Thermal Analysis of a Hybrid Rocket Propulsion System for **Interplanetary CubeSats**

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Abstract

This paper presents results of a thermal analysis performed on a gaseous oxygen/Polymethylmethacrilate hybrid rocket motor for a CubeSat application. This work was focused on deriving temperatures inside the combustion chamber and verifying the selected materials can withstand the high temperatures and stresses. Results show that the fuel starts to break-up before the end of the burn. The addition of a graphite post-combustion chamber was also evaluated but it is not recommended for long burn times due to high temperature reached in the steel. This work is an important intermediate step in the design process to determine system performance.

Nomenclature

a_o	regression rate parameter	α	empirical consta
c_p	specific heat at constant pressure kJ/kgK	ϵ	emissivity
$\frac{dT}{dt}$	heating rate K/s	$ ho_f$	fuel density kg/
FOS	S factor of safety	σ	Stefan-Boltzman
G_{ox}	oxidizer mass flow rate kg/m^2s		
GO_2	_x gaseous oxygen		
H	enthalpy kJ/kgK		
h_g	effective heat of gasification kJ/kgK	Sub	script
\dot{m}_f	fuel mass flow rate kg/s		
m_i	mass fraction of pyrolysis product i	С	convective
Ν	p for gas systems, n, number density of particles, if	eff	effective
ther	e are radiating solids in the gas	8	gas or gaseous
п	regression rate parameter	8	liquid
\overline{p}_{c}	mean chamber pressure MPa	lit	literature
PM	MA polymethylmethacrilate	mel	melting
ŕ	regression rate mm/s	out	outside
ī	mean regression rate mm/s	ox	oxygen
\dot{Q}_c	convective heat transfer without radiation W/m^2	pyr	pyrolysis
\tilde{Q}_r	radiative heat transfer W/m^2	r	radiative
\widetilde{T}_0	ambient temperature K	SW	SolidWorks
T_r	effective radiation temperature K	vap	vaporization
T_w	surface temperature K	w	wall
t_b	burning time		
z	radiation path length m		

- constant
- ity kg/m³
- oltzmann constant

1. Introduction

Hybrid rocket motors represent a promising propulsion solution for future missions, as described in Ref. 13, 6, 5, since they are higher performance and safer than conventional solid and liquid bi-propellant propulsion systems and therefore permit reduced cost.⁴ The typical hybrid motor configuration consists of a solid fuel grain and a liquid or gaseous oxidizer stored physically separately from the fuel. The fuel has a cylinder form with one or more channels called ports hollowed out along its axis and is contained within the combustion chamber. The oxidizer is delivered to the combustion chamber through a single fluid feed system controlled by a main run valve.

In 2013, research started at the NASA Jet Propulsion Laboratory with the aim of developing a hybrid propulsion system for Solar System exploration. As part of this work, a hybrid propulsion test facility was built, making it possible to perform many hotfire tests with different propellant combinations and motor configurations in order to support preliminary design and to evaluate performance.¹⁴ Current research is focused on designing a hybrid motor that is capable of providing sufficient impulse to enable a stand-alone CubeSat/SmallSat interplanetary mission. Some details of the mission and system characteristics are summarized in Table 1. The the hybrid motor of 2016 test facility is shown in Figure 1, with the position of the three thermocouples indicated.

Table 1: CubeSat mission requirements and system characteristics. Further details are present in Ref. 14, 12.

$\Delta V [m/s]$	1000
Total mass [kg]	25
Max thrust [N]	222
Fuel outer diameter [m]	0.05
Maximum expected operating chamber pressure [MPa]	2.41
Maximum oxidizer upstream pressure [MPa]	



Figure 1: Hybrid motor with thermocouples locations.

It is essential to investigate and control thermal effects inside the hybrid rocket motor, in order to verify that adopted materials can withstand the high temperatures reached during combustion (adiabatic flame temperature 3300 K), minimize ablation of insulation and determine if, during the burn time, combined thermal and pressure stresses in the fuel grain exceed the yield strength of the fuel. This analysis will then provide the limits of safety for the motor design. CFD analysis has also been conducted concurrently, with results presented in Ref. 9.

Data have been collected from more than 30 tests and have been subsequently post-processed through an inhouse tool. Two tests will be used throughout this paper to validate the model: test 50 and test 59. These tests were selected because they present a $GO_X/PMMA$ combination, which is current baseline for the CubeSat design. Data from these reference tests are listed in Table 2.

Table 2: Data from test #50 and test #59. See Ref. 14, 12 for further information about test data. \bar{r}_{lit} is calculated using the regression rate coefficients present in literature ($a_o = 2.11 \times 10^{-5}$, n = 0.62).¹¹

Test #	Fuel	t_b [sec]	r [mm/s]	$\overline{\dot{r}}_{lit} \ [mm/s]$	\overline{p}_c [MPa]
50	clear PMMA	20	0.48	0.26	1.38
59	blackened PMMA	20	0.44	0.28	1.42

SolidWorks Simulation tool is used for this analysis. This work initially focuses on analysis of a 0.3048 m (12 inch) fuel grain (see Figure 2). A configuration with a graphite post-combustion chamber is then explored, in order to understand the changes in thermal behavior during combustion for a different motor design. Results are validated and substantiated by data from the hotfire tests carried out at JPL.



Figure 2: Basic motor configuration (test 50).

2. Thermal analysis

In the classical hybrid configuration using liquid or gaseous oxidizer, the combustion process occurs in a turbulent boundary layer through diffusive mixing between oxidizer flowing through the port and fuel evaporating from the solid surface. The flame sits within the boundary layer and is generally assumed to be a thin flame sheet. The flame zone is relatively deep in the boundary layer and the flame sheet is fed from below by vaporized fuel and from above by the port oxidizer flow. The fuel is vaporized by convective and radiative heat transfer from the flame sheet to the fuel surface. A simplified model of the hybrid combustion process is shown in Figure 3.



Figure 3: Simplified model that show boundary layer combustion, including temperature distribution in the fuel grain and the energy balance on the surface. Figure adapted from Ref. 17, 10, 19.

The model considers the solid phase regression rate to be controlled and limited by convective and radiative heat transfer to the solid grain from a relatively thin diffusion flame in a turbulent boundary layer. All chemical reactions are considered to occur very rapidly in an infinitely thin flame sheet and therefore, kinetic effects are neglected. The basic model assumes the surface responds to the heat flux instantaneously by decomposing to yield gases.¹⁵

This assumption is applicable to the steady-state case. Thus, in the steady-state case, the total heat flux \dot{Q} can be written as:

$$\dot{Q} = \dot{m}_f h_g = \rho_f \dot{r} h_g \tag{1}$$

where \dot{m}_f is the fuel mass flow rate, h_g is the effective heat of gasification of the fuel, ρ_f is the fuel density and \dot{r} is the regression rate.

Usually, in the steady state case, radiation from gas-phase products is small compared to convection and can be neglected. In the case in which this is not negligible, the radiative heat transfer, \dot{Q}_r can be defined as:

$$\dot{Q}_r = \sigma \epsilon_w (\epsilon_g T_r^4 - T_w^4) \tag{2}$$

where σ is the Stefan-Boltzmann constant, ϵ_w is the emissivity of the surface, ϵ_g is the emissivity of the gas and T_r and T_w are the effective radiation temperature and the surface temperature respectively. Alternatively, the radiative heat transfer can be considered as a percentage of convective heat transfer.⁷

Marxman and coworkers proposed a correction factor to model the total heat flux \dot{Q} ,¹⁶ taking into account the contribution of both convective, \dot{Q}_c , and radiative, \dot{Q}_r , heat transfer:

$$\dot{Q} = \dot{Q}_c \left[\left(\frac{\dot{Q}_r}{\dot{Q}_c} \right) + e^{\left(-\frac{\dot{Q}_r}{\dot{Q}_c} \right)} \right].$$
(3)

This expression considers that there is a coupling effect between the radiative flux \dot{Q}_r and the convective flux \dot{Q}_c . The radiative flux enhances the blocking effect due to blowing, whereby the vaporized fuel leaving the fuel surface decreases convective heat transfer to the surface.

2.1 Definition of the heat of gasification

The hybrid rocket modeled here uses a polymeric material as fuel. Per equation 1, it is important to define the heat of gasification for this type of material. The heat of gasification is the amount of heat required to bring one kilogram of a polymer from the initial state, usually taken at room temperature, to the pyrolytic state at the final pyrolysis temperature. It includes the energy required to raise the temperature of the polymer from the initial temperature (T_0) to the pyrolysis temperature (T_{pyr}), and to melt it. It also includes the energy needed for polymer pyrolysis and the energy required to vaporize the pyrolizing material and to raise the temperature further to the final pyrolysis temperature (T_e).¹⁰ The heat of gasification is thus strongly related to the material pyrolysis process.

Due to the heat flux from the flame to the solid fuel, the temperature of the fuel increases and a temperature profile in the fuel grain is established, characterized by a high surface temperature (T_w) at the combustion side and a much lower temperature (T_0) at the outer wall. As we can see in Figure 2, it is possible to distinguish a zone where the temperature of the material is not affected and the material remains unchanged, and a heating zone in which the temperature of the material increases. This latter can be divided into two regions:¹⁰

- a transition zone $(T_0 < T < T_{pyr})$
- a pyrolysis layer $(T_{pyr} \le T \le T_w)$.

In the transition zone only the physical properties of the material are changed, not its composition, while in the pyrolysis layer also the chemical composition of the material is changed. The transition between the pyrolysis process in the solid and gaseous phase of the fuel grain is characterized by the surface temperature, that is the temperature at which the pyrolyzing fuel is no longer in a solid or liquid state, but is transformed in a gaseous state. Once pyrolysis starts the material transforms solely to the gas phase, the front face regresses and the surface temperature remains at the final pyrolysis temperature.

The above mentioned processes can be described mathematically, allowing us to write the effective heat of gasification as:

$$h_{g} = \int_{T_{0}}^{T_{mel}} (c_{p}(T)_{s}) dT + H_{mel} + \int_{T_{mel}}^{T_{pyr}} (c_{p}(T))_{l} dT + \frac{1}{\sum_{i=1}^{n} m_{i}} \left\{ \sum_{i=1}^{n} \left[\int_{T_{pyr}}^{T_{vap}} m_{i} \left(T_{w} \frac{dT}{dt} \right) (c_{pi}(T))_{l} dT + m_{i} \left(T_{w} \frac{dT}{dt} \right) H_{vapi} + m_{i} \left(T_{w} \frac{dT}{dt} \right) H_{pyr} + \int_{T_{vapi}}^{T_{w}} m_{i} \left(T_{w} \frac{dT}{dt} \right) (c_{pi}(T))_{g} dT \right] \right\}.$$
(4)

The effective heat of gasification induces the transformation of the solid fuel into gaseous pyrolysis products at the surface temperature.

2.2 Assumptions

A number of simplifying assumptions had to be made in order to analyze the heat transfer process using SolidWorks. First of all, it is assumed that the polymeric fuel completely pyrolyzes into a gaseous monomer, which escapes at the surface and diffuses into the flame. The temperature of the gaseous fuel is assumed to be constant and equal to the surface temperature (T_w , where $700K \le T_w \le 850K$).¹⁰

Moreover, it is assumed that pyrolysis and the evaporation of the pyrolysis products only take place at the final pyrolysis temperature, in this case the surface temperature¹⁰

$$T_{pyr} = T_{vap} = T_e = T_w \tag{5}$$

According with these assumptions, relation 4 results in:

$$h_{g} = \int_{T_{0}}^{T_{mel}} (c_{p}(T))_{s} dT + H_{mel} + \int_{T_{mel}}^{T_{w}} (c_{p}(T))_{l} dT + H_{vap} + H_{pyr} = \int_{T_{0}}^{T_{w}} c_{p}(T) dT + H_{mel} + H_{vap} + H_{pyr}.$$
(6)

The first term on the the final expression accounts for the heat required to increase the temperature of a unit mass from ambient temperature up to the final pyrolysis temperature, and is referred to as the *sensible heat*, while H_{mel} , H_{vap} and H_{pyr} are respectively the latent heat of fusion, the latent heat of vaporization and the latent heat of pyrolysis. The *latent heat*, or enthalpy, is defined as the heat required to convert a unit mass of the material from an initial state at a defined temperature into a unit mass in an another state at the same temperature, i.e. it does not contribute to temperature rise.

Typically, steady-state analysis of hybrid combustion neglects radiative heat transfer for non-metalized fuels. Howewer, in non-metalized fuels, the contribution of radiation can be significant,⁷ and thus is taken into account here. To do this, we assume that the total steady-state heat flux in equation 1 can be treated as convective heat transfer,¹⁵ keeping the regression rate constant, and a percentage of the convective heat flux (1%, 20% and 40%) is considered as radiative heat transfer. This approach allows us to see the effect of different amounts of radiative heat transfer on the fuel grain stresses. The total heat transfer is then calculated through equation 3. This assumption leads to a difference in the total flux of less than 10% even for the extreme case of 40% of radiative heat transfer.

It was observed during test 50 and 59 that the regression rate of the fuel grain is significantly higher (35-45%) than that predicted using the regression rate coefficients in Ref. 11. It appeared that this increased regression rate was due to mechanical break-up of the fuel grain. Further, looking at the plume of test 50 (see Ref. 8), it appeared that much of the fuel breaking off of the surface leaves the combustion chamber without combusting.

Therefore, convective heat transfer has been calculated, for test 50, as follows:

$$\dot{Q}_c = \rho_f \left(\int_{T_0}^{T_w} c_p(T) dT \right) \dot{r}_{test} + \rho_f (H_{mel} + H_{vap} + H_{pyr}) \dot{r}_{lit}$$
(7)

where \dot{r}_{lit} is the regression rate calculated with coefficients of Ref. 11 and \dot{r}_{test} corresponds to the regression rate of test 50. In other words, we assumed that only part of the fuel is vaporized completely and only the vaporized fuel is burnt inside the combustion chamber.

Since we are interested to know the temperature reached by the fuel surface during the combustion process and SolidWorks cannot directly account for the fuel pyrolysis process, we apply the heat flux considering only the term representing the sensible heat in the equation 7, assuming that the contribution of the latent heats goes into the heat flux responsible of the material ablation ($\dot{Q}_{phase \ change}$), as represented in Figure 2.

$$\dot{Q}_{cSW} = \rho_f \left(\int_{T_0}^{T_w} c_p(T) dT \right) \dot{r}_{test}$$
(8)

The assumption used in the SolidWorks model are summarized below:

- $T_{pyr} = T_{vap} = T_e = T_w$
- $h_g = \int_{T_0}^{T_w} c_p(T) dT + H_{mel} + H_{vap} + H_{pyr}$

•
$$\dot{Q}_w = \dot{Q}_c$$

- $\dot{Q}_r = 0.01 \dot{Q}_c \text{ or } 0.2 \dot{Q}_c \text{ or } 0.4 \dot{Q}_c$
- $\dot{Q}_c = \rho_f \left(\int_{T_0}^{T_w} c_p(T) dT \right) \dot{r}_{test} + \rho_f (H_{mel} + H_{vap} + H_{pyr}) \dot{r}_{lit}$

•
$$\dot{Q}_{cSW} = \rho_f \left(\int_{T_0}^{T_w} c_p(T) dT \right) \dot{r}_{test}$$

2.3 Heat flux calculation

In order to calculate the total amount of heat flux provided by the flame, first we have to define the heat of gasification for the analyzed case.

In the equation 6, the first term contains the specific heat at constant pressure that has to be integrated over temperature. Figure 4 shows the temperature dependency of the specific heat of PMMA.



Figure 4: Specific heat as a function of temperature for PMMA. Data derived from Ref. 10

A linear extrapolation of the $c_p - T$ curve from 500 K to the final pyrolysis temperature has been performed, using data from Ref. 10, to calculate the heat of gasification. Then, in order to calculate the integral term, the curve has been split into three parts and a linear piece-wise interpolation has been realized for each one.⁸ The integral has been calculated in each of the three ranges using the respective curves of interpolation and then the three contributions have been added together. Results for the integral of c_p are collected in Table 3 and compared with Ref. 10 for validation. There is a maximum error less than 3% introduced by this interpolation on top of any errors in the original data.

T (K)	$\int_{T_0}^{T_w} c_p(T) dT \text{ (kJ/kgK)}$	$\int_{T_0}^{T_w} c_p(T) dT^{10} \text{ (kJ/kgK)}$
700	887	885
750	1024	1030
800	1165	1180
850	1310	1340

Using data from Ref. 10:

- $H_{mel} = 180 \text{ kJ/kgK}$
- $H_{vap} = 360 \text{ kJ/kgK}$
- $H_{pyr} = H_{pol} = 580 \text{ kJ/kgK}$

we obtain an effective heat of gasification that is consistent with their data:

Table 4: Heats of gasification for PMMA at different temperatures.

T (K)	$h_g (kJ/kgK)$	h_g^{10} (kJ/kgK)
700	2007	2035
750	2144	2150
800	2285	2330
850	2430	2490

The interpolated values of the heat of gasification will be used, to be consistent with all other calculations carried out during the analysis. Convective heat flux has then been calculated through equation 7, using the regression rate of test 50, r_{test} for the first term and that calculated through regression rate constants of Ref. 11, \dot{r}_{lit} , for the second term. A heat of gasification that brings the PMMA from the ambient temperature to a final pyrolysis temperature of 800 K has been used for this model, since this latter is a reasonable reference temperature, half of the range predicted by theory.¹⁰ The radiative heat transfer has been calculated, as described previously, considering different percentages of the convective heat transfer. Finally, the total heat flux provided by the flame is calculated through equation 3 and is equal to:

$$\dot{Q}_{test50} = 1.0175 \times 10^6 \text{ W/m}^2$$

This is the total heat flux transferred to the fuel surface during the ignition process, to bring it from ambient temperature to the final pyrolysis temperature, in our case 800 K with 20% of radiation. This value was found to be consistent with the results of a concurrent CFD study, see Ref. 9.

As explained in section 2.2, since we have only taken into account the term in the heat of gasification responsible for heating up the fuel, the total amount of heat flux that has to be input in SolidWorks is different from the total heat flux calculated above, in which all terms in the heat of gasification are considered.

2.4 Mesh

A curvature-based mesh has been selected for the entire model, applying different mesh controls to each component in the assembly. The mesh is finer inside the fuel grain, in which we have to resolve high temperature gradients, and is more coarse in the insulation and the combustion chamber tube, since the temperature doesn't change much in these components during the simulation. In this way, we can obtain a good compromise between accuracy and computational time. Different sizes of mesh were tested until convergent results were achieved. The center mesh of Figure 5 was chosen, as it produces similar temperature values, with a maximum relative difference less than 3% and with less computational time than the finest mesh.

2.5 Analysis

A first steady-state analysis has been performed in order to reproduce the ambient conditions that have to be inserted as initial conditions in the subsequent transient analyses. All the surfaces are initialized at the ambient temperature (300 K) and a bonded contact is used between the insulation and the combustion chamber tube, in order to permit conduction through contacting areas. This contact is used for all the thermal analyses discussed here.

The ignition process is reproduced by performing a transient thermal analysis with a very short time (~ 0.2 sec) and applying the total heat flux needed to rise the temperature from 300 K to 800 K to the inner fuel surface. As explained in section 2.2, only the sensible heat has been considered in SolidWorks. A cylinder of 1 mm of thickness has been created in order to represent the pyrolysis layer that originates inside the material during this ignition process. The real thickness of the pyrolysis layer was later determined from the results of the analysis based on temperature.



Figure 5: Temperature sensitivity to mesh density.

Convection to the ambient temperature is set as boundary condition, with a heat transfer coefficient *h* of 100 W/ m^2 K,² chosen through an iterative process to obtain a consistent result with observations. Convection to the ambient temperature is applied to the part of the combustion chamber tube exposed to the ambient air and this condition has been kept for all thermal analyses.

The total burn time for test 50 is 20 seconds. During this time, the surface regresses with a velocity equal the regression rate \dot{r} . Since SolidWorks Simulation tool is not able to simulate the regression of the surface during time, an expedient has been adopted to have a better match with the physical reality, performing a quasi steady-state analysis. The total burn time has been split into five steps of 2 seconds, 7 seconds, 3 seconds and the last 1 second. For each step, initial and final surfaces have been calculated multiplying the regression rate value (see Table 1) and the initial and final time of each analysis, and a total of five concentric cylinders (including the fuel grain) have been created (see Figure 6). This assumes a constant regression rate throughout the burn. To perform a transient analysis, initial temperatures have to be inserted or taken from a previous analysis. For the first analysis (ignition process) the initial temperatures are taken from the steady-state analysis, while for the other analyses the initial temperatures are taken from the steady-state analysis. Passing from one analysis to the subsequent one, the fuel surface that is transformed into gases is excluded. The heat flux to bring the surface at the final pyrolysis temperature has been calculated for each step through equation 3 and has been applied in the inner surface of each cylinder.

After performing all the thermal analysis, a temperature trend for the last fuel surface has been determined (see Figure 9).



Figure 6: Cylinders and surfaces considered.

3. Stress evaluation

The results of the thermal analysis are used to inform a stress analysis of the fuel. The fuel for this test was observed to burn significantly faster than the theoretical burn rate. It was conjectured that this may be due to mechanical break up of the fuel under the thermal and potentially pressure loads. A stress analysis was therefore performed in order to verify whether the fuel could break during the combustion process and, if so, determine the thickness of the mechanically compromised region.

Stresses have been calculated on the final fuel surface at ignition and at selected time steps during the burn. The process through which the mesh has been chosen is the same as that applied for thermal analysis. Constraints on cylindrical and flat faces have been applied to the model, constraining translations and rotations where appropriate. Expansion is allowed with a constraint of no penetration between cylindrical faces in contact.

Von Mises stresses have been calculated at the selected time steps, considering initially only thermal loads and applying different amounts of radiation to evaluate its influence on the stresses. Then, an interpolation has been completed to see the stresses trend with time and compared to the temperature trend on the same surface. Finally, pressure stresses have been added and the material behavior has been evaluated. However, the Von Mises equation computes the net energy stored by element distortion and returns that values as an equivalent stress without direction, so it is important to evaluate directional stresses to understand the behavior of the material in each direction. As we are considering a cylindrical model, stresses are evaluated along radial, tangential and axial directions. First of all, stresses have been evaluated considering thermal and pressure stresses present alone, then a superposition has been applied and results have been checked with SolidWorks simulating pressure and thermal stresses acting simultaneously (see Table 5).

Finally, we calculated the factor of safety, since it is important to know where applied stresses exceed the yield strength of the material, conservatively assumed to be 52 MPa.³ The factor of safety is calculated using the Von Mises criterion and the thickness in which the factor of safety is less than 1 is calculated. It is assumed that this is where the material may break up during combustion.

4. Post-combustion chamber analysis

The configuration of the motor was altered to add a post-combustion chamber, in order to give more residence time and permit the unburned fuel to burn more completely, thus improving performance. The configuration with a graphite post-combustion chamber of 0.0254 m (1 inch) inserted at the aft end of the fuel (see Figure 7) and the graphite nozzle has been investigated. The goal of this analysis was to verify the temperature reached at the interface of the steel case and the post-combustion chamber during a long burn operation. Since the reference test for this analysis uses blackened PMMA as fuel grain (see Table 1), a different total heat flux has been used, assuming that all the fuel vaporizes completely and is burnt inside the combustion chamber. This assumptions is still being verified through testing, but appears accurate for test ≤ 20 seconds (see Ref. 8). Convective heat transfer is thus calculated through equation 1 using the regression rate of test 59 and, considering 20% of radiation, the total heat flux is then calculated through equation 3. We obtained:

$$\dot{Q}_{test59} = 1.1971 \times 10^6 \ W/m^2$$

Two long burn simulations have been performed: in the first simulation, the total heat flux has been applied both in the post-combustion chamber and in the nozzle, while in the second the total heat flux has only been applied to the nozzle. This latter simulation is equivalent to assuming that an ablative post-combustion chamber material is used instead of graphite. A bonded contact has been applied between the post-combustion chamber and the combustion chamber tube, between the nozzle and the aft end cap, the nozzle and the nozzle retaining plate and the combustion chamber tube and the aft end cap.



Figure 7: Configuration with post-combustion chamber and nozzle. The highlighted area is what is considered for the post-combustion chamber analysis.

5. Results discussion

In this section results will be illustrated, discussed, and compared with theory.

5.1 Thermal results

The temperature profile inside the fuel grain at the ignition and the mean fuel surface temperature with respect to time are plotted in Figure 8. As we can see, the trend of temperature profile is similar to that expected by theory (Figure 3) and the surface temperature remains in the range predicted $(700K \le T_w \le 850K)$ during the combustion process. Moreover, the pyrolysis layer thickness has been calculated and it can be seen that it remains more or less constant during the combustion process, with a value between 0.5 mm and 1 mm (see Ref. 8). The trend of temperature during combustion on the final fuel surface is shown in Figure 9.



Figure 8: Temperature profile inside the fuel grain at ignition, at L = 0.0254 m (2 in), the location of the first thermocouple. The highlighted area is the range predicted by theory for the surface temperature.¹⁰



Figure 9: Temperature trend on fuel surface during the burn time. On the left we can see the final fuel surface, which is considered for this analysis, highlighted in blue.

Results of the two long burn simulations are shown in Figures 10 and 11. Applying the total heat flux only to the nozzle results in the steel edge reaching a temperature of ~ 600 K. Also applying the heat flux in the post-combustion chamber causes the temperature to reach a value of ~ 1100 K. Since the maximum service temperature (in air) of the AISI Type 316 Stainless Steel is 925° C (~ 1200 K)¹ for continuous service and the temperature reached is near this value, an ablative material is recommended for the post-combustion chamber during long duration burns (> 1 min). Thus, different types of material, as nylon or nyatron (glassfilled nylon), due being investigated for the post-combustion chamber, in order to achieve a temperature distribution similar to that shown in Figure 11, in which the heat flux is only applied to the nozzle. Results of the long burn simulations have been verified, using test data and looking at the temperatures reached during the first 20 seconds, as presented in Figure 12.



Figure 10: Results applying the total heat flux both in the post-combustion chamber and in the nozzle.



Figure 11: Results applying the total heat flux in the nozzle.



Figure 12: Comparison between the SolidWorks simulation and test data from test 59. The relative error between temperatures is about 3%. On the left, the location of the thermocouple on the aft end is highlighted. Test data is taken at 6 Hz, causing the stair pattern visible in the figure. Details are provided in Ref. 8.

5.2 Stress results

Von Mises stresses have been investigated considering initially only thermal loads and applying different percentages of radiative heat flux, as discussed in section 2.2. Results show that the fuel starts to break apart before it is heated to the final pyrolysis temperature. As expected, increasing the amount of radiation coming from the flame, increases the thickness of the structurally compromised layer. Figure 13 shows the area between 10 and 20 seconds, since the stresses before 10 seconds are negligible.



Figure 13: Von Mises stresses with different amount of radiation. The black line represents the yield strength of PMMA, equal to 52 MPa. The figure to the right shows the zoomed in area represented by a red box in the figure on the left.

Results for directional stresses are presented in Table 5. According to the theory of the design of ductile pressure vessels, for which pressure and thermal stresses can reinforce or oppose,¹⁸ our results show that pressure and thermal stresses reinforce in radial and axial directions, creating a more compressive stress, while in the tangential direction pressure loads create initially a compressive stress that reinforces the thermal one and after it becomes tensile, attenuating the compressive effect of the thermal stress. This mitigation effect can be seen also in the Von Mises stresses. In fact, applying simultaneously thermal and pressure loads, the total Von Mises stress are a little lower than the case in which only thermal loads are applied (see Table 6). In this case, 20% of radiation is considered.

Table 5: Results for directional stresses after 20 seconds of combustion, including a comparison of the superposition of thermal and pressure stresses. Mean values of stresses on the fuel surface are considered. Trends of directional stresses during burn time are presented in Ref. 8.

	Radial stresses [MPa]	Tangential stresses [MPa]	Axial stresses [MPa]
Thermal	-8.63	-158.8	-152.9
Pressure	-1.35	0.9162	-0.1343
Superimposition (th+pr)	-9.98	-157.88	-153.03
Solidworks	-9.99	-157.9	-153

Table 6: Effect of chamber pressure on Von Mises stresses during the last second of the burn.

Von Mises stresses		
	Thermal	Thermal+pressure
stress [MPa]	194.6	194

Finally, the factor of safety has been evaluated to verify where and how much of the fuel breaks during the combustion process. Regions under a factor of safety equal to 1 are shown in Figure 14. The case with 20% of radiation is considered.



Figure 14: Regions under FOS=1.

The thickness of the broken fuel has been calculated to be between 1.3 mm and 2 mm for the front face and between 0.5 and 1.3 mm for the aft face and the inner fuel surface (see Ref. 8). The thickness of the pyrolysis layer is assumed to be ~ 1 mm. Therefore the areas where the factor of safety is less than 1 for thickness greater than 1 mm are potentially breaking up.

6. Conclusions

A preliminary thermal analysis has been conducted to evaluate the temperatures inside the combustion chamber. Temperature profiles inside the fuel grain have been derived and the thickness of the pyrolysis layer has been calculated. A post-combustion chamber has been added and a long burn has been simulated, to verify the temperature reached by the stainless steel at the interface between the post-combustion chamber and the combustion chamber tube. These simulations results are found to compare favorably with test data. First results show that, during a long duration burn, graphite adopted in the current configuration brings the stainless steel of the combustion chamber tube near its maximum service temperature. Results presented in this paper provide a first overview of the fuel thermal behavior during the combustion process and inform the selection of appropriate materials for long combustion times. Different percentages of radiative heat flux have then been considered to evaluate the influence on stresses in the fuel grain and verify at what time during the burn the fuel breaks in the worst case (40% of radiation). Stresses in radial, tangential and axial directions have been evaluated considering both thermal and pressure loads acting simultaneously and results are in line with theory of the design of ductile vessels. The factor of safety has been calculated using the Von Mises stress criterion and regions under a factor of safety equal to 1 have been represented to evaluate the thickness in which the fuel breaks during the combustion process.

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