Spalart-Allmaras one-equation model⁴⁰ is used to compute the turbulent viscosity. Turbulent thermal conductivity and turbulent mass diffusivity are computed from the turbulent viscosity, specific heat at constant pressure, turbulent Prandtl number, and turbulent Schmidt number. The chemistry is modeled assuming a zero-dimensional perfectly stirred reactor sub-model. The code solves the time-dependent conservation equations of mass, momentum, and energy for the chemical non-equilibrium flowfield and adopts a standard finite volume Godunov-type formulation. It is second order accurate in space and uses multi-block structured meshes. The system of ordinary differential equations is advanced in time by means of an explicit Runge-Kutta integration.

Radiation energy exchange from the hot combustion gases to the pyrolyzing fuel surface is accounted for through a separate code for generic axisymmetric gray/diffuse boundaries and inhomogeneous gray/non–scattering media, based on the Discrete Transfer Method (DTM).³³ The equations of the problem are written in finite form by discretizing the solid angle, at each node, and the path length, for each ray. The discretized equations are then integrated by means of a summation over the path length along each ray, to evaluate the relevant radiative intensity, and over the solid angle at each node, to evaluate the associated radiative wall heat flux. The field and wall local parameters needed by the code are recovered from the CFD simulations, with which the DTM computations are coupled. A suitable ray–tracing procedure is also implemented in the radiation code.

2.1 Gas-Surface Interface for Pyrolyzing Fuels

To complete the formulation of the theoretical model, boundary conditions must be specified at the gas/solid interface ("wall"), which describe the physics of the surface phenomena. The wall boundary conditions applicable to pyrolyzing fuels (such as HDPE) are detailed in the following.

If it is assumed that no material is being removed in a condensed phase (solid or liquid), then the general conservation laws at the gas-solid interface over a pyrolyzing fuel grain can be written as follows.²⁷ The overall mass balance is:

$$\dot{m}_b = (\rho v)_w = \rho_s \dot{r} \tag{1}$$

where \dot{m}_b is the overall mass flow rate of the injected pyrolysis products per unit surface area of the wall, ρ and v are the gas density at wall and its normal-to-wall velocity component, and finally ρ_s and \dot{r} are the fuel density and regression rate, respectively. The mass balance for each generic species is:

$$\rho D_{im} \frac{\partial y_i}{\partial \eta} + \dot{\omega}_i = (\rho v)_w y_i \qquad i = 1, 2, ..., N$$
⁽²⁾

where D_{im} is the *i*-th species-to-mixture diffusion coefficient, y_i the gas phase mass fraction of the *i*-th species at the wall, *N* the number of species, and $\dot{\omega}_i$ is the rate of production of gas-phase species *i* at the surface due to fuel pyrolysis, and η is a coordinate normal to surface oriented from solid to gas. The overall energy balance is:

$$k\frac{\partial T}{\partial \eta} + \sum_{i=1}^{N} h_i \rho D_{im} \frac{\partial y_i}{\partial \eta} + \dot{m}_b h_s + \dot{q}_{rad_{in}} = \dot{m}_b h_w + \dot{q}_{cond}^{ss} + \dot{q}_{rad_{out}}$$
(3)

where h_w is the enthalpy of the gas mixture at wall, h_i the enthalpy of the single gas species at the wall temperature, h_s the enthalpy of the solid grain at the wall temperature, k the gas thermal conductivity, T the gas temperature, and \dot{q}_{cond}^{ss} the conduction heat flux inside the grain. Note that the term $\dot{m}_b h_s$ is the energy flux entering the surface due to grain regression. Finally, radiation emitted from the hot combustion products and re-radiation from the propellant surface are included in the surface energy balance, Eq. (3). The terms $\dot{q}_{rad_{in}}$ and $\dot{q}_{rad_{out}}$ represent the absorbed and emitted radiation from the grain surface, respectively.

Concerning the energy transfer into the fuel grain, it is assumed that heat conduction is dominant in the direction normal to the grain surface. Although axial temperature gradients certainly exist along the grain wall, for the negligible surface temperature variation they are generally small if compared with those in the radial direction and can be considered to represent a second-order effect. In a local coordinate system moving with the receding fuel surface, the steady-state conduction term \dot{q}_{cond}^{ss} can be expressed as:

$$\dot{q}_{\rm cond}^{ss} = k_s \frac{\partial T_s}{\partial \eta} = \rho_s \dot{r} c_s (T_w - T_{si}) = \dot{m}_b c_s (T_w - T_{si}) \tag{4}$$

where c_s and k_s indicate the fuel heat capacity per unit mass and the fuel thermal conductivity, respectively, and T_w and T_{si} are the fuel grain wall temperature and initial temperature, respectively. Note that the steady-state conduction heat flux is only affected by the fuel specific heat and not by its thermal conductivity that only influences the in-depth grain temperature profile. The steady-state assumption is a reasonable approximation when the thermal lag in the solid is sufficiently small, which actually occurs in the operating conditions to be simulated, i.e. moderately high fuel regression rate and low fuel thermal diffusivity.

The energy balance expression can be recasted using Eq. (2) recalling that, by definition of the enthalpy at the wall, $h_w = \sum_{i=1}^N y_i h_i$, emphasizing the contributions due to convection from the gas, net radiation, pyrolysis chemical reactions, and conduction into the solid propellant grain, as:

$$\underbrace{k\frac{\partial T}{\partial \eta}}_{\text{convection}} + \underbrace{\dot{q}_{\text{rad}_{\text{in}}} - \dot{q}_{\text{rad}_{\text{out}}}}_{\text{net radiation}} = \underbrace{\sum_{i=1}^{N} \dot{\omega}_i h_i - \dot{m}_b h_s}_{\text{pyrolysis}} + \underbrace{\dot{q}_{\text{cond}}^{ss}}_{\text{solid conduction}}$$
(5)

The chemical heat flux due to pyrolysis surface reactions can be expressed as:

$$\dot{q}_{\text{chem}} = \dot{m}_b \left(\sum_{i=1}^N \frac{\dot{\omega}_i}{\dot{m}_b} h_i - h_s \right) = \dot{m}_b \Delta h_p \tag{6}$$

where the term in parentheses is the heat absorbed by the pyrolysis chemical reactions and hence it represents the so-called heat of pyrolysis (more correctly a heat of thermal degradation or depolymerization), Δh_p . Note that, upon substitution of Eq. (4) and Eq. (6) in Eq. (5), one obtains

$$k\frac{\partial T}{\partial \eta} + \dot{q}_{\rm rad_{in}} - \dot{q}_{\rm rad_{out}} = \dot{q}_{\rm conv} + \dot{q}_{\rm rad_{net}} = \rho_s \dot{r} \left[\Delta h_p + c_s (T_w - T_{si}) \right]$$
(7)

which, by defining $h_v = [\Delta h_p + c_s(T_w - T_{si})]$ as the effective heat of gasification of the fuel, coincides with the classical regression rate expression derived by Marxman and Gilbert³⁴

$$\dot{r} = \frac{\dot{q}_{\rm conv} + \dot{q}_{\rm rad_{net}}}{\rho_s h_v} \tag{8}$$

which shows that the regression rate is proportional to the incoming heat flux (convective plus radiative) and inversely proportional to the fuel density and effective heat of gasification.

2.2 HDPE Pyrolysis Model

The rate of production of gas-phase species *i* at the surface, $\dot{\omega}_i$, appearing in Eqs. (2) and (5), has to be estimated on the basis of the fuel pyrolysis model. In this work, according to Ref. [39], a single one-step irreversible Arrhenius-type equation is used to model \dot{r} assuming that the only product of HDPE pyrolysis is the monomer ethylene (C₂H₄). In particular, the solid fuel regression rate is obtained as:

$$\dot{r} = \frac{\dot{m}_b}{\rho_s} = A \cdot \exp(-E_a/2RT_w) \tag{9}$$

where A and E_a are the pre-exponential factor and the activation energy for the pyrolysis reaction, respectively, and R is the universal gas constant. Note that this technique for correlating surface temperature and regression rate is quite customary.^{17,18} The values assumed for these coefficients are reported in Table 1, according to Lengellè.³¹ Note that, for the large value of the activation energy E_a (Table 1), significant regression rate changes can be produced with minor changes in the surface temperature.

Table 1: Heterogeneous rate constants for HDPE (Ref. [31])

Surface reaction	A, mm/s	E_a , kJ/mol
$HDPE \rightarrow C_2H_4$	4.78 x 10 ⁶	251.04

As a single pyrolysis product is considered, the species production rate at the fuel surface, $\dot{\omega}_i$, is equal to \dot{m}_b for the species C₂H₄ and is equal to zero for all the remaining gas-phase species. The heat of pyrolysis could be, thus, directly evaluated from Eq. (6); nevertheless, since the assumption that the pyrolysis gas is composed of ethylene alone is necessarily an approximation, in this work the heat of pyrolysis has been derived from the available experimental data. According to Ref. [31] a value of 2.72 MJ/kg has been assigned to the heat of pyrolysis of HDPE. In Table 2 all the HDPE properties used for the simulations are listed.

Table 2: HDPE properties (Ref. [31])

Density ρ_s	Specific heat c_s	Thermal conductivity k_s	Heat of pyrolysis Δh_p
kg/m ³	J/(kg K)	W/(m K)	MJ/kg
960	1255.2	0.1549	2.72

Note that, although ethylene is assumed to be the only pyrolysis product, the gas mixture at the fuel surface is not entirely composed of C_2H_4 , as the other gaseous species (both oxygen and combustion gases) can actually reach/leave the surface due to diffusion and convection induced by blowing, as shown in Eq. (2). The mixture composition at the HDPE wall, in fact, is determined by solving the species surface mass balance, Eq. (2), coupled with the surface energy balance, Eq. (5), to guarantee that the correct amount of fuel is injected in the flowfield from the grain surface.

2.3 Gas Phase Reactions

Finite-rate gas phase reactions are modeled by global reaction mechanisms, because detailed chemical kinetics mechanisms would include many species and would be, on the one hand, computationally heavy and, on the other, beyond the scope of the present study, whose major purpose is to focus on gas-surface interaction. Therefore, finite-rate gas phase reactions are modeled by a simplified two-step global reaction mechanism. The first irreversible global reaction step is between C_2H_4 , which is the HDPE pyrolysis product, and molecular oxygen to form CO and H_2O and is considered first order in both fuel and oxidizer. The second reversible global step accounts for the formation of CO_2 .

$$C_2H_4 + 1.5O_2 \quad \Rightarrow \quad 2CO + H_2O + H_2 \tag{10}$$

$$CO + 0.5O_2 \Leftrightarrow CO_2$$
 (11)

According to Refs. [16] and [2], the net reaction rates of these two global reactions can be expressed as follows, where $[\cdot]$ indicates the species concentration:

$$\dot{w}_1 = k_{f_1} [C_2 H_4] [O_2]$$

$$\dot{w}_2 = k_{f_2} [CO] [H_2 O]^{0.5} [O_2]^{0.25} - k_{b_2} [CO_2] [H_2 O]^{0.5} [O_2]^{-0.25}$$

The resulting rates of production and destruction of species *i*, \dot{w}_i , are obtained from the reaction stoichiometry and the net reaction rates \dot{w}_i :

$$\begin{split} \dot{w}_{C_{2}H_{4}} &= -M_{C_{2}H_{4}}\dot{w}_{1} \\ \dot{w}_{O_{2}} &= M_{O_{2}}\{-1.5\dot{w}_{1} - 0.5\dot{w}_{2}\} \\ \dot{w}_{CO} &= M_{CO}\{2\dot{w}_{1} - \dot{w}_{2}\} \\ \dot{w}_{H_{2}O} &= M_{H_{2}O}\dot{w}_{1} \\ \dot{w}_{H_{2}} &= M_{H_{2}}\dot{w}_{1} \\ \dot{w}_{CO_{2}} &= M_{CO_{2}}\dot{w}_{2} \end{split}$$

The forward and backward reaction rates k_f and k_b for the two reactions are expressed as Arrhenius functions in the form $k = A_k T^{n_k} \exp(-E_{a,k}/RT)$, and the values of the constants used in this study are tabulated in Table 3. The first

Table 3: Reaction rates constants for reactions (10) and (11)

Reaction rate	A_k	n_k	$E_{a,k}/R$, K
k_{f_1} [16]	4.9486 x 10 ⁹	0.0	15200
k_{f_2} [2]	2.2400 x 10 ⁶	0.0	5032.7
k_{b_2} [2]	$1.1000 \ge 10^{13}$	-0.97	39456.5

global step reaction rate is taken from the work of Ref. [16] and the second step is taken from Ref. [2], which is a modified version of the one used in Refs. [39] and [16] to account for oxy-fuel combustion conditions.

2.4 Radiation Model

Thermal radiation is modeled separately in order to compute its contribution to the total wall heat flux. The basic hypotheses adopted include gray/diffuse wall and gray/non–scattering medium. The gray/diffuse wall assumption is deemed acceptable since thermal radiation from the wall is not a major contribution. The gray/non–scattering medium assumption, on the other hand, allows to enormously reduce the computational time, thanks to a spectrally–averaged treatment. It is also assumed that radiation does not affect the flow field significantly,^{23,25} a hypothesis which is justified in view of the relative small weight (1% at most for the analyzed test cases) of total wall heat transfer, and the radiative one in particular, as compared to the power generated in the combustion chamber.

The radiative wall heat flux is defined by the following integral of the radiative intensity at wall, I_w , over the hemispherical solid angle facing the incoming radiation, Ω :

$$\dot{q}_{\rm rad_{net}} = \int_{2\pi} \mathcal{I}_w \sin\theta \, d\Omega = \int_0^{2\pi} \int_0^{\frac{\pi}{2}} \mathcal{I}_w \sin\theta \cos\theta \, d\theta \, d\psi \tag{12}$$

The radiative intensity at the wall from a generic line–of–sight can be computed by integrating the radiative transfer equation (RTE) along the whole radiation path length. The RTE expresses the balance of radiative intensity along a generic direction, including contributions due to absorption/emission and, potentially, in/out–scattering. Under the assumption of gray/non–scattering medium it reduces to the form:

$$\frac{dI}{ds} = j^e - \kappa I = \kappa I_b - \kappa I \tag{13}$$

The RTE formal solution is given by:

$$\mathcal{I}_{w} = \mathcal{I}_{0} \exp\left(-\int_{s_{0}}^{s_{w}} \kappa \, ds\right) + \int_{s_{0}}^{s_{w}} j^{e} \exp\left(-\int_{s}^{s_{w}} \kappa \, ds'\right) ds \tag{14}$$

where it is assumed that the line–of–sight originates from another wall point. The radiative intensity I_w reaching a generic wall point from a given line–of–sight is therefore due to the contributions I_0 from the origin of the line–of–sight, and j^e from each point in the medium along the line–of–sight. The exponential terms account for radiation absorption by the medium from the origin (first term), and through the medium itself (second term). Equations (12,14) require the radiative intensity at the origin I_0 and the absorption coefficient κ ; the emission coefficient j^e is specified in terms of the latter quantity in Eq. (13), with the black–body term $I_b = \sigma T^4/\pi$.

The radiative intensity at the origin, under the assumption of gray/diffuse wall,

$$I_0 = \frac{\epsilon_0 \sigma T_0^4 + \rho_0 \dot{q}_{0,\text{rad}}}{\pi} \tag{15}$$

includes the radiative intensity emitted (with surface emissivity ϵ_0) and reflected (with surface reflectivity ρ_0) by the wall. Since such boundary condition depends on the radiative power flux reflected by the wall, then on the incoming radiative intensity, an iterative computational procedure is required.

The absorption coefficient of the gas mixture is derived by means of a global model, typically used for hightemperature combustion mixtures under vibrational equilibrium conditions,

$$\kappa = p \sum_{i=1}^{N_{\text{rad}}} X_i \kappa_{p,i} \tag{16}$$

indicating that the absorption of radiative energy is proportional to the pressure p, i.e., the number of molecules per unit volume, and to the absorption coefficients of the N_{rad} participating species, weighted with their concentrations in terms of molar fraction, X_i . In particular, water vapor, carbon dioxide and carbon monoxide^{35,44} are most relevant in the process of thermal radiation exchange. The absorption coefficients of the relevant participating species, known as Planck–mean absorption coefficients, are averaged over the whole spectrum. Their temperature dependence is given up to 5000 K and at atmospheric pressure in Ref. [38], by fitting and extending the curves reported in Refs. [35,44]. This model is adopted here, although not explicitly accounting for high–pressure effects, admittedly leaving some uncertainty, which is deemed of a weight comparable to those implied by other aspects of the model.

3. Motor Configuration

A brief description of the lab-scale motor and of the test facility employed to carry out the firing tests referred to in this paper is reported here; exhaustive details can be found in Ref. [12].



Figure 1: Lab-scale hybrid rocket motor schematic

The lab-scale motor (shown in Fig. 1) has an axisymmetric combustion chamber, with a 720 mm length and a 133 mm inner-case diameter. Several static engine firings have been performed with this motor configuration using HDPE grains. All the experimental firing test data used in this paper have been obtained with an axial injector which employs a converging subsonic conical nozzle whose exit diameter is 8 mm. A pre-chamber in stainless steel and a thermally insulated aft-mixing chamber (Fig. 1) are set up ahead and aft of the fuel grain, respectively. Gaseous oxygen is supplied to the motor feed line with a pressure regulator. This system allows one to set the oxygen feeding pressure and control the oxygen mass flow rate over a firing test. The oxygen mass flow rate is imposed by means of a choked venturi, independently of the conditions attained in the downstream combustion chamber. A water-cooled conical converging-diverging nozzle with 16 mm throat diameter, an 82 mm length, and a 2.4 area ratio, made of copper alloy, ensures long-duration firings without throat erosion. Chamber pressure is measured by two capacitive transducers set up in the pre-chamber and in the aft-mixing chamber.

Cylindrical HDPE grains with a single central bore have been employed with a 560 mm length and four different inner diameters (16, 25, 50, and 75 mm) were used to achieve a wide range of average mass fluxes and grain length to diameter ratios at a limited expense of oxygen mass flow rate (the maximum achievable flow rate being 350 g/s). The test cases selected in this work are summarized in Table 4 which report the average parameters measured over the static firings. More details can be found in Ref. [13]. The firing test parameters are derived as follows.

The time-space-averaged regression rate has been calculated with the fuel-mass loss method:

$$\bar{\dot{r}} = \frac{\dot{m}_f}{\rho_s \pi \bar{D}L} \tag{17}$$

where \overline{m}_f is the time-averaged fuel mass flow rate that is determined by dividing the measured fuel mass loss ΔM by the burning time t_b :

$$\overline{\dot{m}}_f = \frac{\Delta M}{t_b} \tag{18}$$

The port diameter averaged over the entire burning, $\overline{D} = (D_0 + \widehat{D}_2)/2$, is determined starting from the initial port diameter D_0 and the final one \widehat{D}_2 , averaged over the grain axis. \widehat{D}_2 is estimated by means of the fuel mass consumed as follows:

$$\widehat{D}_2 = \sqrt{D_0^2 + \frac{4\Delta M}{\pi \rho_s L}} \tag{19}$$

The determination of the burning time follows from the pressure versus time trace, following a well-assessed procedure.¹³

Test no.	t_b , s	\bar{G}_{ox} , kg/m ² s	\bar{D} , mm	\bar{r} , mm/s	\bar{p}_c , atm	O/F
2	54.9	27.68	75.84	0.47	16.85	2.05
4	43.0	87.14	43.48	0.64	15.64	2.70
9	31.5	28.41	89.84	0.47	22.61	2.50
10	21.2	126.00	42.33	0.82	20.25	3.02
13	26.2	28.19	60.04	0.38	10.09	2.05
18	44.1	78.07	40.32	0.55	11.78	2.64
19	44.3	54.03	47.19	0.50	11.11	2.37

Table 4: Firing test average results (HDPE)¹³

Finally, the time-space-averaged mass flux (whether it is the oxidizer or the total one) is calculated based on the average port diameter:

$$\bar{G} = \frac{4}{\pi} \frac{\bar{m}}{\bar{D}^2} \tag{20}$$

This averaging definition has been demonstrated to be the most accurate in describing the classical regression-rate/mass flux power law.²⁶

4. Results and Discussion

The computational domain is a simplified geometrical representation of the physical domain where the pre-chamber, fuel grain and post-chamber have been schematized with constant cross section (see Fig. 2). Based on the available

experimental data, the selected inflow boundary condition enforces oxidizer mass flow rate and temperature. More specifically, a single temperature value is imposed for all simulations (T = 300 K) whereas different oxygen mass flow rate values are considered according to the experimental data. Note that the oxidizer mass flow rate is controlled by the choked venturi and held constant during each firing, regardless of the chamber pressure and grain diameter variations over time. The pyrolysis gas injected from the fuel wall is C_2H_4 , whose amount is determined by iteratively solving Eq. (5) first for the wall temperature, and, then, calculating the gaseous chemical composition at the grain surface through Eqs. (2) and (9). Incoming net radiative heat flux is computed from the DTM radiation code as a post-processing of the CFD solution. CFD code and radiation code are loosely coupled until convergence is reached. Some iterations between the two codes are required to reach convergence as the radiation computed by the DTM code affects the fuel regression rate which, in turns, affect the CFD solution. All the computations presented here are two-dimensional (axisymmetric) and at the steady-state condition obtained by iterating in time until residuals drop by five orders of magnitude. For each value of oxygen mass flow rate enforced, the CFD computation provides the fuel mass flow rate (from the surface balances) and the chamber pressure level as a consequence of the choked condition at the nozzle throat. Therefore, the chamber pressure level attained in the motor depends on the fuel mass flow rate and on the mixing and combustion process.



Figure 2: Numerical grid schematic (not in scale)



Figure 3: Temperature field and detail of the recirculation zone for firing #19 test

Figure 2 shows the numerical grid at the average port diameter condition of firing #10 test (Table 4). A detail of the injection region is also shown which highlights the corresponding grid refinement needed to capture the strong recirculation zone induced by the axial injector. The computational domain is subdivided into 170 x 60 grid points in the axial and radial directions, respectively. Cells are clustered towards the wall in such a way to ensure a value of y+ of the order of 1 at the wall-adjacent cell all along the motor length to accurately describe the boundary layer. Additional axial clustering of cells is placed in the regions near the grain leading and trailing edges and in the nozzle region. A grid sensitivity analysis has been carried out to ensure that the presented results are grid independent and a grid convergence study has been performed to assess that the selected grid size is sufficiently refined to obtain an accurate solution. Figure 3 shows the temperature field and the detail of the strong recirculation region induced by the oxidizer axial injection. The hot recirculation region favors pyrolyzed fuel and oxidizer mixing and combustion.

4.1 Sensitivity Analysis to the Pyrolysis Model

In order to understand how the law of pyrolysis and the heat of pyrolysis used in the surface energy balance can affect the regression rate and chamber pressure prediction, a sensitivity analysis has been performed. For such study the firing #19 test has been taken as reference because of its intermediate port diameter (47.19 mm) and oxidizer mass flux (54.03 kg/m²s) conditions with respect to the considered firing tests (see Table 4).

The laws of pyrolysis available in literature are widely scattered because of significantly different pre-exponential factors and activation energies. At a given temperature this leads to significantly different values of regression rate, as it is shown in Fig. 4.



Figure 4: HDPE regression rate vs temperature from various pyrolysis laws

Five different laws of pyrolysis have been compared in Fig. 4 for HDPE fuel grain temperatures ranging approximatively from 750 to 950 K. The analyzed laws of pyrolysis are taken from different references in the open literature and they have all been expressed in the form of Eq. (9). However, as the pyrolysis reaction rates have been obtained from thermogravimetric analyses and are expressed in terms of reaction speeds with a pre-exponential factor in sec⁻¹, the pre-exponential term A of Eq. (9) is, in general, a function of temperature.

Table 5: Activation energies and pre-exponential factors for different HDPE pyrolysis laws

Pyrolysis law	Source	A, mm/s	E_a , kJ/mol
r1 (reference)	Ref. [13]	4.78 x 10 ⁶	251.04
r2	Ref. [32]	2.68 x 10 ⁶	251.20
r3	Ref. [31]	-	251.04
r4	Ref. [41]	-	349.00
r5	Ref. [22]	-	130.00

Table 5 lists the activation energy and the pre-exponential factor (only when this is a constant and not function of

temperature) for the different HDPE pyrolysis laws. As shows in Fig. 4 a significant scattering is evidenced between the various laws. Note that r5 pyrolysis law, according to its authors,²² has been obtained with a very low heating rate of about 20 K/min that is several orders of magnitude lower than the heating rate of the fuel in actual hybrid rocket combustion chamber conditions, that is estimated around 10^5 K/s.²² For this reason, r5 pyrolysis law has been discarded from this analysis. The behavior of r1, r2, and r3 laws shows a very similar slope (Fig. 4) due to the fact that the activation energies are practically the same. Differently, r4 pyrolysis law show a significantly higher activation energy. Finally, although the regression rate of r1, r2, r3, and r4 pyrolysis laws are comparable, they can show differences in *r* at the same fuel temperature up to 40%. Therefore, it is of importance to analyze the effect of different activation energies and pre-exponential factors on the regression rate prediction in actual hybrid rocket combustion chamber conditions.

Starting from the reference law of pyrolysis r1 (blue curve in Fig. 4), the following parameters have been changed: first the pre-exponential factor has been modified by using the corresponding (lower) value from the law r2 (green curve in Fig. 4) and second the activation energy has been modified by using the corresponding (higher) value from the law r4 (cyan curve in Fig. 4), in order to analyze separately the effect of a change in each of the parameters A and E_a of the pyrolysis law.

Sensitivity to the Pyrolysis Law Parameters

Figure 5 shows the effect of the 44% reduction in the pre-exponential factor (from 4.78 x 10^6 to 2.68 x 10^6 mm/s) on both the fuel regression rate and the fuel temperature. Table 6 shows the results in terms of space-averaged regression rate and motor chamber pressure. Not surprisingly, once the widely different pre-exponential factors are used in gas-



Figure 5: Effect of the pre-exponential factor on the fuel regression rate and the fuel temperature

Table 6	: Results	for firing	#19 test y	with the e	ffect of a	change in	the pre-e	xponential f	factor.	case #19a

Firing test	$\bar{p_c}$, bar	$\bar{\dot{r}}$, mm/s
#19	9.78	0.320
#19a	9.69 (-0.9%)	0.316 (-1.3%)

surface interaction wall boundary condition, very similar values of fuel regression rate are found, with a difference of the order of 1%. This is due to the fact that the wall temperature behaves as a compensating factor while the regression rate is dominantly dictated by the energy balance, Eq. (8). In fact, when a lower pre-exponential factor is used, a higher wall temperature results from the energy balance (see Fig. 5(b)), leading to a computed regression rate that remains practically unaffected (see Fig. 5(a)). Thus, the pyrolysis process is definitely controlled by the heat flux to the fuel surface, and reaction kinetics only play a minor role.³⁴ Hence, as shown in Figure 5 and in Table 6, the 44% reduction in the pre-exponential factor of the pyrolysis law is almost completely compensated by the increased grain

wall temperature (of the order of 30-40 K), leading to negligibly small variations of both regression rate and motor chamber pressure.

Figure 6 shows the effect of the 39% increase in the activation energy (from 251.04 to 349.00 kJ/mol) on both the fuel regression rate and the fuel temperature. Table 7 shows the results in terms of space-averaged regression rate and motor chamber pressure. The effect of the increase of the activation energy is qualitatively similar to the



Figure 6: Effect of the activation energy on the fuel regression rate and the fuel temperature

Table 7: Results for firing #19 test with the effect of a change in the activation energy, case #19b

Firing test	$\bar{p_c}$, bar	\bar{r} , mm/s
#19	9.78	0.320
#19b	9.25 (-5.4%)	0.283 (-11.6%)

effect of a decrease in the pre-exponential factor, although quantitative results are significantly different. As shown in Figure 6 and in Table 7, the 39% increase in the activation energy is only partially compensated by the increased grain wall temperature, causing limited but non-negligible variations of both regression rate and chamber pressure. Although the analyzed percentage variations in the pre-exponential factor and activation energy are similar (44% and 39%, respectively), the effect of the activation energy on the overall results is much stronger than that of the preexponential factor, due to its exponential effect on the pyrolysis law. It must be noted that the grain temperature increase is also much higher (of the order of 300-350 K) than in the previous case. Finally, it has to be stressed that the present sensitivity analysis has been carried out for the sake of investigation to understand the specific role of the two parameters (A and E_a) of the pyrolysis law. However, pyrolysis laws should be obviously used taking all the relevant parameters from a single reference, avoiding mismatches between parameters taken from different data. Therefore, a direct comparison between r1 and r4 pyrolysis laws will be later presented. We conclude this sensitivity analysis highlighting the fact that both the pre-exponential factor and the activation energy have a direct effect on the grain wall temperature, while they are only mildly affecting the grain regression rate and hence the motor chamber pressure. This is especially true for the pre-exponential factor.

Sensitivity to the Heat of Pyrolysis

The last sensitivity that has been analyzed, before comparing directly one complete set of pyrolysis law data to another complete set (r1 vs r4), is the effect of the heat of pyrolysis, Δh_p . A significant scattering is found in the literature also for the heat of pyrolysis of HDPE. For such a reason, the heat of pyrolysis has been reduced by the arbitrarily amount of 30% with respect to the reference value (2.72 MJ/kg) listed in Table 2 in order to measure the direct effect of its change on the results in terms of regression rate and chamber pressure. The results presented in Figure 7 and Table 8 show a strong sensitivity to the value assumed for the heat of pyrolysis. A modest rise of the grain wall temperature (of

the order of 6-12 K) is also found because of the increased available energy associated to the lower heat of pyrolysis. However, differently from the two previous cases (#19a and #19b), the effect of the heat of pyrolysis on the grain wall temperature is minimal (of the order of 1%) while the effect on the fuel regression rate (and hence on the chamber pressure) is definitely more pronounced (+21% on the regression rate). This, again, confirms that the regression rate



Figure 7: Effect of the heat of pyrolysis on the fuel regression rate and the fuel temperature

Table 8: Results for firing #19 test with the effect of a change in the heat of pyrolysis, case #19c

Firing test	$\bar{p_c}$, bar	\bar{r} , mm/s
#19	9.78	0.320
#19c	10.37 (+6.0%)	0.387 (+20.9%)

is dictated by the energy balance at the grain surface and by the competition between incoming heat fluxes (convection and radiation) and outgoing heat fluxes (fuel pyrolysis and solid grain heating), as shown in Eq. (8). A modification of the heat of pyrolysis without any change in the pyrolysis law (pre-exponential factor and activation energy), hence, causes a direct effect on the fuel regression rate, that is increased as the heat of pyrolysis is decreased, and generates only a very limited effect on the grain temperature.

According to the results of the previous sensitivity analyses (cases #19a, #19b, and #19c), it can be concluded that the heat of pyrolysis is the parameter most affecting the fuel regression rate, followed by the activation energy with a relatively weak effect and the pre-exponential factor showing negligibly small variations.

Sensitivity to the Pyrolysis Law Data Set

This last sensitivity analysis has been performed comparing directly one complete set of pyrolysis law data to another complete set (r1 vs r4 in Table 5). The heat of pyrolysis Δh_p associated with r1 law is 2.72 MJ/kg and the corresponding average specific heat c_s is 1255.2 J/(kg K)³¹ while the heat of pyrolysis associated with r4 law is 1.138 MJ/kg and the corresponding average specific heat is 3428.5 J/(kg K).⁴² For such study the firing #2 and #10 tests have been analyzed because they are characterized by the minimum and maximum oxidizer mass flux, respectively (see Table 4).

As shown in Table 9, the results in terms of space-averaged regression rate and motor chamber pressure are moderately affected by a change in the adopted law of pyrolysis and associated heat of pyrolysis. In conclusion, it can be stated that it has been not evidenced a significant dependence of the solution from the law of pyrolysis adopted, provided that each data set is consistently taken from a single reference.

Table 9: Results for firing #2 and #10 tests with the effect of a change in the pyrolysis law data (r1 vs r4), case #2d and #10d

Firing test	$\bar{p_c}$, bar	\bar{r} , mm/s
#2 (r1)	12.78	0.248
#2d (r4)	13.23 (+3.5%)	0.265 (+6.9%)
#10 (r1)	16.26	0.541
#10d (r4)	17.01 (+4.6%)	0.589 (+8.9%)

4.2 Comparison with Experimental Data

All the firing tests are considered in this section in order to understand the ability of the numerical setup to reproduce the experimental data. For such comparison, all simulations have been performed neglecting radiative energy exchange. The resulting deviation between predicted and measured data of regression rate and chamber pressure over all the test cases are summarized in Fig. 8. When fuel regression rate is computed by solving the gas-surface interaction wall boundary condition (neglecting radiative energy exchange), results (see Fig. 8(a)) indicate a quite evident underestimation of the averaged regression rate for all firing tests. This underestimation in the regression rate reflects more or less directly (depending on the average O/F ratio and deriving theoretical c^* of the firing test) into an underestimation of the motor operating chamber pressure (see Fig. 8(b)). In particular the regression rate is underestimated in between 31 and 47%, while the chamber pressure is underestimated in between 13 and 25%. It is important to underline here that the predicted chamber pressure is not only the result of the fuel regression rate but also of the mixing and combustion process within the combustion chamber, that can strongly affect the characteristic velocity. Hence, a correct prediction of the fuel regression rate does not necessarily grant that the chamber pressure is correctly predicted. If the mixing and combustion processes are not correctly modeled, in fact, a mismatch between predicted and measured chamber pressure will result.



(a) Time-and spatially-averaged regression rates

(b) Time-averaged chamber pressures

Figure 8: Comparison of calculated values from simulations (with calculated or imposed regression rate) with measured data from the motor firings

Since the sensitivity analysis showed that the results can be affected by the law of pyrolysis adopted just for a few percentage points (less than 10% for regression rate and less than 5% for the chamber pressure), a study to evaluate the goodness of the combustion model has been carried out. To check that the mixing and combustion processes are adequately modeled in the current approach, simulations for 5 firing tests (namely #2, #4, #10, #13 and #19) have been computed by directly imposing the experimentally measured averaged regression rate. Results are shown in Fig. 8 and show that, once the experimental regression rate value is correctly imposed in the simulation, a very good prediction of motor chamber pressure (hence of the characteristic velocity) is obtained. This confirms that mixing and combustion processes are adequately modeled in the present approach. Nevertheless, when fuel regression rate is predicted from

the gas-surface interaction wall boundary condition, the averaged regression rate values from the various experimental firing tests are underestimated by 31% up to 47%. Considering the analyses that have been carried out so far, it is clear that such a significant underestimation must be attributed to another source of error, as the absence of the radiative contribution to the wall heat flux. The regression rate general trend (Fig. 8(a)), in fact, seems to suggest the lack of a radiation modeling, whose effect is known to be more significant at the lower oxidizer mass fluxes.

4.3 Effect of Thermal Radiation Exchange

In order to analyze the effect of radiative energy exchange, CFD simulations have been loosely coupled to radiative simulations to take into account the net incoming radiative heat flux to the fuel grain and its effect on the fuel regression rate, through Eq. (8). A wall emissivity equal to 0.85 has been assumed according to typical values for polymers exposed to flames. The wall and flow field local conditions needed by the radiative code are taken from the CFD solution, with which the radiative solution is coupled until convergence is attained. The resulting deviation between predicted (with and without radiative energy exchange) and measured data of regression rate and chamber pressure over all the test cases are summarized in Fig. 9.



Figure 9: Comparison of calculated values from simulations (with or without radiative energy exchange) with measured data from the motor firings

Looking at the numerical results obtained with the radiation modeling, the errors previously found without including thermal radiation are much mitigated. Despite a noticeable improvement with respect to previous solutions, regression rate and chamber pressure for firing tests characterized by the larger port diameters (firing #2 and #9 tests) are overestimated when including thermal radiation. In particular, firing #9 test shows a huge overestimation with respect to measured data on both regression rate and chamber pressure data, requiring a deeper analysis of the port diameter effect on the net radiative heat flux to the grain surface. The rest of the analyzed tests, however, falls within $\pm 15\%$ error on both regression rate and chamber pressure. For firing #9 test, the radiative wall heat flux is found to be much higher with respect to the other test cases as a consequence of the high value of the product between chamber pressure and port diameter. One of the responsible for this significant error on the computed thermal radiation at wall could be the simplified chemical mechanism, which lacks radical and hence tends to overestimate flame temperature and radiation emitting species mass fractions. Probably, this error is also amplified in the coupling procedure between CFD and radiative simulations. In addition, it has been confirmed by radiative simulations that the value assumed for the wall emissivity has a differential impact on the net computed radiative wall heat flux, strictly depending on the medium optical thickness, i.e. the product between chamber pressure and port diameter. In particular, for high values of the medium optical thickness, as in the case firing #9 test, a high sensitivity to the wall emissivity is found, hence requiring more carefulness in the selection of the HDPE emissivity value.

5. Conclusions

A CFD approach to the internal ballistics of a GOX/HDPE hybrid motor with integrated gas-surface interaction modeling capabilities has been presented. The fluid dynamic equations are the Reynolds averaged Navier-Stokes equations with additional transport equation for chemical species and turbulence. Suitable sub-models are included in order to describe the homogeneous combustion in the gas phase, the radiative energy exchange toward the fuel surface, and the fluid-surface interaction in the combustion chamber (solid fuel pyrolysis model). The gas-phase equations are solved coupled to the solid fuel-phase using customized surface balances of mass and energy which enable the determination of the fuel regression rate and surface temperature as part of the flowfield solution. A test case, represented by the combustion in a lab-scale hybrid rocket burning gaseous oxygen and HDPE, has been simulated. A sensitivity analysis has been carried out on the literature pyrolysis models for HDPE. Obtained results confirmed that both the pre-exponential factor and the activation energy of the pyrolysis law have a direct effect on the grain wall temperature, while they are only mildly affecting the grain regression rate and hence the motor chamber pressure. This is due to the fact that the wall temperature behaves as a compensating factor while the regression rate is dominantly dictated by the surface energy balance. Differently, the obtained results showed a much stronger sensitivity of fuel regression rate and chamber pressure to the value assumed for the heat of pyrolysis. On the other hand, the effect of the heat of pyrolysis on the grain wall temperature appears to be minimal. Thus, the pyrolysis process appears to be definitely controlled by the energy balance at the grain surface and by the competition between incoming heat fluxes (convection and radiation) and outgoing heat fluxes (fuel pyrolysis and solid grain heating), with reaction kinetics only playing a minor role. The heat of pyrolysis is the parameter most affecting the fuel regression rate, followed by the activation energy with a relatively weak effect and the pre-exponential factor showing negligibly small variations.

The numerical approach has been validated by direct comparison of numerical and experimental data. Comparison shows that, despite a number of simplifying assumptions, the CFD approach is fairly able to capture the main features of the motor internal ballistics both in terms of average chamber pressure and regression rate trends with oxidizer mass flux and port diameter. When the experimentally measured regression rate is imposed as a boundary condition in the simulation, the model is correctly able to predict the motor chamber pressure, hence the mixing and combustion process, with errors ranging from +4% to -8%. When the regression rate is computed from the gas-surface interaction wall boundary condition, however, the regression rate is underestimated in between 31 and 47%, while the chamber pressure is underestimated in between 13 and 25%. Considering the sensitivity analyses that have been carried out so far, such a significant underestimation must be attributed to another source of error, as the absence of the radiative contribution to the wall heat flux. When a thermal radiation model is included, the errors previously found are much mitigated. Despite a noticeable improvement with respect to solutions without radiation, regression rate and chamber pressure for firing tests characterized by the larger port diameters are overestimated when including thermal radiation, requiring a deeper analysis of the port diameter and grain wall emissivity effects on the net radiative heat flux to the grain surface. The rest of the analyzed tests, however, falls within $\pm 15\%$ error on both regression rate and chamber pressure.

References

- D. Altman and A. Holzman. Overview and history of hybrid rocket propulsion. In K.K. Kuo and M.J. Chiaverini, editors, *Fundamentals of Hybrid Rocket Combustion and Propulsion*, volume 218 of *Progress in Astronautics* and Aeronautics, pages 1–36. AIAA, 2007.
- [2] J. Andersen, C. L. Rasmussen, T. Giselsson, and P. Glarborg. Global combustion mechanisms for use in cfd modeling under oxy-fuel conditions. *Energy & Fuels*, 23:1379–1389, 2009.
- [3] H. Arisawa and T. B. Brill. Flash pyrolysis of hydroxyl terminated poly-butadiene (htpb) ii: Implications of the kinetics to combustion of organic polymers. *Combustion and Flame*, 106:131–143, 1996.
- [4] N. Bellomo, M. Lazzarin, F. Barato, A. Bettella, D. Pavarin, and M. Grosse. Investigation of effect of diaphragms on the efficiency of hybrid rockets. *Journal of Propulsion and Power*, 30(1):175–185, 2014. doi: 10.2514/1.B34908.
- [5] B. Betti, D. Bianchi, F. Nasuti, and E. Martelli. Chemical reaction effects on wall heat flux in liquid rocket thrust chambers. AIAA Paper 2014-3675, 2014. doi: 10.2514/6.2014-3675.
- [6] B. Betti, F. Nasuti, and E. Martelli. Numerical evaluation of heat transfer enhancement in rocket thrust chambers by wall ribs. *Numerical Heat Transfer Part A: Applications*, 66(5):488–508, 2014. doi: 10.1080/10407782.2014.885233.

- [7] D. Bianchi, B. Betti, F. Nasuti, and C. Carmicino. Simulation of gaseous oxygen/hydroxyl-terminated polybutadiene hybrid rocket flowfields and comparison with experiments. *Journal of Propulsion and Power*, 31(3):919–929, 2015. doi: 10.2514/1.B35587.
- [8] D. Bianchi and F. Nasuti. Numerical analysis of nozzle material thermochemical erosion in hybrid rocket engines. *Journal of Propulsion and Power*, 29(3):547,558, 2013. doi: 10.2514/1.B34813.
- [9] D. Bianchi, F. Nasuti, and C. Carmicino. Hybrid rockets with axial injector: port diameter effect on fuel regression rate. *Journal of Propulsion and Power*, 32(4):984–996, 2016. doi: 10.2514/1.B36000.
- [10] D. Bianchi, F. Nasuti, R. Paciorri, and M. Onofri. Computational analysis of hypersonic flows including finite rate ablation thermochemistry. 7th European Workshop on Thermal Protection Systems and Hot Structures, ESA-ESTEC, Noordwijk, The Netherlands, 8-10 April 2013.
- [11] D. Bianchi, A. Urbano, B. Betti, and F. Nasuti. Cfd analysis of hybrid rocket flowfields including fuel pyrolysis and nozzle erosion. AIAA Paper 2013-3637, 2013. doi: 10.2514/6.2013-3637.
- [12] C. Carmicino. Acoustics, vortex shedding, and low-frequency dynamics interaction in an unstable hybrid rocket. *Journal of Propulsion and Power*, 25(6):1322–1335, 2009. doi: 10.2514/1.42869.
- [13] C. Carmicino and A. Russo Sorge. Role of injection in hybrid rockets regression rate behavior. *Journal of Propulsion and Power*, 21(4):606–612, 2005. doi: 10.2514/1.9945.
- [14] C. Carmicino and A. Russo Sorge. Influence of a conical axial injector on hybrid rocket performance. *Journal of Propulsion and Power*, 22(5):984–995, 2006. doi: 10.2514/1.19528.
- [15] Y. S. Chen, T. H. Chou, B. R. Gu, J. S. Wu, B. Wu, Y. Y. Lian, and L. Yang. Multiphysics simulations of rocket engine combustion. *Computers & Fluids*, 45(1):29–36, 2011.
- [16] G. C. Cheng, R. C. Farmer, H. S. Jones, and J. S. McFarlane. Numerical simulation of the internal ballistics of a hybrid rocket motor. AIAA Paper 94-0554. 32nd AIAA Aerospace Sciences Meeting & Exhibit, Reno, NV, Jan. 10-13, 1994.
- [17] M. J. Chiaverini, K. K. Kuo, A. Peretz, and G. C. Harting. Regression-rate and heat-transfer correlations for hybrid rocket combustion. *Journal of Propulsion and Power*, 17(1), 2001.
- [18] M. J. Chiaverini, N. Serin, D. K. Johnson, Y. C. Lu, K. K. Kuo, and G. A. Risha. Regression rate behavior of hybrid rocket solid fuels. *Journal of Propulsion and Power*, 16(1), 2000.
- [19] A Coronetti and W. A. Sirignano. Numerical analysis of hybrid rocket combustion. *Journal of Propulsion and Power*, 29(2), 2013.
- [20] G. D. Di Martino, C. Carmicino, and R. Savino. Transient computational thermofluid-dynamic simulation of hybrid rocket internal ballistics. *Journal of Propulsion and Power*, 2017.
- [21] G. Gariani, F. Maggi, and L. Galfetti. Simulation code for hybrid rocket combustion. AIAA Paper 2010-6872, 2010. 46th AIAA/ASME/SAE/ASEE Joint Propulsion Conference.
- [22] N. Gascoin, G. Fau, P. Gillard, and A. Mangeot. Experimental flash pyrolysis of high density polyethylene under hybrid propulsion conditions. *Journal of Analytical and Applied Pyrolysis*, 101:45–52, 2013.
- [23] F. Göbel and C. Mundt. Implementation of the p1 radiation model in the cfd solver nsmb and investigation of radiative heat transfer in the ssme main combustion chamber. AIAA Paper 2011-2257, San Francisco, CA, April 2011. doi:10.2514/6.2011-2257.
- [24] S. Gordon and B. J. McBride. Computer program for calculation of complex chemical equilibrium compositions and applications. NASA RP 1311, 1994.
- [25] L. Jiwen and S. N. Tiwari. Radiative heat transfer effects in chemically reacting nozzle flows. *Journal of Ther*mophysics and Heat Transfer, 10(3):436–444, July-September 1996. doi:10.2514/3.808.
- [26] M. A. Karabeyoglu, B. J. Cantwell, and G. Zilliac. Development of scalable space-time averaged regression rate expressions for hybrid rockets. *Journal of Propulsion and Power*, 23(4):737–747, 2007. doi: 10.2514/1.19226.

- [27] R. M. Kendall, E. P. Bartlett, R. A. Rindal, and C. B. Moyer. An analysis of the chemically reacting boundary layer and charring ablator. part i: Summary report. NASA CR 1060, 1968.
- [28] C. P. Kumar and A. Kumar. Effect of diaphragms on regression rate in hybrid rocket motors. *Journal of Propulsion and Power*, 29(3):559–572, 2013. doi: 10.2514/1.B34671.
- [29] K. K. Kuo and R. W. Houim. Theoretical modeling and numerical simulation challenges of combustion processes of hybrid rockets. AIAA Paper 2011-5608, 2011.
- [30] M. Lazzarin, F. Barato, A. Bettella, and D. Pavarin. Computational fluid dynamics simulation of regression rate in hybrid rockets. *Journal of Propulsion and Power*, 29(6):1445–1452, 2013. doi: 10.2514/1.B34910.
- [31] G. Lengelle, B. Fourest, J. Godon, and C. Guin. Condensed-phase behavior and ablation rate of fuels for hybrid propulsion. AIAA Paper 1993-2413, 1993.
- [32] C. Li, G. Cai, and H. Tian. Numerical analysis of combustion characteristics of hybrid rocket motor with multisection swirl injection. Acta Astronautica, 123:26–36, 2016.
- [33] F. C. Lockwood and N. G. Shah. A new radiation solution method for incorporation in general combustion prediction procedures. *Symposium (International) on Combustion*, 18(1):1405–1414, 1981. doi:10.1016/S0082-0784(81)80144-0.
- [34] G. A. Marxman and M. Gilbert. Turbulent boundary layer combustion in the hybrid rocket. *Ninth International Symposium on Combustion*, pages 371–383, 1963. Academic Press, New York.
- [35] M. F. Modest. Radiative Heat Transfer, pages 288–410. Academic Press, San Diego, 2013.
- [36] M. Motoe and T. Shimada. Numerical simulations of combustive flows in a swirling-oxidizer-flow-type hybrid rocket. AIAA Paper 2014-0310, 2014. doi: 10.2514/6.2014-0310.
- [37] K. Ramohalli and J. Yi. Hybrids revisited. AIAA Paper 90-1962. 26th AIAA/ASME/SAE/ASEE Joint Propulsion Conference, Orlando, FL, July 16-18, 1990.
- [38] P. Rivière and A. Soufiani. Updated band model parameters for h₂o, co₂, ch₄ and co radiation at high temperature. *International Journal of Heat and Mass Transfer*, 55(13):3349–3358, 2012. doi:10.1016/j.ijheatmasstransfer.2012.03.019.
- [39] V. Sankaran. Computational fluid dynamics modeling of hybrid rocket flowfields. In K.K. Kuo and M.J. Chiaverini, editors, *Fundamentals of Hybrid Rocket Combustion and Propulsion*, volume 218 of *Progress in Astronautics and Aeronautics*, pages 323–349. AIAA, 2007.
- [40] P. R. Spalart and S. R. Allmaras. A one-equation turbulence model for aerodynamic flows. *La Recherche Aerospatiale*, 1:5–21, 1994.
- [41] S. I. Stoiliarov, S. Crowley, R. E. Lyon, and G. T. Linteris. Prediction of the burning rates of non-charring polymers. *Combustion and Flame*, 156(5):1068–1083, 2009.
- [42] S. I. Stoiliarov and R. N. Walters. Determination of the heats of gasification of polymers using differential scanning calorimetry. *Polymer Degradation and Stability*, 93(2):422–427, 2008.
- [43] G. P. Sutton and O. Biblarz. Rocket Propulsion Elements. John Wiley and Sons, Inc., New York, NY, 2001.
- [44] C. L. Tien. Thermal radiation properties of gases. In T. F. Irvine and J. P. Hartnett, editors, *Advances in Heat Transfer*, volume 5, pages 253–324. Academic Press, New York, 1968.