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Dynamical evolution of space debris in vicinity of GNSS regions

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Abstract
Dynamical evolution of space debris in a vicinity of orbits of the global navigation satellite systems: GLONASS, GPS, BeiDou, Galileo was studied for 240 years. The time intervals to space debris objects get into the region of navigation satellites motion were estimated. Initial orbits of the objects were choosing in the GNSS orbit regions as well as 500 km above and below with respect to nominal semi-major axes of the navigation orbits. The results significantly depend on orbital plane position. The objects pass through the GNSS region due to long period oscillations of eccentricities and inclinations of orbits, which are produced by inclination dependent luni-solar resonances and the radiation pressure.

1. Introduction

The long-term dynamics of space debris need to be investigated to provide a good estimate a satellite lifetime and to ensure the stability of a disposal orbit through centuries. The global navigation satellite systems (GNSS): GLONASS, GPS, BeiDou, Galileo consist of 24 and more controlled satellites of each. All systems have large inclinations, what lead to necessary to take into account the Lidov-Kozai effect for long-time dynamical evolution. Coordination of disposal strategies becomes a common task and a general problem.

The dynamical evolution of satellites in the Medium Earth Orbits (MEO) region is complex due to inclination dependent luni-solar resonances. This was the subject of a number of previous papers, for instance [1], [5], [7]–[9]. Chao and Gick [5] used analytical approximations through doubly-averaged equations reveal that the cause is due to the resonance induced by Sun/Moon and the Earth's oblateness secular perturbations. They propagated dynamical evolution of a total of 105 non-operational GLONASS satellites and upper stages and 10 GPS/Block I satellites for 200 years and showed that the GLONASS satellites will start to enter the operating GPS constellation after 40 years. Strategies to minimize the significant eccentricity growth was studied.

Rossi [9] for the MEO region considered two possible disposal strategies involving either stable or unstable orbits from the eccentricity growth point of view. Anselmo and Pardini [1] investigated long lifetime orbits for high area to mass ratio debris released into GPS orbits. It showed that the lifetime can exceed 100 years up to area to mass ratio 45 m^2/kg, decreasing rapidly to a few months above such a threshold. Deleflie et al. [7] studied the stability of disposal orbits and defined areas in space where disposal orbits should not be located, if the eccentricity has to be kept as a minimum.

Kuznetsov et al. [8] considered light pressure effect on satellites moving in vicinity low-order resonances for different area to mass ratios from small corresponding satellites to high corresponding space debris. Secular perturbations of semimajor axes of orbits, caused by the Poynting–Robertson effect, are estimated in the neighborhood of low-order resonance zones at different area-to-mass ratios.

Daquin et al. [6] present analytical and semi-analytical models that accurately reflect the true nature of the resonant interactions, and trace the topological organization of the manifolds on which the chaotic motions take place in the MEO region.

In the present paper, the dynamical evolution of space debris in a vicinity of orbits of the GNSS (GLONASS, GPS, BeiDou, Galileo) was studied for 240 years. The aim of the research was estimation of time intervals which necessary to space debris objects get into the region of navigation satellites motion.

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2. Methods

Initial conditions for the objects are choosing in the GNSS orbit regions (GLONASS, GPS, BeiDou, Galileo) as well as 500 km above and below with respect to nominal semi-major axes of the navigation orbits. Initial data correspond near circular orbits with the eccentricity e = 0.001. Initial inclination depends on the navigation system. The dynamical evolution is propagated for 240 years. Initial moment T_0 is $00^h\ 00^m\ 00^s$ UTC 21.03.1958. Initial values of the longitude of ascending node Ω are equal to 0, 90, 180, and 270 degrees. The argument of pericenter g = 0.45804°. The pericenter is directed toward the Sun when Ω = 0°. An area to mass ratio γ is varied from small values corresponding to the satellites γ = 0.02 m^2/kg to high values which correspond to the space debris.

The orbital evolution of objects is modeled in the "Numerical Model of Motion of Artificial Satellites" [2], [3] developed at the Tomsk State University. The model of perturbing forces takes into account the major perturbing factors: the gravitational field of the Earth (EGM96 model, harmonics up to the 27th order and degree, inclusive), the gravitation of the Moon and the Sun, the tides of the Earth, the direct radiation pressure taking into consideration the shadow of the Earth (coefficient of reflection of the satellite surface k = 1.44), the Poynting–Robertson effect, and the atmospheric drag. The equations of motion are integrated by the Everhart's method of the 19th order.

3. Results

The results depend on orbital plane position significantly. The objects pass through the region of navigation satellites motion due to long period oscillations of eccentricities and inclinations of orbits which are produced by the inclination dependent luni-solar resonances and the radiation pressure.

Tables 1–8 give interval (in years) after the initial moment T_0 when an object with area to mass ratio γ the first-time pass through the navigation systems regions (GLONASS, GPS, BeiDou, Galileo) or the geostationary orbit (GEO) region for initial value of the longitude of ascending node Ω.

Table 1 presents results of numerical simulation for initial conditions corresponding the GLONASS satellite near the 8:17 resonance region. The initial value of the semi-major axis is 25 509 km. The initial inclination is 64.8 degrees. The object does not pass any regions of navigation satellites motion for the initial value of the longitude of ascending node is 90° and 180° and the area to mass ratio less than 0.02 m^2/kg. The initial Ω = 0° and the area to mass ratio γ = 0.02 m^2/kg lead to pass through the GPS, BeiDou and Galileo regions after 149, 177 and 192 years correspondently. Increase in the area to mass ratio leads to decrease of time intervals before passing through the GNSS regions. The passages through the GEO region are started for γ = 0.8 m^2/kg and the initial Ω = 270°. The initial Ω = 0° permits orbital evolution during 240 years with maximal value of the area to mass ratio 43 m^2/kg.

Evolution of the orbital eccentricity depends on the initial value of the longitude of ascending node. Figure 1 shows evolution of the eccentricity for the area to mass ratio 0.02 m^2/kg and the initial value of the semi-major axis 25 509 km near the GLONASS region. Variations of the eccentricity are minimal for the initial Ω = 180° and maximal for the initial Ω = 0°.

Figure 2 presents evolution of the distances in pericenter r_p and apocenter r_a for γ = 0.02 m^2/kg and a = 25 509 km near the GLONASS region. Behavior of the distance in apocenter is like the eccentricity because mean value of semi-major axis is constant. Variations of the distances r_p and r_a are minimal for the initial Ω = 180° and maximal for the initial Ω = 0°. The pericenter is directed toward the Sun when Ω = 0° and the apocenter is directed toward the Sun when Ω = 180°.

Table 2 gives results for the object is above the GLONASS orbit near the 14:29 resonance region. The initial value of the semi-major axis is 25 947 km. The initial inclination is 64.8 degrees. The object does not pass any regions of navigation satellites motion for the initial value of the longitude of ascending node is 180° and area to mass ratio less than 0.02 m^2/kg. The initial Ω = 0° leads to pass through the GLONASS, GPS, BeiDou and Galileo regions after 109, 124, 160 and 181 years correspondently. The passages through the GEO region are started for γ = 1 m^2/kg and the initial Ω = 90°. The initial Ω = 180° permits orbital evolution during 240 years with maximal value of the area to mass ratio 76 m^2/kg.

Figure 3 shows evolution of the distances in pericenter r_p and apocenter r_a for the area to mass ratio 0.02 m^2/kg and the initial value of the semi-major axis 25 947 km. It is above the GLONASS orbit near the 14:29 resonance region. Variations of the distances r_p and r_a are minimal for the initial Ω = 180° and maximal for the initial Ω = 0°.

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Table 1: Intervals for the first time of passage through the navigation systems regions or the GEO region for the GLONASS satellite

γ, m^2/kg	Ω, deg	Time, years			
		GPS	BeiDou	Galileo	GEO
0.02	0.00	148.86	176.51	192.17	—
38.00	0.00	0.03	0.05	0.08	9.20
43.00	0.00	0.03	0.05	0.05	7.28
0.02	90.00	—	—	—	—
0.03	90.00	221.66	—	—	—
0.07	90.00	189.82	238.96	—	—
0.20	90.00	134.67	186.75	213.94	—
5.00	90.00	0.22	0.38	32.20	213.31
12.00	90.00	0.11	0.22	0.30	50.51
0.02	180.00	—	—	—	—
4.00	180.00	0.16	—	—	—
8.00	180.00	0.08	0.25	—	—
14.00	180.00	0.05	0.11	0.22	—
20.00	180.00	0.05	0.08	0.14	—
0.02	270.00	221.14	—	—	—
0.05	270.00	207.12	239.95	—	—
0.07	270.00	175.96	207.89	231.79	—
0.80	270.00	45.72	84.38	106.64	213.66
8.00	270.00	0.14	0.22	0.36	63.08

Table 3 presents results of numerical simulation for initial conditions corresponding the GPS satellite near the 1:2 resonance region. The initial value of the semi-major axis is 26 562 km. The initial inclination is 55 degrees. The object does not pass any regions of navigation satellites motion for the initial value of the longitude of ascending node is 0° and the area to mass ratio less than 0.02 m^2/kg. The initial Ω = 90° and the area to mass ratio γ = 0.02 m^2/kg lead to pass through the GLONASS, BeiDou and Galileo regions after 73, 95 and 138 years correspondently. The passages through the GEO region are started for γ = 0.02 m^2/kg and the initial Ω = 180°. The initial Ω = 0° permits orbital evolution during 240 years with maximal value of the area to mass ratio 37 m^2/kg.

Figure 4 shows evolution of the distances in pericenter r_p and apocenter r_a for the area to mass ratio 0.02 m^2/kg and the initial value of the semi-major axis 26 562 km near the GPS region. Variations of the distances r_p and r_a are minimal for the initial Ω = 0° and maximal for the initial Ω = 180°.

Table 4 gives results for the object is above the GPS orbit near the 15:29 resonance region. The initial value of the semi-major axis is 27 168 km. The initial inclination is 55 degrees. The object does not pass any regions of navigation satellites motion for the initial value of the longitude of ascending node is 0 degrees and area to mass ratio less than 0.2 m^2/kg. The initial value of the longitude of ascending node 180 degrees and area to mass ratio 0.02 m^2/kg lead to pass through the GLONASS, GPS, BeiDou and Galileo regions after 194, 159, 163 and 216 years correspondently. When the area to mass ratio is 1 m^2/kg, time intervals 75, 0.4, 21 and 98 years correspond to pass through the GLONASS, GPS, BeiDou and Galileo regions. The passages through the GEO region are started for γ = 0.02 m^2/kg and the initial Ω = 90° and 180°. The initial Ω = 0° permits orbital evolution during 240 years with maximal value of the area to mass ratio 37 m^2/kg.

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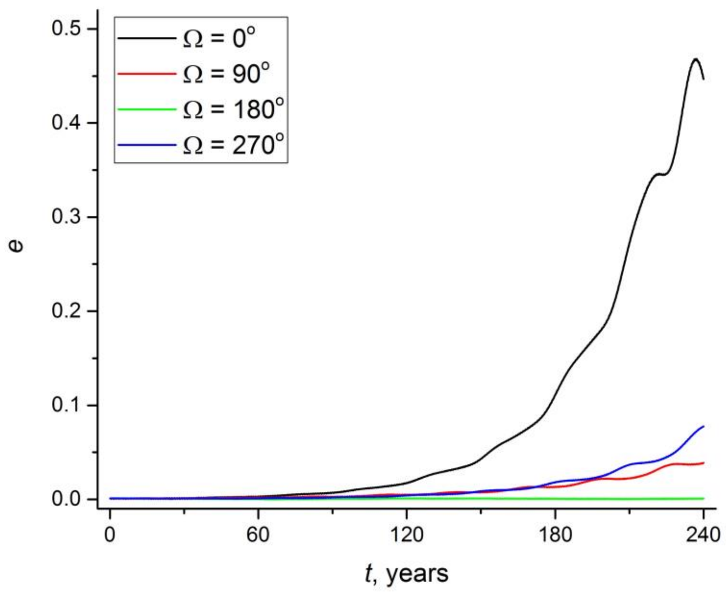

Figure 1: Evolution of the orbit eccentricity e of the GLONASS satellite depends on initial value of the longitude of ascending node **Ω** for area to mass ratio $\gamma = 0.02$ m^2/kg (a = 25 509 km, i = 64.8°).

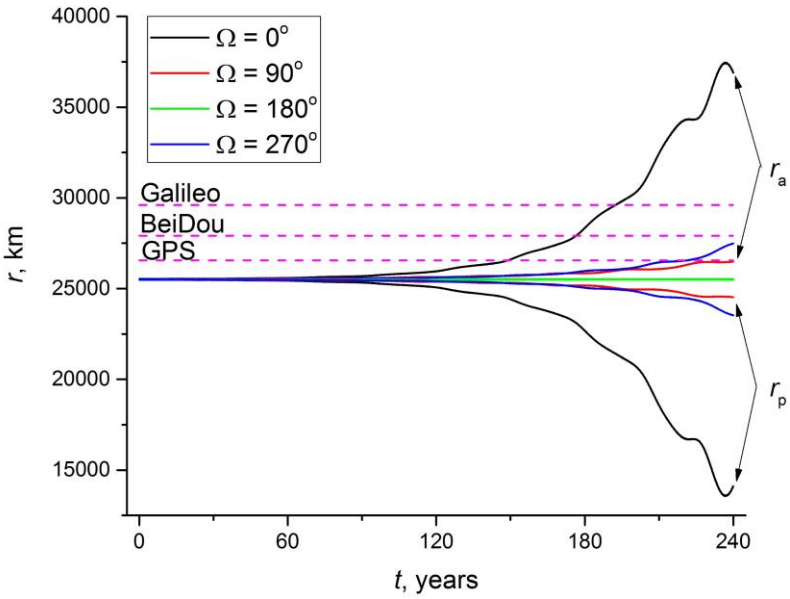

Figure 2: Evolution of the distances in pericenter r_p and apocenter r_a of the GLONASS satellite depends on initial value of the longitude of ascending node **Ω** for area to mass ratio $\gamma = 0.02$ m^2/kg (a = 25 509 km, i = 64.8°).

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Table 2: Intervals for the first time of passage through the navigation systems regions or the GEO region for a satellite above the GLONASS orbit near the 14:29 resonance region (a = 25 947 km, i = 64.8°)

γ, m²/kg	Ω, deg	Time, years				
		GLONASS	GPS	BeiDou	Galileo	GEO
0.02	0.00	109.38	123.78	159.67	181.46	—
35.00	0.00	0.03	0.03	0.05	0.05	9.36
43.00	0.00	0.03	0.03	0.03	0.05	3.07
0.02	90.00	145.90	167.89	223.00	—	—
0.20	90.00	75.92	93.20	142.53	170.73	—
1.00	90.00	0.36	0.52	74.91	104.89	214.07
13.00	90.00	0.05	0.08	0.16	0.25	41.15
0.02	180.00	—	—	—	—	—
7.00	180.00	0.05	0.05	0.19	—	—
12.00	180.00	0.03	0.03	0.11	0.22	—
63.00	180.00	0.03	0.03	0.03	0.05	1.37
76.00	180.00	0.03	0.03	0.03	0.03	0.16
0.02	270.00	153.79	164.27	212.65	236.44	—
7.00	270.00	0.08	0.08	0.22	0.36	57.77
10.00	270.00	0.05	0.08	0.16	0.25	50.73

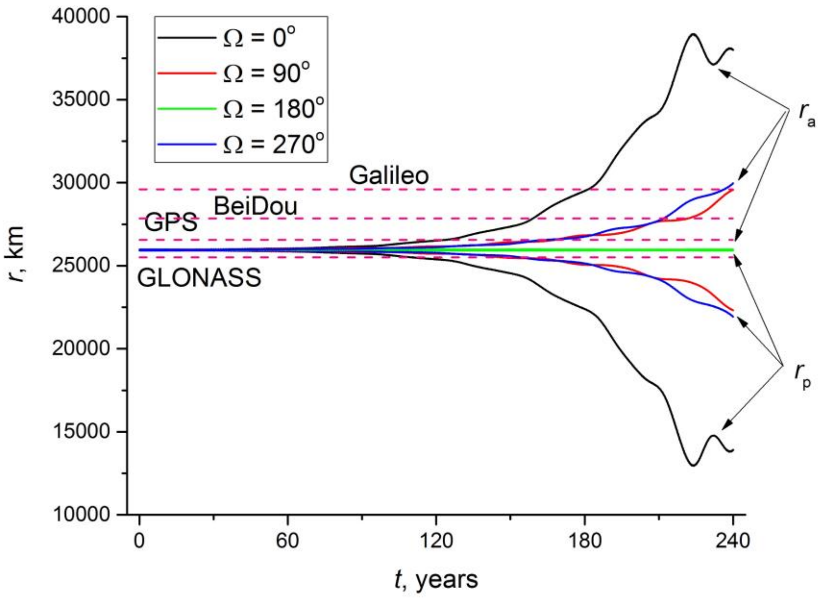

Figure 3: Evolution of the distances in pericenter r_p and apocenter r_a for a satellite above the GLONASS orbit near the 14:29 resonance region depends on initial value of the longitude of ascending node Ω for area to mass ratio γ = 0.02 m²/kg (a = 25 947 km, i = 64.8°).

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Table 3: Intervals for the first time of passage through the navigation systems regions or the GEO region for the GPS satellite

γ, m^2/kg	Ω, deg	Time, years			
		GLONASS	BeiDou	Galileo	GEO
0.02	0.00	—	—	—	—
2.00	0.00	0.41	201.18	—	—
5.00	0.00	0.11	0.16	5.45	—
28.00	0.00	0.03	0.03	0.05	36.17
37.00	0.00	0.03	0.03	0.05	0.30
0.02	90.00	73.02	95.14	138.18	—
4.00	90.00	0.22	0.27	17.25	123.56
0.02	180.00	104.70	122.87	154.44	235.73
18.00	180.00	0.05	0.05	0.11	—
0.02	270.00	207.31	224.15	—	—
0.30	270.00	180.53	199.23	229.51	—
4.00	270.00	0.16	0.22	76.74	—

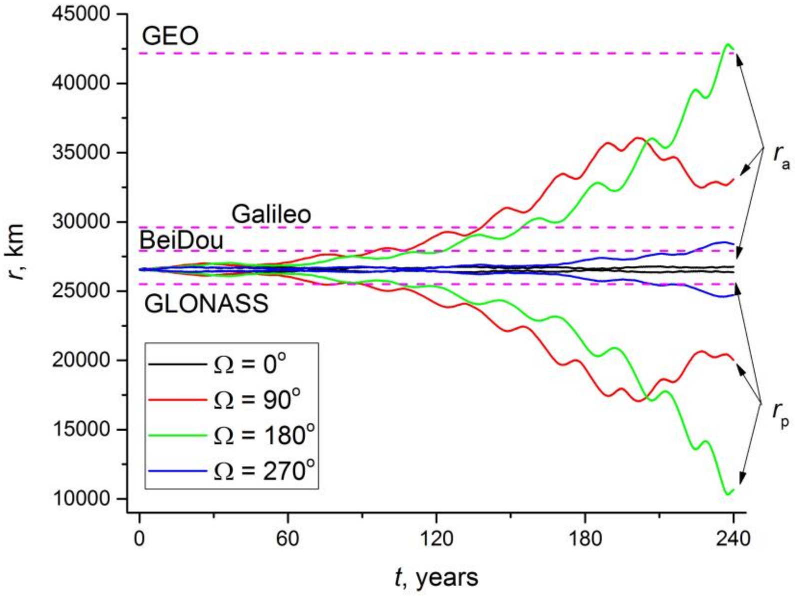

Figure 4: Evolution of the orbit distances in pericenter r_p and apocenter r_a of the GPS satellite depends on initial value of the longitude of ascending node Ω for area to mass ratio $\gamma = 0.02$ m^2/kg (a = 26 562 km, i = 55°).

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Table 4: Intervals for the first time of passage through the navigation systems regions or the GEO region for a satellite above the GPS orbit near the 15:29 resonance region (a = 27 168 km, i = 55°)

γ, m²/kg	Ω, deg	Time, years				
		GLONASS	GPS	BeiDou	Galileo	GEO
0.02	0.00	—	—	—	—	—
1.00	0.00	—	3.53	—	—	—
2.00	0.00	—	0.16	0.22	—	—
3.00	0.00	0.41	0.11	0.14	—	—
26.00	0.00	0.05	0.03	0.03	0.05	4.27
37.00	0.00	0.03	0.03	0.03	0.05	0.25
0.02	90.00	—	205.48	210.57	—	—
0.20	90.00	156.47	104.50	122.30	178.21	—
1.00	90.00	75.13	0.44	21.16	98.73	222.67
2.00	90.00	25.05	0.25	0.27	52.51	173.74
0.02	180.00	193.95	159.10	162.82	215.66	—
1.00	180.00	137.19	62.45	86.05	160.74	239.73
12.00	180.00	0.08	0.03	0.05	0.11	77.23
0.02	270.00	187.00	136.95	141.68	196.96	—
3.00	270.00	0.33	0.14	0.16	84.71	202.03

Figure 5 shows evolution of the distances in pericenter r_p and apocenter r_a for the area to mass ratio 0.02 m²/kg and the initial value of the semi-major axis 27 168 km. It is above the GPS orbit near the 15:29 resonance region. Variations of the distances r_p and r_a are minimal for the initial Ω = 0° and maximal for the initial Ω = 180°.

Table 5 presents results of numerical simulation for initial conditions corresponding the BeiDou satellite near the 7:13 resonance region. The initial value of the semi-major axis is 27 907 km. The initial inclination is 55 degrees. The object does not pass any regions of navigation satellites motion for the initial value of the longitude of ascending node is 0° and the area to mass ratio less than 0.02 m²/kg. The initial Ω = 90° and the area to mass ratio γ = 0.02 m²/kg lead to pass through the GLONASS, GPS and Galileo regions after 73, 95 and 138 years correspondently. The passages through the GEO region are started for γ = 1 m²/kg and the initial Ω = 90° and 180°. The initial Ω = 0° permits orbital evolution during 240 years with maximal value of the area to mass ratio 37 m²/kg.

Figure 6 shows evolution of the distances in pericenter r_p and apocenter r_a for the area to mass ratio 0.02 m²/kg and the initial value of the semi-major axis 27 907 km near the BeiDou region. Variations of the distances r_p and r_a are minimal for the initial Ω = 0° and maximal for the initial Ω = 270°.

Table 6 gives results for the object is above the BeiDou orbit near the 4:7 resonance region. The initial value of the semi-major axis is 29 034 km. The initial inclination is 55 degrees. The object does not pass any regions of navigation satellites motion for the initial values of the longitude of ascending node are 0 and 180 degrees and area to mass ratio is less than 0.2 m²/kg. The initial value of the longitude of ascending node 90° and area to mass ratio 0.02 m²/kg lead to pass through the BeiDou and Galileo regions after 221 and 190 years correspondently. The initial value of the longitude of ascending node 270 degrees and area to mass ratio 0.02 m²/kg lead to pass through the GLONASS, GPS, BeiDou and Galileo regions after 195, 173, 140 and 110 years correspondently. The passages through the GEO region are started for γ = 0.2 m²/kg and the initial Ω = 90°. The initial Ω = 0° permits orbital evolution during 240 years with maximal value of the area to mass ratio 36 m²/kg.

Figure 7 shows evolution of the distances in pericenter r_p and apocenter r_a for the area to mass ratio 0.02 m²/kg and the initial value of the semi-major axis 27 168 km. It is above the BeiDou orbit near the 15:29 resonance region. Variations of the distances r_p and r_a are minimal for the initial Ω = 0° and 180°, and maximal for the initial Ω = 270°.

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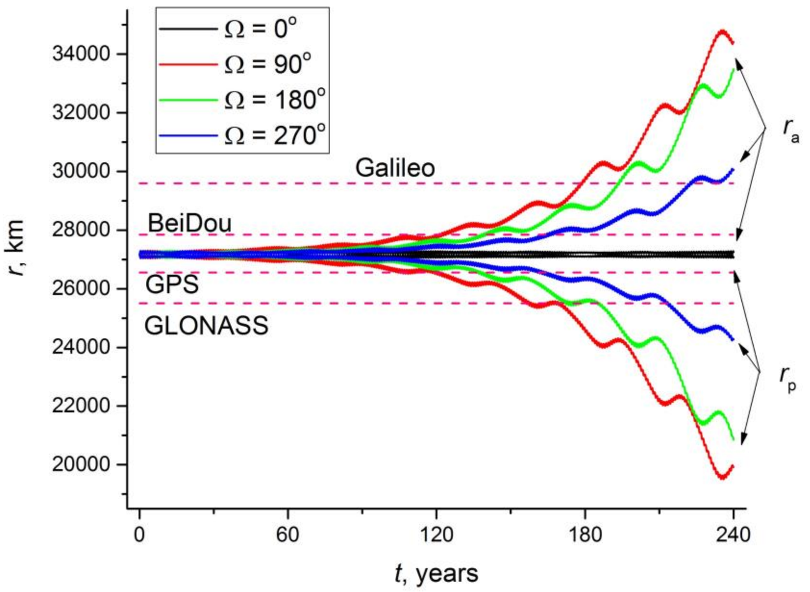

Figure 5: Evolution of the distances in pericenter r_p and apocenter r_a for a satellite above the GPS orbit near the 15:29 resonance region depends on initial value of the longitude of ascending node **Ω** for area to mass ratio γ = 0.02 m^2/kg (a = 27 168 km, i = 55°).

Table 5: Intervals for the first time of passage through the navigation systems regions or the GEO region for the BeiDou satellite

γ, m^2/kg	Ω, deg	Time, years			
		GLONASS	GPS	Galileo	GEO
0.02	0.00	—	—	—	—
2.00	0.00	—	5.48	—	—
3.00	0.00	—	0.25	0.38	—
4.00	0.00	0.44	0.19	0.25	—
23.00	0.00	0.05	0.03	0.05	5.26
37.00	0.00	0.05	0.03	0.03	0.22
0.02	90.00	—	227.54	—	—
0.20	90.00	164.52	137.19	142.92	—
1.00	90.00	84.98	54.57	76.25	179.08
3.00	90.00	22.18	0.33	0.38	127.04
0.02	180.00	213.33	185.11	206.08	—
1.00	180.00	154.96	124.54	131.36	238.36
13.00	180.00	0.11	0.05	0.08	81.26
0.02	270.00	184.34	157.92	175.88	—
2.00	270.00	115.35	0.41	87.86	216.21
4.00	270.00	0.36	0.19	0.25	164.16

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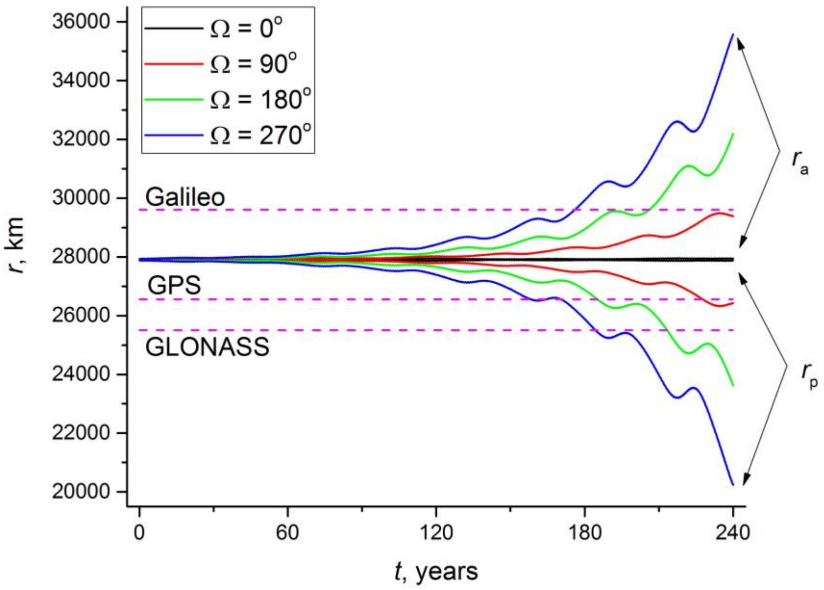

Figure 6: Evolution of the distances in pericenter r_p and apocenter r_a of the BeiDou satellite depends on initial value of the longitude of ascending node **Ω** for area to mass ratio γ = 0.02 m^2/kg (a = 27 907 km, i = 55°).

Table 7 presents results of numerical simulation for initial conditions corresponding the Galileo satellite near the 10:17 resonance region. The initial value of the semi-major axis is 29 601 km. The initial inclination is 56 degrees. The object does not pass any regions of navigation satellites motion for the initial value of the longitude of ascending node is 90° and 180° and the area to mass ratio less than 0.02 m^2/kg. The initial **Ω** = 270° and the area to mass ratio γ = 0.02 m^2/kg lead to pass through the GLONASS, GPS, BeiDou and the GEO regions after 187, 181, 151 and 240 years correspondently. The initial **Ω** = 0° permits orbital evolution during 240 years with maximal value of the area to mass ratio 40 m^2/kg.

Figure 8 shows evolution of the distances in pericenter r_p and apocenter r_a for the area to mass ratio 0.02 m^2/kg and the initial value of the semi-major axis 27 907 km near the Galileo region. Variations of the distances r_p and r_a are minimal for the initial **Ω** = 180° and maximal for the initial **Ω** = 0° and 270°.

Table 8 gives results for the object is above the Galileo orbit near the 3:5 resonance region. The initial value of the semi-major axis is 29 994 km. The initial inclination is 56 degrees. The satellite is subjected to significant luni-solar perturbations. The object returns to the Galileo region after 99, 170, 201 and 85 years for the initial values of the longitude of ascending node are 0, 90, 180 and 270 degrees correspondently when area to mass ratio is 0.02 m^2/kg. Increase in the area to mass ratio leads to decrease of time intervals. When the area to mass ratio is 1 m^2/kg, time intervals 100, 95, 67 and 0.27 years correspond to pass through the GLONASS, GPS, BeiDou and Galileo regions. The passages through the GEO region are started for γ = 0.5 m^2/kg and the initial **Ω** = 0°. The initial **Ω** = 180° permits orbital evolution during 240 years with maximal value of the area to mass ratio 80 m^2/kg.

Figure 9 shows evolution of the distances in pericenter r_p and apocenter r_a for the area to mass ratio 0.02 m^2/kg and the initial value of the semi-major axis 29 994 km. It is above the Galileo orbit near the 3:5 resonance region. Variations of the distances r_p and r_a are minimal for the initial **Ω** = 270° and maximal for the initial **Ω** = 0°.

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Table 6: Intervals for the first time of passage through the navigation systems regions or the GEO region for a satellite above the BeiDou orbit near the 4:7 resonance region (a = 29 034 km, i = 55°)

γ, m²/kg	Ω, deg	Time, years				
		GLONASS	GPS	BeiDou	Galileo	GEO
0.02	0.00	—	—	—	—	—
0.80	0.00	—	—	—	7.47	—
2.00	0.00	—	—	0.33	0.14	—
4.00	0.00	—	0.41	0.14	0.08	—
5.00	0.00	4.49	0.27	0.11	0.05	—
14.00	0.00	0.14	0.08	0.05	0.03	160.55
36.00	0.00	0.05	0.05	0.03	0.03	0.19
0.02	90.00	—	—	221.05	190.39	—
0.04	90.00	—	222.09	189.35	159.01	—
0.05	90.00	225.41	216.92	183.76	154.22	—
0.20	90.00	163.81	155.95	122.66	91.17	235.78
0.70	90.00	120.63	96.97	59.66	22.31	178.48
0.02	180.00	—	—	—	—	—
0.70	180.00	—	—	—	84.33	—
2.00	180.00	—	—	82.38	0.14	—
5.00	180.00	—	17.22	0.11	0.05	—
7.00	180.00	21.05	0.22	0.08	0.05	—
9.00	180.00	1.70	0.14	0.05	0.03	85.78
10.00	180.00	0.22	0.14	0.05	0.03	68.34
0.02	270.00	195.46	172.70	140.37	110.12	—
2.00	270.00	109.57	99.82	0.30	0.16	187.21
5.00	270.00	0.41	0.27	0.14	0.08	131.99

Figure 10 presents evolution of the orbit inclination i (fig. 10a) and the orbital eccentricity e (fig. 10b) for a satellite above the Galileo orbit near the 3:5 resonance region for the area to mass ratio γ = 60 m²/kg. Initial values are a = 29 994 km, i = 56°, Ω = 180°. The orbit has flips. The orbital inclination changes from 56° to 109° with period less than 3 years. Instead of the area to mass ratio has high value γ = 60 m²/kg, the maximal value of the eccentricity does not exceed 0.65. This effect is rose by equality between the initial longitude of pericenter π and the longitude of the Sun λ_S. In [4] the existence of a stationary point (e_0, π_0) was demonstrated in the phase plane "eccentricity e and longitude of pericenter π," corresponding to the following initial conditions

$$e_0 = \frac{3}{2} k\gamma P \frac{(\cos(\varepsilon/2))^2}{ann_S} \approx 0.01k\gamma, \quad \pi_0 = \lambda_S. \tag{1}$$

Here $P = 4.56 \cdot 10^{-6}$ Nm^{-2} is the radiation pressure, ε is the obliquity of the ecliptic to the equator, n and n_S are mean motions of a satellite and the Sun, and λ_S is the ecliptic longitude of the Sun.

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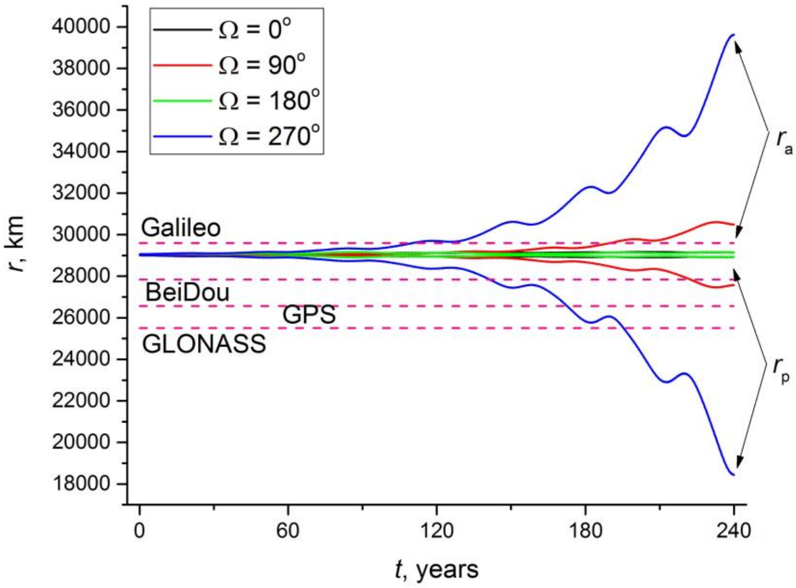

Figure 7: Evolution of the distances in pericenter r_p and apocenter r_a for a satellite above the BeiDou orbit near the 4:7 resonance region depends on initial value of the longitude of ascending node **Ω** for area to mass ratio γ = 0.02 m^2/kg (a = 29 034 km, i = 55°).

Table 7: Intervals for the first time of passage through the navigation systems regions or the GEO region for the Galileo satellite

γ, m^2/kg	Ω, deg	Time, years			
		GLONASS	GPS	BeiDou	GEO
0.02	0.00	196.11	190.97	161.81	—
0.30	0.00	187.57	164.00	153.26	224.01
40.00	0.00	0.05	0.05	0.03	0.16
0.02	90.00	—	—	—	—
0.05	90.00	—	—	232.20	—
0.06	90.00	—	234.93	206.19	—
0.07	90.00	234.72	229.21	199.62	—
0.20	90.00	170.98	163.23	133.52	225.60
5.00	90.00	25.22	0.38	0.25	92.02
0.02	180.00	—	—	—	—
4.00	180.00	—	199.95	5.61	—
5.00	180.00	87.34	47.45	0.19	154.39
9.00	180.00	16.24	0.19	0.08	69.57
0.02	270.00	187.13	180.62	151.43	239.81
2.00	270.00	83.01	75.26	40.77	141.14

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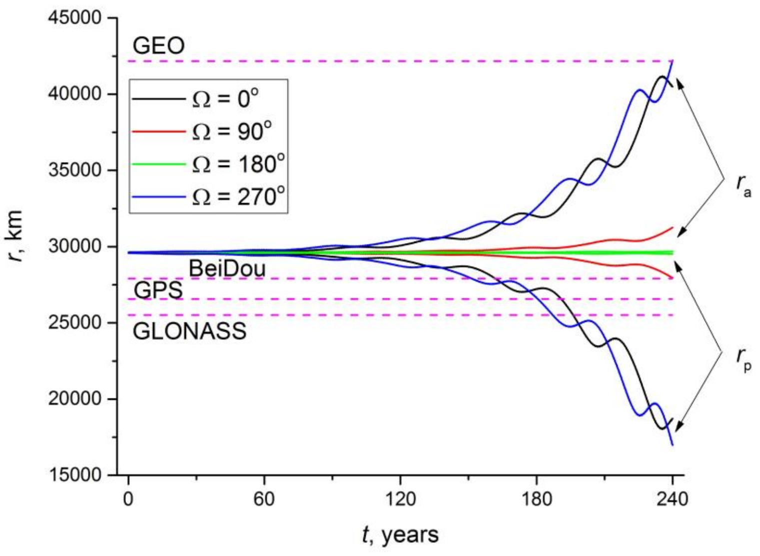

Figure 8: Evolution of the distances in pericenter r_p and apocenter r_a of the Galileo satellite depends on initial value of the longitude of ascending node **Ω** for area to mass ratio $\gamma = 0.02$ m^2/kg (a = 29 601 km, i = 56°).

Table 8: Intervals for the first time of passage through the navigation systems regions or the GEO region for a satellite above the Galileo orbit near the 3:5 resonance region (a = 29 994 km, i = 56°)

γ, m^2/kg	Ω, deg	Time, years				
		GLONASS	GPS	BeiDou	Galileo	GEO
0.02	0.00	220.78	214.13	173.22	99.27	—
0.50	0.00	164.00	135.41	126.57	24.09	212.35
0.70	0.00	132.46	127.58	97.96	0.33	175.77
0.02	90.00	—	—	—	169.66	—
0.06	90.00	—	—	216.34	140.67	—
0.08	90.00	—	213.44	182.86	114.85	—
0.09	90.00	218.51	206.82	177.22	108.94	—
0.40	90.00	136.70	131.72	104.15	31.13	193.65
2.00	90.00	67.65	62.70	31.10	0.16	120.82
0.02	180.00	—	—	—	200.88	—
0.40	180.00	—	—	235.48	160.55	—
2.00	180.00	—	231.07	200.47	0.08	—
5.00	180.00	86.46	51.36	2.68	0.05	—
6.00	180.00	48.41	32.85	0.19	0.03	111.54
80.00	180.00	0.03	0.03	0.03	0.03	0.08
0.02	270.00	—	185.76	154.25	85.39	—
0.40	270.00	187.65	160.96	147.27	53.52	—
2.00	270.00	84.13	78.28	46.68	0.14	141.22

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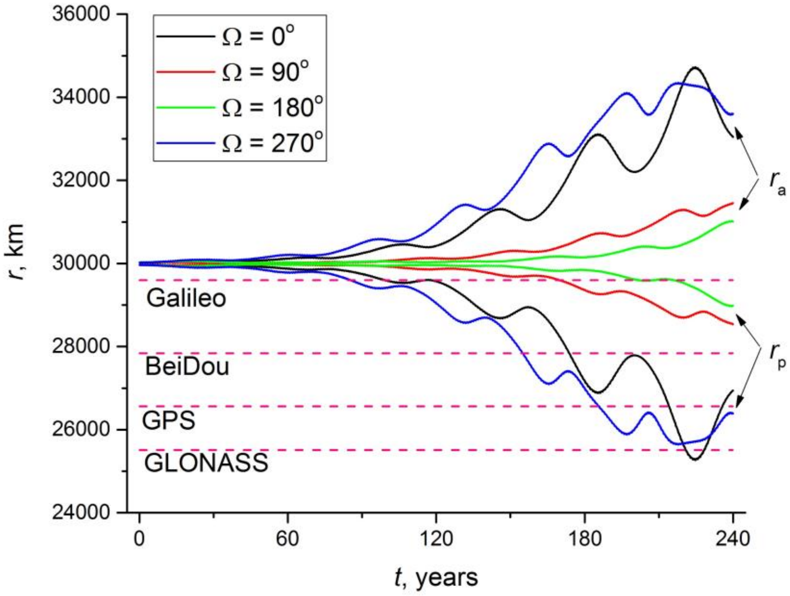

Figure 9: Evolution of the distances in pericenter r_p and apocenter r_a for a satellite above the Galileo orbit near the 3:5 resonance region depends on initial value of the longitude of ascending node **Ω** for area to mass ratio $\gamma = 0.02$ m^2/kg (a = 29 994 km, i = 56°).

Flips of the orbits due to effect of solar radiation pressure are also observed for the GLONASS satellites with the area to mass ratios from 16 to 20 m^2/kg and initial value of the longitude of ascending node **Ω** = 180°; for the object above the GLONASS orbit near the 14:29 resonance region with the area to mass ratios from 16 to 76 m^2/kg and initial value of the longitude of ascending node **Ω** = 180°; for the object above the Galileo orbit near the 14:29 resonance region with the area to mass ratios from 18 to 80 m^2/kg and initial value of the longitude of ascending node **Ω** = 180°. Flips are fixed for initial value of the longitude of ascending node **Ω** = 180° only.

The long-time evolution is realized for the high area to mass ratio objects depend on initial value of the longitude of ascending node. Table 9 gives maximal values of product of the coefficient of reflection of the satellite surface on the area to mass ratio $k\gamma$ and corresponding initial values of the longitude of ascending node which allow long term evolution of an object for time interval 240 years. Initial values of the longitude of ascending node **Ω** = 0° or 180° allow orbital evolution of the high area to mass objects for 240 years. In this case the Sun lie in the initial orbital plane.

4. Conclusions

Orbits of MEO satellites are characterized by an eccentricity growth which can become very large over long time scales. Strategies to minimize this growth are necessary, in order to avoid collisions with navigation or geostationary orbits.

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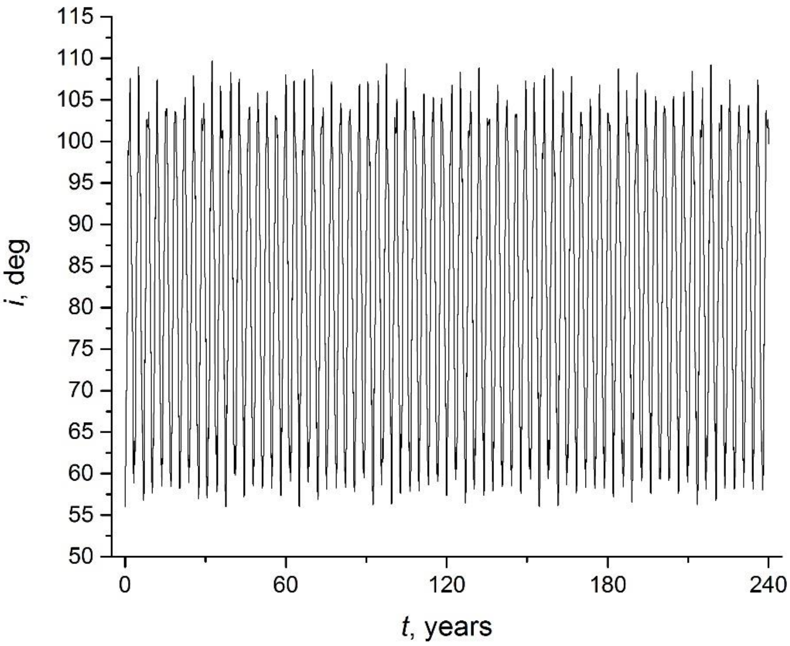
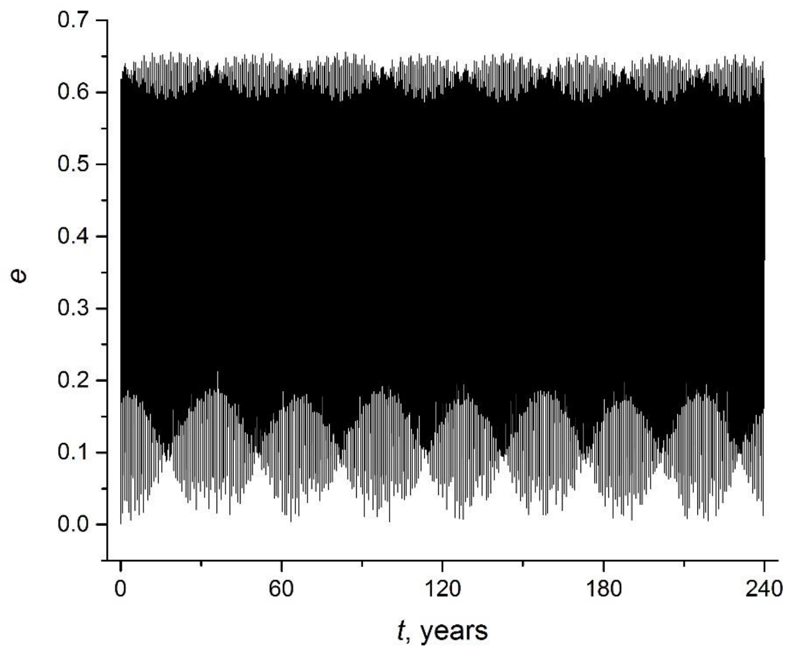


a)



b)

Figure 10: Evolution of a) the inclination i and b) the eccentricity e for a satellite above the Galileo orbit near the 3:5 resonance region for area to mass ratio γ = 60 m^2/kg (a = 29 994 km, i = 56°, Ω = 180°)

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Table 9: Maximal values of product of the coefficient of reflection of the satellite surface on the area to mass ratio $k\gamma$ allowing long term evolution of object for 240 years.

Name of region	a, km	Ω, deg	$k\gamma$, m^2/kg
GLONASS	25 509	0	62
Above GLONASS	25 947	180	109
GPS	26 562	0	53
Above GPS	27 168	0	53
BeiDou	27 907	0	53
Above BeiDou	29 034	0	52
Galileo	29 601	0	58
Above Galileo	29 994	180	115

The orbital evolution of satellites in vicinity of the GNSS regions significantly depends on orbital plane position. The objects pass through the GNSS and the GEO regions due to long period oscillations of eccentricities and inclinations of orbits which are produced by luni-solar effect. Increase in the area to mass ratio leads to decrease of time intervals before passing through the GNSS and the GEO regions due to the effect of solar radiation pressure. The long-time evolution is realized for the high area to mass ratio objects depend on initial value of the longitude of ascending node when the Sun lie in the initial orbital plane. Flips of the orbits are fixed for the GLONASS region, above the GLONASS region and above the Galileo region but only for initial value of the longitude of ascending node $\Omega = 180°$.

5. Aknowledgements

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