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Optimal Control of Spacecraft Using Electrostatic forces

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The Lorentz force acting on an electrostatically charged spacecraft may provide a useful thrust for controlling a spacecraft's orbit and attitude control. In this work, the Lorentz force has been developed for two terms, a) first term which experienced with magnetic field in the case of absolute charging of the spacecraft; b) the second term which is experienced with electric field in the case of electric charging of the spacecrfat. The orbital perturbations of a charged spacecraft by Lorentz force in the Earth's magnetic field, which is modeled as a titled dipole is investigated using the Gauss variation of the Lagrange planetary Equations. The perturbations in the orbital elements depend on the value of the charge to mass ratio (q/m). The dynamical models of the Relative motion are developed that leads to approximate analytical solutions for the motion of a charged spacecraft subject to Lorentz force. The modeled derived when the chief spacecraft's reference orbit is either circular or elliptical, and the deputy spacecraft is capable of established electrostatic charge. The numerical results show that the effects of the Lorentz force on the spacecraft are to change in track position or/and plane orbit. The results investigated the approximation of the trajectory an estimate the reachable of the Lorentz spacecraft for short time intervals with different ration (q/m) for different orbits in LEO.

Keywords: Lorentz Augmented orbit; Formation flying control; relative motion.

1. Introduction

Spacecraft charging is a naturally phenomenon which occurring in the space plasma environment. Based on the fundamental physical principle that a moving charged particle experiences the Lorentz force in a magnetic field, one can deduce that an electrostatically charged spacecraft in Earth orbit is subjected to the Lorentz force in the Earth's magnetic field. Early studies of spacecraft charging conclude that the Natural spacecraft charging level may reach to about 10-8 C/kg and the induced Lorentz force with such charging level is insufficient to perturb the orbit and attitude of satellite significantly. The concept of Lorentz-augmented

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orbits is analogous to the motion of charged dust grains in planetary magnetic fields. After the launch of artificial satellites, the phenomenon of spacecraft surface charging was discovered and found to be omnipresent, and therefore the motion of electrically charged artificial satellites affected by the Lorentz force. However, much research relating to charged spacecraft are conducted by space-plasma physicists, and the primary purpose of their research is to attenuate the hazardous electromagnetic radiation effect caused by surface charging.

Contrary to previous studies that concentrate on passive mitigation of the charge, a new concept of active application of the charge of spacecraft has been proposed by Peck (2005). Such conception spacecraft is referred to as Lorentz spacecraft, an artificially charged space vehicle that intentionally generates net charge on its surface to induce Lorentz force via interaction with the planetary magnetic field. If the charging level is several orders of magnitude larger than natural charging level or even higher, the induced Lorentz force could be utilized as propellant less electromagnetic propulsion for orbital maneuvers and attitude control.

Relative motion between a chief and a chaser spacecraft has been extensively studied over past several decades. The well-known Clohessy-Wiltshire (CW) equations [1] originally known as Hills equations [2] used to study the linearized equation of motion around the orbit of the chief satellite, which is circular and subject to the Keplerian motion only. Other models have been introduced in which the chief orbit is eccentric subject to the non-Keplerian perturbation forces [3], [4], [5], [6], [7], [8].

The Lorentz spacecraft is a nascent concept that artificially generates a net electrostatic charge on a spacecraft to provide propulsive accelerations for orbit control. Therefore, the Lorentz force can be used to change and control the orbit of the spacecraft without consuming propellant (Peck 2005). Abdel-Aziz and et al.[8-14] studied the effects of an Lorentz force on the orbital motion in Low Earth Orbit (LEO) and on the attitude control of spacecraft . Streetman & Peck (2005) investigated the Lorentz-augmented orbits and used them to accomplish a variety of complex orbital behaviors for new types of geosynchronous orbits. Pollock et al. (2011) studied the relative motion of a charged spacecraft subject to perturbations from the Lorentz force due to interactions with the planetary magnetosphere. In the present paper, the total Lorentz force is developed in two cases: (1) the Lorentz force experienced by a

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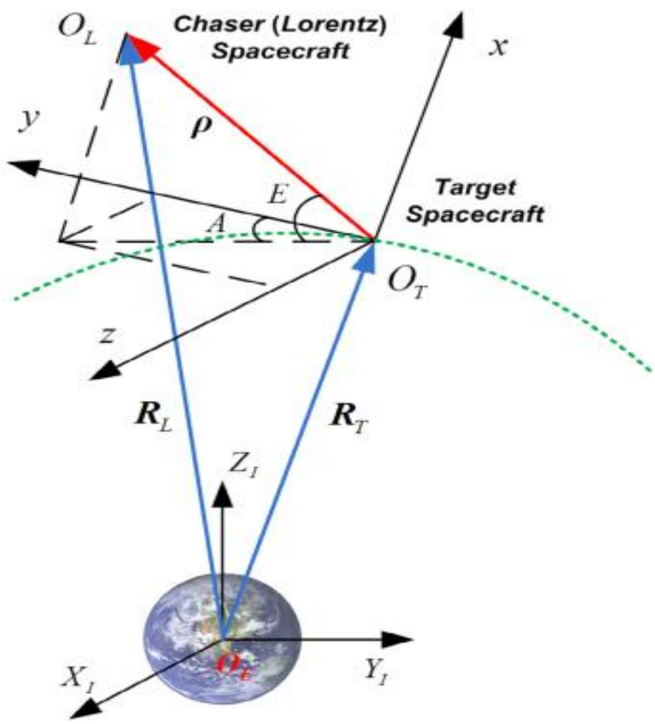
geomagnetic field and (2) the Lorentz force experienced by an electric dipole moment in the presence of an electric field. The numerical results show that the effect of the Lorentz force due to its magnetic component is greater than the effect of the Lorentz force due to its electrical component. In addition. The results confirm that the magnitude of the Lorentz force depends on the charge to mass ratio. This means we can use artificial charging to create a desired force which is needed to control the attitude and orbital motion of a spacecraft. In Section 2 we develop the relative motion of formation flying consisting of two Satellites and describes Lorentz force perturbations. Section 4 introduces numerical results, which show the effects of the Lorentz force for two different satellites.

2. Nonlinear Equations of Relative Motion

Recall the nomenclature used to distinguish the satellites: one is often called the chief, and the other is referred to as the deputy. Note that the chief satellite is not necessarily a physical object; in the case of a satellite formation, it could be a useful reference point to describe the relative motion. The relative motion equations developed in this section utilize a Cartesian local-vertical, local-horizontal (LVLH) frame attached to the chief satellite, as shown in Figure1. This coordinate frame rotates with the chief's radius vector and is a convenient reference frame to describe the relative motion[10]. This reference frame is also sometimes referred to as the Hill frame or the CW frame. In this coordinate frame, x lies in the



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chief's radial direction, z lies in the direction of the chief's orbital angular momentum, and y completes the right-handed orthogonal triad. Note that the x and y directions correspond to the in-plane motion, and z corresponds to the out-of-plane motion. In the chief's LVLH frame, the position of the deputy satellite is given by the following.

Figure 1: Local-vertical local-horizontal (LVLH) coordinate frame

$$r_d = r_c + \rho = (r_c + x)\hat{i} + y\hat{j} + z\hat{k} \tag{1}$$

The angular velocity and acceleration of the LVLH frame are given by $\dot{\theta} = \frac{h}{r_c^2}\hat{k}$ and $\ddot{\theta} = -\frac{2\dot{r}_c\dot{\theta}}{r_c}\hat{k}$ respectively. In the subsequent discussion, the subscript c will be omitted from r and f. From kinematics, the equation of motion for the deputy in the chief's frame is given by the following.

$$\ddot{r}_d = \ddot{r} + \ddot{\theta}\times\rho + \dot{\theta}\times\left(\dot{\theta}\times\rho\right) + 2\dot{\theta}\times\dot{\rho} + \ddot{\rho} \tag{2}$$

allows the chief's acceleration to be written as the following.

$$\ddot{r}_c = -\frac{\mu}{r^2} \tag{3}$$

Substituting Equation (2) into Equation (3) gives the following

$$\ddot{\mathbf{r}}_{\mathbf{d}} = \left(\frac{\mu}{r^2} - \ddot{\theta}y - \dot{\theta}^2 x - 2\dot{y}\dot{\theta} + \ddot{x}\right)\hat{\mathbf{i}} + \left(\ddot{\theta}x - \dot{\theta}^2 y + 2\dot{x}\dot{\theta} + \ddot{y}\right)\hat{\mathbf{j}} + \ddot{z}\hat{\mathbf{k}} \tag{4}$$

From Equation (4), the acceleration of the deputy is given by $\ddot{\mathbf{r}}_d = -\frac{\mu}{r_d^{\ 3}}\mathbf{r}_d$ where $\mathbf{r}_d = \left[\left(x + r\right)\ y\ \ z\right]^T$. Equating coefficients in Equation (4) gives the full nonlinear equations of relative motion (NERM)

$$\begin{aligned}
\ddot{x} &= 2\dot{y}\dot{\theta} + \ddot{\theta}y + \dot{\theta}^2 x + \frac{\mu}{r^2} - \frac{\mu}{r_d^{\ 3}}(x + r) + a_{Lx} \\
\ddot{y} &= -2\dot{x}\dot{\theta} - \ddot{\theta}x + \dot{\theta}^2 y - \frac{\mu}{r_d^{\ 3}}y + a_{Ly} \\
\ddot{z} &= -\frac{\mu}{r_d^{\ 3}}z + a_{Lz}
\end{aligned} \tag{5}$$

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$$a_L = \begin{bmatrix} a_{Lx} & a_{Ly} & a_{Lz} \end{bmatrix}^T = a_m + a_e \tag{6}$$

Where a_L is total Lorentz force, $\boldsymbol{a}_{\mathrm{m}}$ is the Lorentz force due to the geomagnetic field and $\boldsymbol{a}_{\mathrm{e}}$ is the Lorentz force.

The only assumption made in deriving Equation (6) was that both satellites obeyed Keplerian motion, i.e. the only force acting on each satellite was gravity.

3. Lorentz Augmented Orbit (LAO)

The propellant less propulsion technique discussed herein allows one to realize a Lorentz Augmented Orbit (LAO). An LAO-capable Spacecraft carries a net electrostatic charge, either an excess of electrons or ions. Such a spacecraft behaves as charged particle subject to interactions with a planetary magnetic field. We begin with a summary of the elementary electrodynamics involved. The total Lorentz force experienced by a particle of charge q (Coulombs) moving through a magnetic field B is given by

$$\mathbf{F}_L = q\left[E + \mathbf{v_r} \times \mathbf{B}\right] = qE + q(\mathbf{v_r} \times \mathbf{B}) = \mathbf{F}_e + \mathbf{F}_m \tag{7}$$

where v_r is the particle velocity with respect to the magnetic field, $\boldsymbol{F}_{\mathrm{m}}$ is the Lorentz force due to the geomagnetic

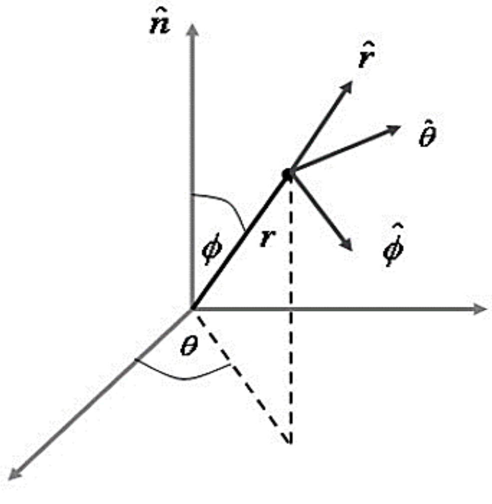

Figure 3: Spherical coordinates used in the derivation of the equations of motion.

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3.1. Equations of Motion of Lorentz force

We describe the motion of a spacecraft in an LAO in the spherical coordinate the frame shown earlier in Fig. 3 The acceleration of the spacecraft including two body gravity and the Lorentz force (per unit mass) in these (inertially referenced) coordinates is given by

$$a_m = \frac{F_L}{m} = \frac{q}{m}\left(\mathbf{v} - \omega_E \hat{\mathbf{n}} \times \mathbf{r}\right) \times B = \lambda\left(\mathbf{v} - \omega_E \hat{\mathbf{n}} \times \mathbf{r}\right) \times B \tag{8}$$

where $\frac{q}{m}$ is the charge-to-mass ratio of the satellite in Coulombs per kilogram (C/kg), and $\hat{\mathbf{n}}$ is a unit vector in the direction of the true north pole.

$$\mathbf{v} = \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} = \begin{bmatrix} \dot{r}_c + \dot{x} - y\left(\dot{\theta} - \omega_E \cos i\right) - z\,\omega_E \cos\theta \sin i \\ \dot{y} - (r_c + x)\left(\dot{\theta} - \omega_E \cos i\right) + z\,\omega_E \sin\theta \sin i \\ \dot{z} - (r_c + x)\omega_E \cos\theta \sin i + y\,\omega_E \sin\theta \sin i \end{bmatrix} \tag{9}$$

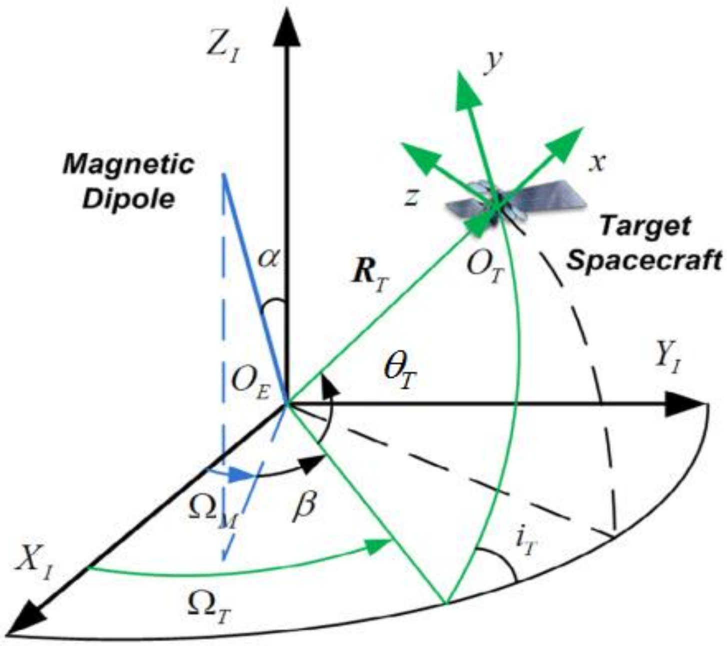

Fig. 2. Definition of angles

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Expressing the Lorentz acceleration in the spherical, inertial frame yields

$$\mathbf{F}_m = \frac{q}{m}\frac{B_0}{r^3}\begin{bmatrix} -r\sin^2\phi + \omega_E r\sin^2\phi \\ 2r\dot{\theta}\sin\phi\cos\phi - 2\omega_E r\cos\phi\sin\phi \\ \dot{r}\sin\phi - 2r\dot{\phi}\cos\phi \end{bmatrix} \tag{10}$$

Hence, by substituting Eq. (7) and Eq. (9) into Eq. (8), the expressions of Lorentz acceleration in RM frame can be derived as

$$a_m = \lambda\mathbf{l} = \lambda\begin{bmatrix} l_x & l_y & l_z \end{bmatrix}^T \tag{11}$$

$$\begin{aligned} l_x &= v_y B_z - v_z B_y, \\ l_y &= v_z B_x - v_x B_z, \\ l_z &= v_x B_y - v_y B_x. \end{aligned} \tag{12}$$

$$a_{mx} = \frac{\lambda B_0}{r^5}\begin{bmatrix} \left(\dot{y} - (r_c + x)\left(\dot{\theta} - \omega_E\cos i\right) + z\,\omega_E\sin\theta\sin i\right)\left(2z^2 - \left(x^2 + y^2\right)\right) - \\ \left(\dot{z} - (r_c + x)\omega_E\cos\theta\sin i + y\,\omega_E\sin\theta\sin i\right)3yz \end{bmatrix}$$

$$a_{my} = \frac{\lambda B_0}{r^5}\begin{bmatrix} \left(\dot{z} - (r_c + x)\omega_E\cos\theta\sin i + y\,\omega_E\sin\theta\sin i\right)3xz - \\ \left(\dot{r}_c + \dot{x} - y\left(\dot{\theta} - \omega_E\cos i\right) - z\,\omega_E\cos\theta\sin i\right)\left(2z^2 - \left(x^2 + y^2\right)\right) \end{bmatrix} \tag{13}$$

$$a_{mz} = \frac{\lambda B_0}{r^5}\begin{bmatrix} \left(\dot{r}_c + \dot{x} - y\left(\dot{\theta} - \omega_E\cos i\right) - z\,\omega_E\cos\theta\sin i\right)3yz - \\ \left(\dot{y} - (r_c + x)\left(\dot{\theta} - \omega_E\cos i\right) + z\,\omega_E\sin\theta\sin i\right)3xz \end{bmatrix}$$

According to Ulaby (2005) and Heilmann et al. (2012), we can write the electric force as the following:

$$\mathbf{F}_e = -\nabla V_e = \left(\frac{\partial v_e}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial v_e}{\partial\phi}\hat{\phi} + \frac{1}{r\sin\Theta}\frac{\partial v_e}{\partial\Theta}\hat{\mathbf{\Theta}}\right) \tag{14}$$

where V_e is the electric potential,

$$V_e = \frac{P.\hat{\mathbf{r}}}{4\pi\varepsilon_0 r^2}, \quad P = qd \tag{15}$$

where P is the electric dipole moment, d is the distance vector from charge +q to charge -q and $\varepsilon_0 = 8.85 \times 10^{-12} C^2 / \left(N - m^2\right)$ is the permittivity of free space.

Then the final form of the Lorentz force experienced by an electric dipole moment in the presence of an electric field is

$$\mathbf{F}_e = \frac{qd}{4\pi\varepsilon_0}\left(2\cos\phi\hat{\mathbf{r}} + \sin\phi\hat{\phi} + 0\hat{\mathbf{\Theta}}\right) \tag{16}$$

Hence, by substituting Eq. (14) and Eq. (16) into Eq. (16), the expressions of Lorentz acceleration in RM frame can be derived as

$$\mathbf{F}_e = \frac{qd}{4\pi\varepsilon_0}\left(3\cos\phi\sin\phi\cos\Theta\hat{\mathbf{X}} + 3\cos\phi\sin\phi\cos\Theta\hat{\mathbf{Y}} + \left(3\cos^2\phi - 1\right)\hat{\mathbf{Z}}\right) \tag{17}$$

$$\mathbf{F}_e = \frac{qd}{4\pi\varepsilon_0}\left(\frac{3xz}{r^2}\hat{\mathbf{X}} + \frac{3yz}{r^2}\hat{\mathbf{Y}} + \left(\frac{3z^2}{r^2} - 1\right)\hat{\mathbf{Z}}\right) \tag{18}$$

$$a_e = \begin{bmatrix} a_{ex} & a_{ey} & a_{ez} \end{bmatrix}^T = \frac{\mathbf{F}_e}{m} = \frac{q}{m}\frac{d}{4\pi\varepsilon_0 r^5}\begin{pmatrix} 3xz \\ 3yz \\ 2z^2 - x^2 - y^2 \end{pmatrix} \tag{19}$$

4. NUMERICAL RESULTS

In this section, we consider the numerical simulation for verification of the derived perturbations in the orbital motion of a spacecraft due to the Lorentz force, using Equations (5), (13) and (19). We can apply those equations to get the perturbation in the separate magnetic and electric components of the Lorentz force. These numerical simulations were performed using MATLAB©. The nonlinear differential equations of motion were solved using 4th/5th order Runge-Kutta method. We note that the components of the Lorentz force due to the magnetic and electric fields are proportional to the charge to mass ratio. This means that with this ratio we can control the orbital motion of a satellite.

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4.1 Spacecraft Physical Parameters

In order to perform numerical simulations, values of spacecraft physicals. For this study take two actual formation flying designs, The *GRACE* twin *satellites*, launched 17 March 2002, are making detailed measurements of Earth's gravity field changes , using program code to compute the orbital elements by MATLAB© packages from initial values TLE (Tow Line Element) for satellites, these parameters will now be presented in table (1) .

Table (1) Spacecraft Physical Parameters

GRACE 1 1 27391U 02012A 13004.06903455 .00003939 00000-0 92932-4 0 2814 2 27391 089.0194 188.7270 0016414 086.1402 274.1671 15.41455555604430		
GRACE 2 1 27392U 02012B 13004.00374406 .00004041 00000-0 95309-4 0 2835 2 27392 089.0168 188.7633 0017245 087.8210 272.5036 15.41454357604428		
Name Satellite	**GRACE 1**	**GRACE 2**
Inclination (deg)	89.0194	89.0168
Eccentricity	0.0016414	0.0017245
Ascending node (deg)	188.7270	188.7633
Argument of perigee (deg)	86.1402	87.8210
Mean motion (rev /day)	15.41455556	15.41454358
Mean anomaly (deg)	274.1671	272.5036
Semi major axis (km)	6819.95385	6819.9573847
r (km)	6819.158694091	6819.46388
r (vector km)	[-6740.15 -1034.88 14.24]	[-6739.798 -1039.21 15.13]
v (vector km/s)	[0.048 -0.125 7.644]	[0.049 -0.125 7.644]
Relative motion (km)	[0.259 0.963 -4.295]	
Norm relative (km)	4.4093197	

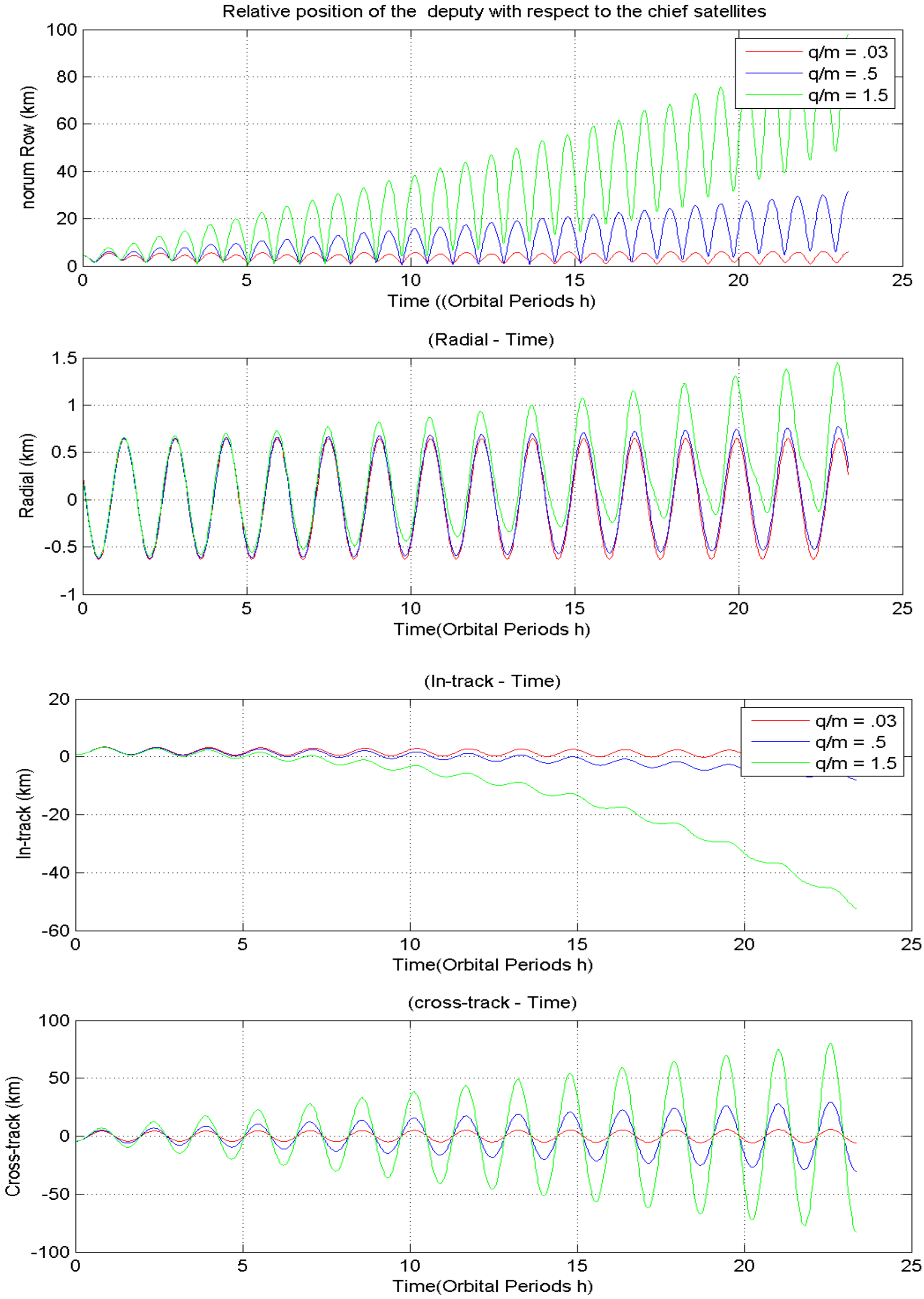

Figure (4): compare relative position of two Satellites at three different value of the charge to mass ratio (q/m) for magnetic case

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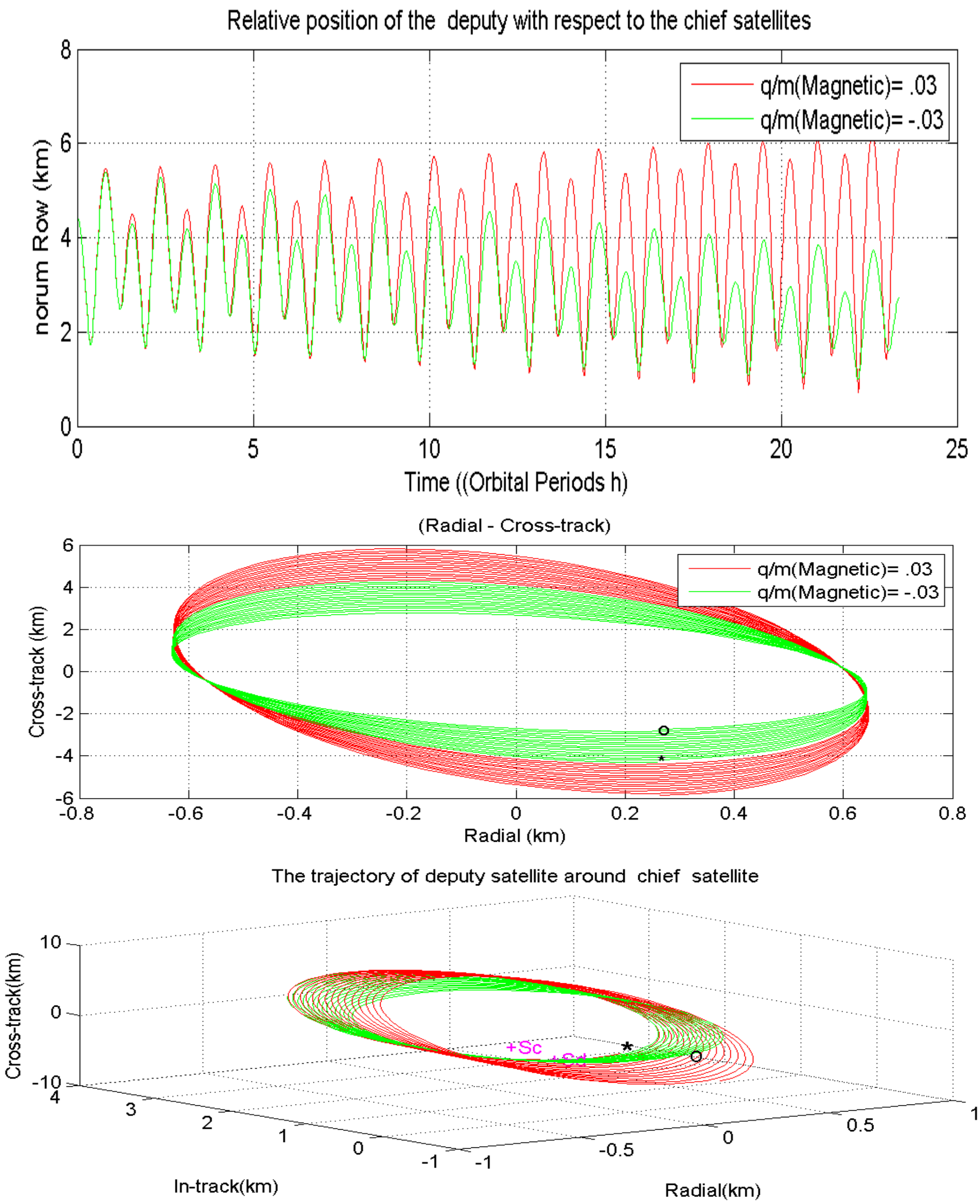

Figure (5): compare positive and negative value of the charge to mass ratio

q/m (0.03) and (-0.03) and dipole magnetic ($\alpha = 0°$)

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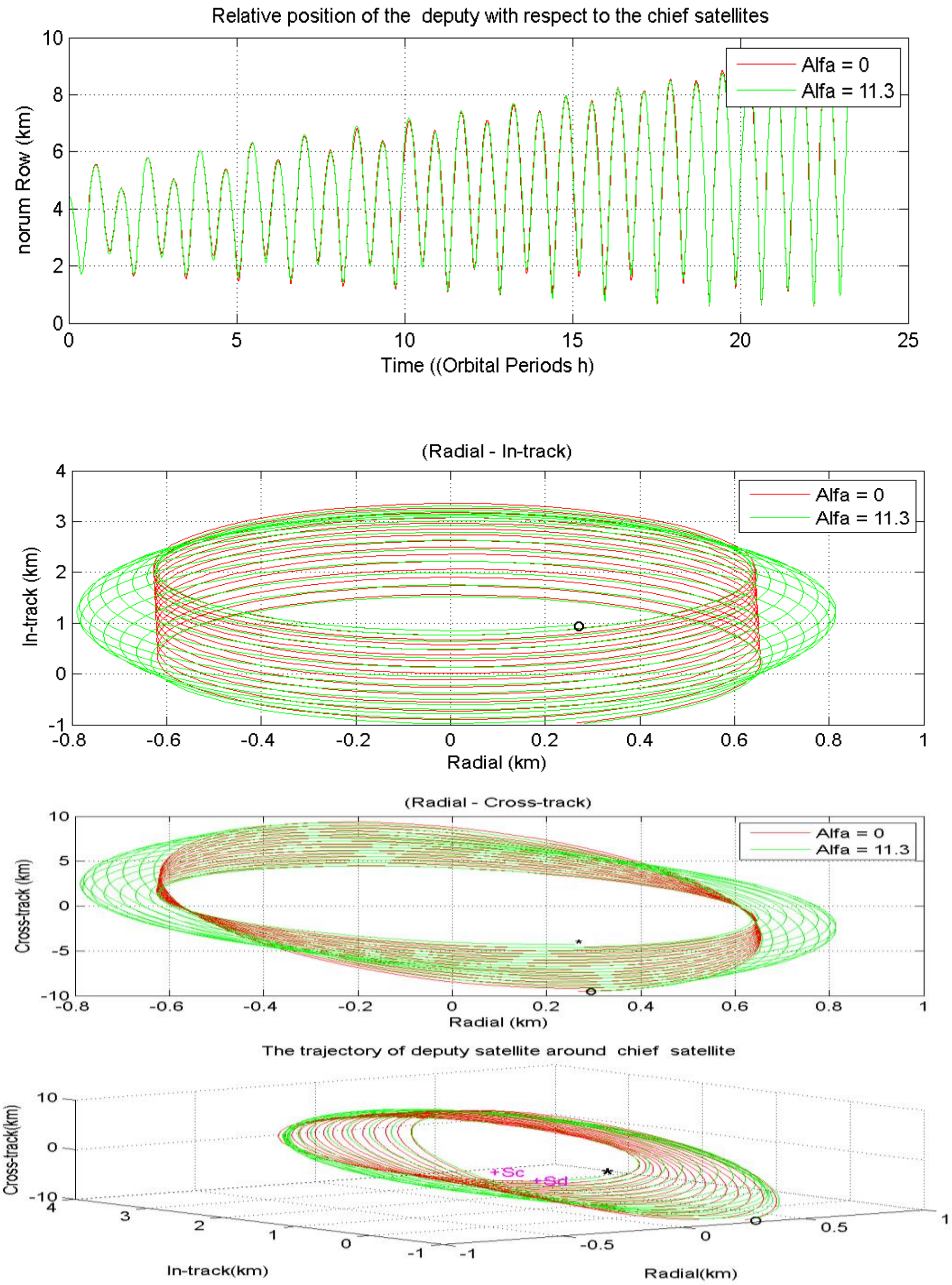

Figure (6): change in the relative position of formation flying at q/m (0.1) and dipole magnetic ($\alpha = 0° - 11.3°$)

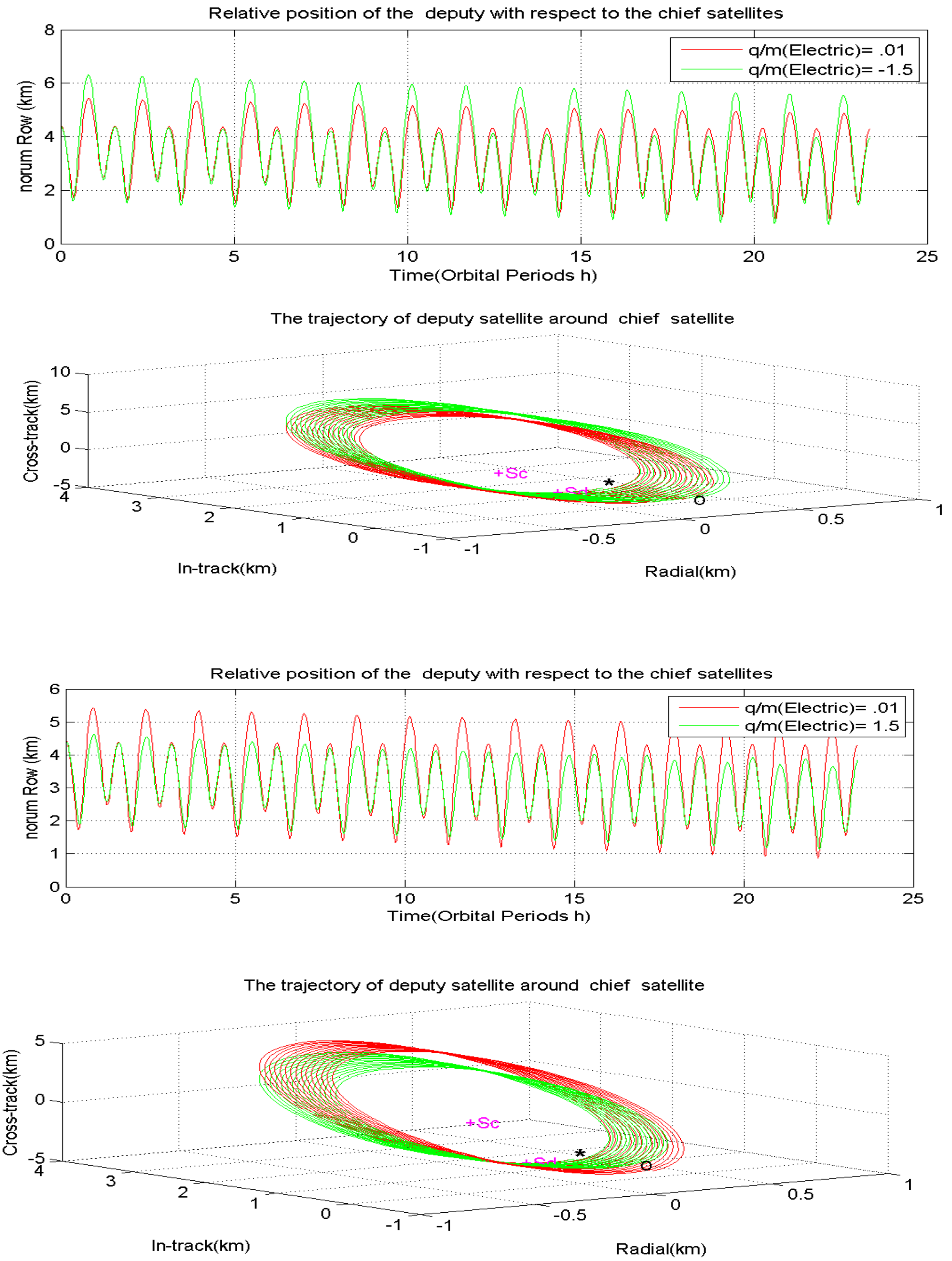

Figure (7): compare positive and negative value of the charge to mass ratio q/m (0.01), (-1.5) and (1.5) for electric case.

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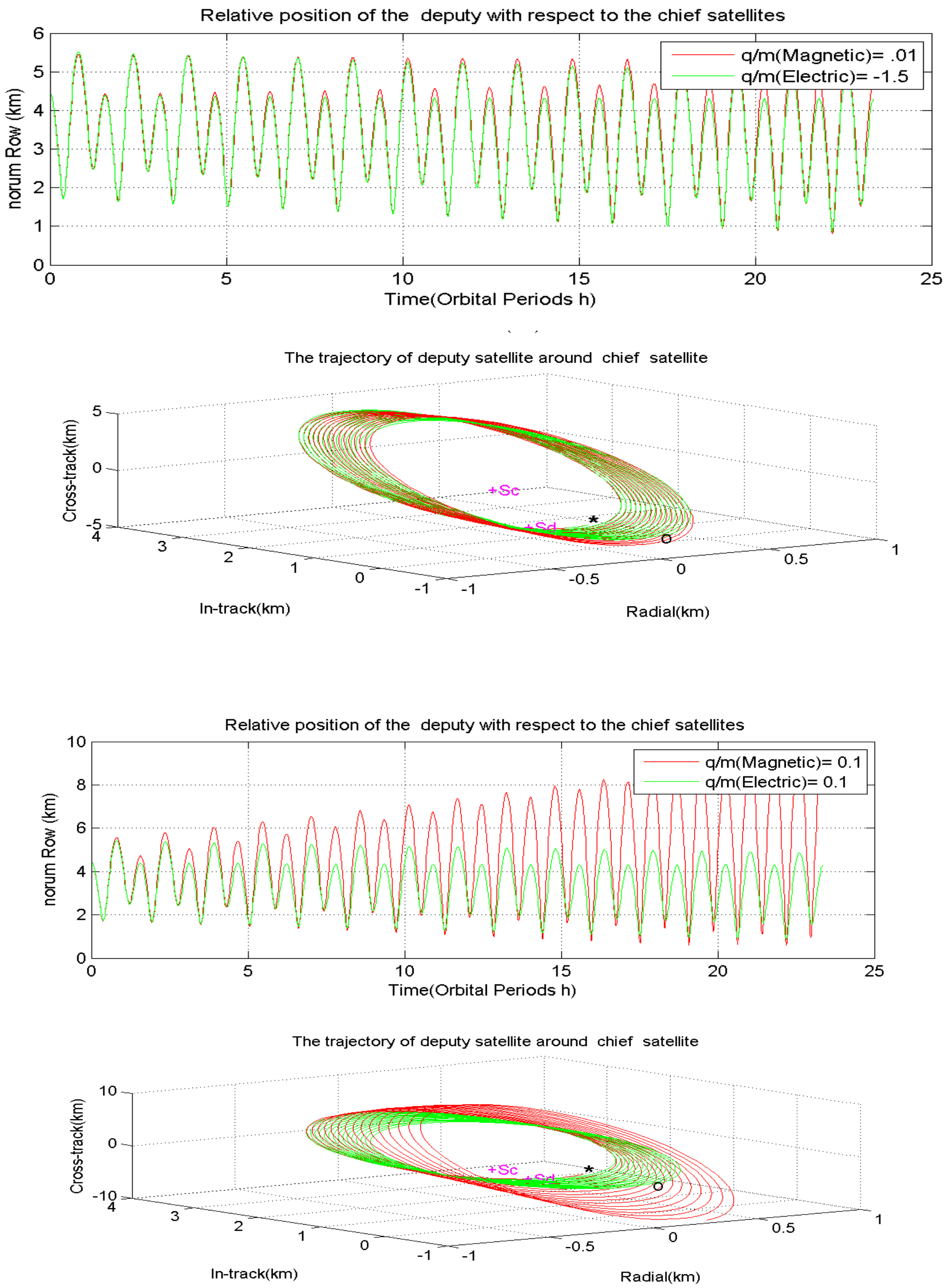

Figure (8): compare magnetic and Electric effects on relative position

at q/m (0.1), and study optimal case

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The results shown in Figure (4): compare three different value of the charge to mass ratio q/m (0.03),(0.5), and (1.5) C/kg for magnetic field only and dipole magnetic ($\alpha = 0°$), and 24 hour period These results are representative of magnetic force effects in the relative position of formation flying consisting of two LEO Satellites, It should be noted that there the change of Redial and cross track are increase after 10 hour for q/m (1.5) but change of value (0.5)is small but value is decrease in- track coordinate .

The total effects of norm relative position at q/m =1.5 about 80 km but decrease to half value at q/m= 0.5 and very small value at the Natural charging q/m = .03c/kg.

In Figure (5) compare positive and negative value of the charge to mass ratio q/m (0.03), and (-0.03) C/kg for magnetic field only and dipole magnetic ($\alpha = 0°$), and 24 hour period from these case It should be noted that the change of total effects of norm relative position at positive q/m same negative q/m but increasing in case positive and decrease in case negative value.

Figure (6) study case of change in the relative position of formation flying of two Satellites at constant value q/m (0.1) and change dipole magnetic α (0°)and (11.3°), It should be noted that there no visible effect on norm relative position but visible on the in-plane (x-y) motion, out-plane (z-y)motion and trajectory motion in this case.

Figure (7) study different value of the charge to mass ratio q/m (0.01), (1.5), and (-1.5) C/kg for electric field only for 24 hour period. It should be noted that effect of electric force at q/m = .01 is very small and increasing with negative value of q/m but value decreasing with positive q/m, and total effect on one day about 2 km.

In Figure (8) study two case first compare the charge to mass ratio q/m (0.01) C/kg for magnetic field with the same value of electric filed the results show that after 24 hour period effect of electric case 500 m but magnetic 2km, second case study the optimal value of q/m to electric field (-1.5) c/kg to equal q/m (.01) in case magnetic field.

Conclusion

The Lorentz acceleration has been developed for two terms, a) first term which experienced with magnetic field in the case of absolute charging of the spacecraft; b) the second term which is experienced with electric field in the case of electric charging of the spacecraft. We have checked the effects of the Lorentz fore on relative motion of chaser spacecraft. The results confirm that

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charge to mass ratio q/m can play the control key for correction in the drift of relative position. In the future work we are going to use feedback control for optimal control of spacecraft formation flying.

The modeled derived when the chief spacecraft's reference orbit is either circular or elliptical, and the deputy spacecraft is capable of established electrostatic charge. The numerical results show that the effects of the Lorentz force on the spacecraft have a significant change in cross and along track position or/and plane orbit. In the other hand results investigated the trajectory an estimate the reachable of the Lorentz spacecraft for short time intervals with different ration (q/m) in case of magnetic or electric part of Lorentz acceleration and the comparable trajectory depend on the optimal q/m for electric part to reach the effects of magnetic part of Lorentz force.

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