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Multi-body modeling for fluid sloshing dynamics investigation in fast spinning rockets

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Abstract

Fuel sloshing dynamics inside space vehicles is a broad topic, still quite open to investigation for modelling, verification and validation. The behaviour of liquids in tanks, subject to a certain spectrum of external accelerations, is hard to predict, no matter the vehicle is either a launcher or a spacecraft. A step over in complexity is added whenever spinning vehicles are considered, as centrifugal actions may play an important role in defining the fluid shape inside the tank; furthermore, if the fluid dissipates energy, the vehicle might lose attitude stability, thus overcoming the benefit of spin stabilization. The numerical modelling of the phenomenon may be faced with different level of accuracy and, therefore, of complexity, even preserving the meaningfulness of the attainable results. The present research investigated the sloshing effects in highly spinning rockets. In particular, also in the light of a possible experimental test to validate the model and characterize this complex phenomenon, a scenario involving a small sounding rocket with a liquid or hybrid propulsion system, hence carrying a significant liquid propellant mass with respect to the overall mass, spun up to enhance the flight stability, was investigated. The liquid mass may undergo sloshing vibrations, coupled with the spin frequency, and affect both the attitude and the trajectory of the rocket. The analysis was performed by exploiting a PoliMi-DAER multi-body numerical tool. The fluid carried on board is modelled with a lumped parameter method, as a set of rigid bodies connected by different types of joints, springs and non-linear dampers. First, the shape of the fluid is obtained with analytical approximations, and its overall inertia properties are estimated. Then, the sloshing modes shapes and natural frequencies are modelled exploiting different configurations embedded in the Simulink-based software, allowing swirling, tangential and longitudinal motion of the fluid mass. Since the sloshing modes themselves were unknown at the beginning of the study, the analysis served as a preliminary investigation on the topic, to investigate the effect of sloshing in a fast-spinning vehicle. Several cases were investigated, corresponding to different tank filling ratios, and different kinematic and dynamic conditions; although preliminary, the numerical results suggest that the high spin rate might trigger an unstable swirling of the fuel, if one of the natural swirling frequency is below the spin frequency. The modelling of the viscous damping is a crucial part, still partially open, of the study. According to the parameters, the viscosity may reveal beneficial in damping the undesired sloshing behaviour, but could also be a source of energy dissipation which leads towards an overall unstable dynamics of the vehicle. The paper presents the adopted model and its implementation and critically discusses the obtained results as potential drivers to the propulsion unit and mission profile design.

1. Introduction

The problem of fluid sloshing, inside the tank of a space or atmospheric vehicle, is well-known to the aerospace community, and has been faced and investigated by a large number of authors. Many models have been devised and employed for such investigation, and their comprehensive collection goes beyond the scope of this work.

While many previous studies focused on sloshing oscillation, few of them considered the sloshing dynamics inside a fast spinning vehicle, with a significant quantity of fluid, in a gravity field; the focus of the present work is, indeed, the dynamical analysis of a spinning atmospheric rocket, which possesses a large quantity of fluid. The spin-stabilization might thus be hindered by the sloshing dynamics. The investigation employs a multi-body approach; some previous studies[1] provided simplified models for the sloshing dynamics, and multi-body methods were analysed

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as well,[5] although for small oscillation analyses. The focus of the current study is to prove the possibility of analysing sloshing phenomena, coupled with a complex dynamical condition such as a spinning motion, and to provide preliminary results where experimental data are still missing. The investigation aims as well at providing some hints for the design of possible experimental procedures.

The analyses were conducted with a multi-body tool[2,3] developed at Aerospace Science and Technology Department of Politecnico di Milano; this tool, initially devised for debris removal studies, was enhanced and enriched of more complex models, making it suitable for a large variety of analyses such as the one at hand.

The paper is structured as follows: the problem is first described in section 2, focusing on the assumptions and simulations performed; a dedicated section 3 described the sloshing model, focusing on its strong points and highlighting its limitations; section 4 presents and discusses the results of the study, and section 5 concludes the analysis, providing remarks for future investigations as well.

2. Problem description

The present investigation focuses on the sloshing dynamics that appear in a spinning rocket; no experimental data are still available, and the main purpose of the study is to analyse if possible couplings may arise between the fluid behaviour and the spinning rigid body.

2.1 Model and assumption

The rocket is modelled as a rigid body, with lumped inertia and mass properties. From the multi-body software point of view, the actual size of the vehicle is not a concern, since the aforementioned physical properties are the only inputs required from the code; the dimensions are in turn necessary for the computation of aerodynamic forces and the inertia properties themselves.

A set of external forces and torques are assumed to act on the vehicle:

- Thrust, constantly aligned with longitudinal body axis;
- Aerodynamic drag, aligned with the anti-velocity direction;
- Aerodynamic lift, perpendicular to the velocity direction and proportional to the angle of attack α;
- Gravity gradient torque, which depends on the inertia matrix and the current orientation of the rocket.

The 6-DOF dynamics are integrated simultaneously, thus coupling rotational and translational motion. The aerodynamic coefficients are interpolated from a set of data, provided by previous analyses. The thrust is assumed constant, in magnitude, throughout the simulation.

The fluid is modelled as another rigid body, linked by flexible components to the rocket body; details of the fluid modelling are provided in Section 3. The capability of the multi-body software allow to increase the complexity of the fluid model, e.g. by increasing the number of bodies and flexible connection. Flexible bodies might be modelled as well, but require the use of external software to perform their dynamical analysis.

2.2 Simulation scenario

The study focuses on three different scenarios, which represent different stages of the rocket trajectory:

1. The first scenario considers the initial ascent phase, where the fluid tank is full and the rocket spins slowly.
2. An intermediate phase is then analysed, considering the fluid tank to be half-empty; in this stage, the spin rate of the rocket has increased.
3. The last scenario simulates the final phase of the ascent, where the fluid is nearly depleted and the spin rate is at its maximum value.

3. Sloshing model

The critical part of the study is the modelling of the fluid and its dynamical behaviour. The choice of the multi-body software was preferred in order to obtain preliminary results, without the need of recurring to computationally expensive and time-consuming CFD simulations. It is thus important to create a multi-body of the fluid which, although preliminary and approximate, is able to yield significant results for the analysis.

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3.1 Fluid shape

The first step to performed, in order to create a realistic model of the fluid, is to investigate the shape it takes in the tank, when subjected to spinning motion. Considering a fluid in a rotating cylindrical tank, in an uniform, vertical gravity field, the free surface of the fluid has a parabolic profile[4]

$$h = \frac{\Omega^2}{2a} r^2 + h_0 \tag{1}$$

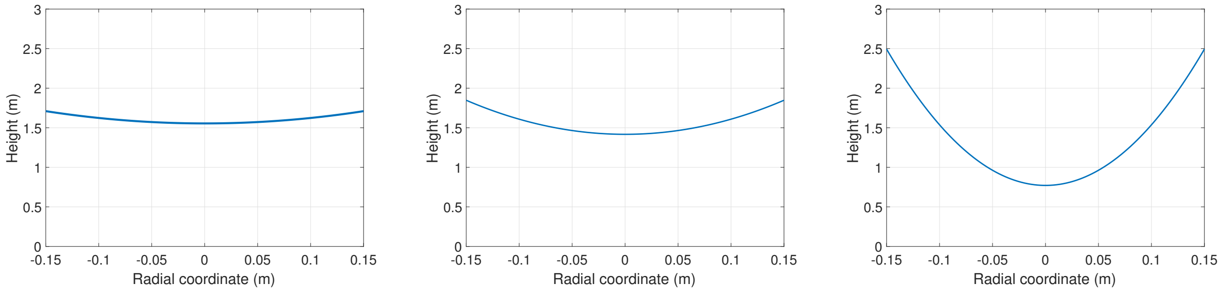
where r is the radial coordinate, outbound from the centre of the cylinder, Ω is the spin rate of the tank, and a is the vertical uniform acceleration. h_0 is the vertical position of the vertex, and may be obtained by knowing the overall fluid mass m_f and its density ρ_f. The fluid mass occupies a volume which is the sum of the base cylindrical shape, plus the upper paraboloid

$$m_f = \rho_f \left(h_0 \pi R^2 + \frac{\Omega^2}{4a} \pi R^4 \right) \tag{2}$$

and thus, the vertex position is located at a height

$$h_0 = \frac{m_f}{\rho_f \pi R^2} - \frac{\Omega^2 R^2}{4a} \tag{3}$$

For the case at hand, the acceleration a includes both the gravitational component and the acceleration induced by the thrust.



(a) Spin rate = 3 Hz (b) Spin rate = 5 Hz (c) Spin rate = 10 Hz

Figure 1: Fluid parabolic profile in spinning tank

Figure 1 portrays a parametric study of the parabolic profile, considering a 0.3 m (diameter) x 3 m (height) cylindrical tank, at sea level, with a vertical thrust of 30 kN. Two important assumptions are necessary for eq. (1) to be applicable:

- The fluid is assumed to be inviscid; such assumption is reasonable, considering the viscosity of common propellants and oxydizers. The viscosity would slightly modify the fluid profile at the interface with the walls, with no significant effect on its overall shape. Furthermore, in the framework of the current preliminary study the viscous behaviour of the fluid is largely approximated, thus justifying once more this assumption.
- The vertical acceleration should be sufficiently greater than the lateral acceleration component, i.e. the acceleration transversal to the spin axis should be negligible. In the case at hand, the flight path angle of the rocket was assumed to be in the rage 90-60 degrees, and the thrust acting always along the spin axis; thus, the lateral gravitational acceleration was always negligible in respect to the overall longitudinal acceleration.

3.2 Limit cases

Two limiting situations may arise, within the context of the presented analytical approximation, at high spin rate:

1. The fuel tank is nearly empty, and the vertex h_0 in eq. (3) results in a negative value; this happens when the spin rate is greater that the limit value

$$\Omega > \sqrt{\frac{4 a m_f}{\pi \rho_f R^4}} \tag{4}$$

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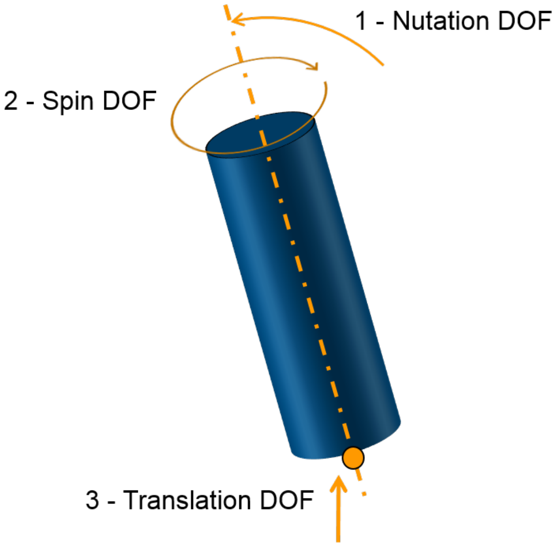

Figure 2: Multi-body fluid model

2. The fuel tank is nearly full, and the analytical parabolic profile would go beyond the maximum tank height H_{tank}; this condition is verified if

$$\Omega > \sqrt{4a\left(\frac{H_{tank}}{R^2} - \frac{m_f}{\pi\rho_f R^4}\right)} \tag{5}$$

The deep investigation of the fluid profile goes beyond the scope of the current analysis, which focuses, in turn, on providing preliminary results and highlighting possible critical situations in the problem at hand. Equations (4) and (5) provide two critical condition, which shall be studied with greater care from the fluid dynamics point of view; within the present framework, such cases are not investigated, and employed exclusively as a condition to be checked for the validity of the results.

3.3 Multi-body model

The modelling of the fluid mass, within the multi-body framework, shall represent the main dynamical features of the liquid body, balancing model complexity and real-world fidelity; the advantages of the multi-body software lie in the simplicity of representation, thus being able to provide preliminary results with a reduced computational effort.

Figure 2 schematically portrays the model employed in the multi-body software, to represent the dynamical behaviour of the fluid mass. The basic model consists in a rigid body, which possesses three degrees of freedom (DOF):

1. A nutation DOF, which models the rotation in a direction perpendicular to the spin axis.
2. A spin DOF, modelling the rotation of the fluid mass about the longitudinal axis, parallel to the spin axis of the vehicle.
3. A translation DOF, which represents the longitudinal movement of the fluid inside the tank.

The spin and the nutation DOFs are modelled through a joint, which allows two consecutive rotations. With this simplified model, the combination of these two DOFs allows to roughly represent the swirling motion that the fluid could undergo, i.e. a spin motion with a slight offset, similar to the movement of a spinning top. Different reaction forces are actuated to the degrees of freedom:

1. A linear spring-damper system is connected to the nutation DOF, whose visco-elastic force represents the vibrations of the fluid, according to the first transversal mode; the damping is assumed to arise from the viscous stress of the fluid.
2. A viscous force, proportional to the angular velocity difference between fluid and tank, acts on the spin DOF, in order to model the viscous stress at the tank wall.

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3. Another linear spring-damper system acts on the translation DOF, representing the first longitudinal sloshing mode, damped by the fluid viscosity.

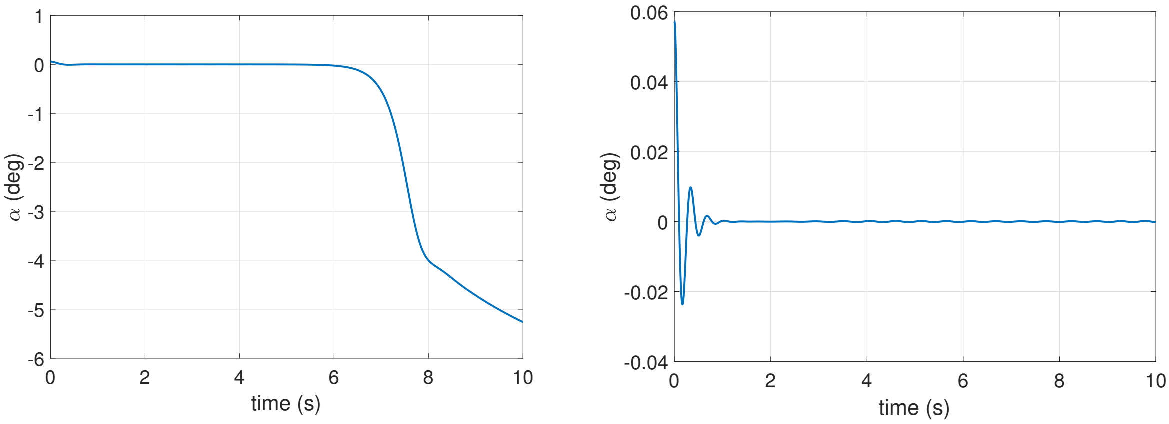
It is underlined that, at the time the present study was performed, no experimental data where available for the fluid modes, neither for the estimation of the modal damping coefficients. The presented analysis was thus performed with a parametric study of such properties, in order to provide general results and focus on the potential applications of the model.

4. Results

The section presents the main results of the study, along with preliminary conclusions and remarks for future analyses. In particular, it is underlined how, notwithstanding the partial lack of experimental data and the strong simplifying assumptions employed, the current research highlighted the possibility of drawing results on the dynamical coupling between a rigid body and a sloshing fuel mass, without the necessity of resorting to computational fluid dynamics simulations. The model employed, although preliminary, and with a large margin of complexity that can be added, proved to be efficient in providing an overview of the problem at hand.Since the actual frequency of the sloshing modes is unknown, different values were investigated, within the range foreseen to be significant for the analysis.

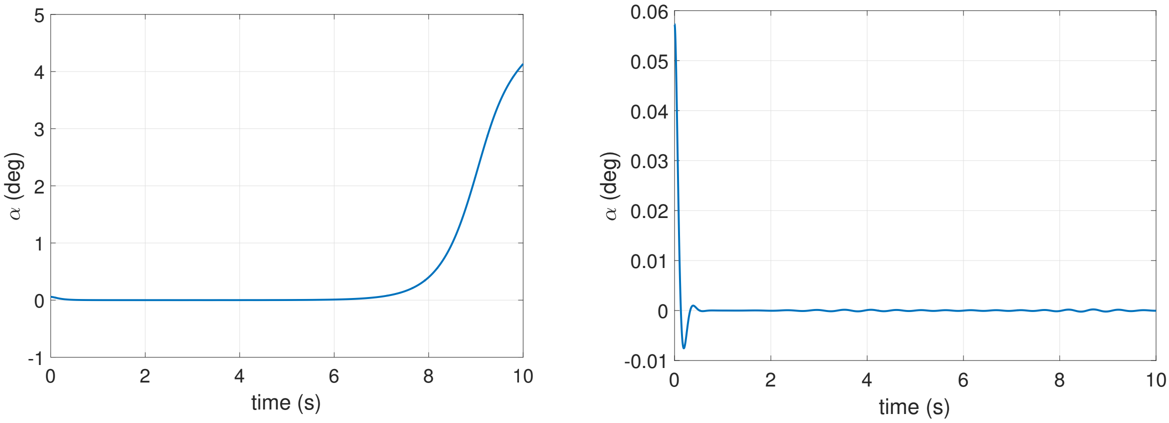
4.1 Transient dynamics

During the first ascent phase, the fluid mass is assumed to be at rest, while the vehicle body starts spinning. No a priori knowledge is possessed about the actual state of the fluid, so such assumption was assumed to be reasonable enough for the purposes of the analysis.



(a) Longitudinal = 2 Hz, swirling = 2 Hz

(b) Longitudinal = 2 Hz, swirling = 4 Hz



(c) Longitudinal = 4 Hz, swirling = 2 Hz

(d) Longitudinal = 4 Hz, swirling = 4 Hz

Figure 3: Fuel "nutation" in transient phase, spin rate = 2.5 Hz

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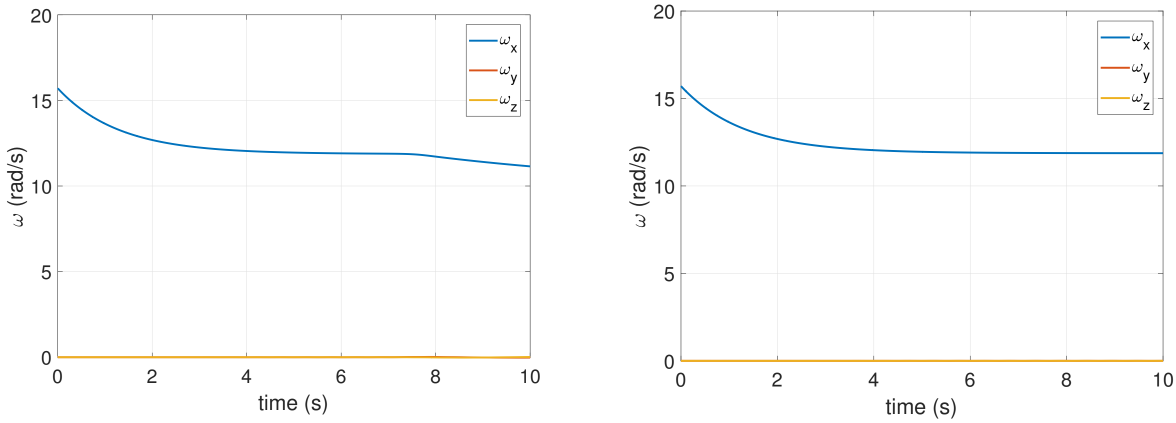
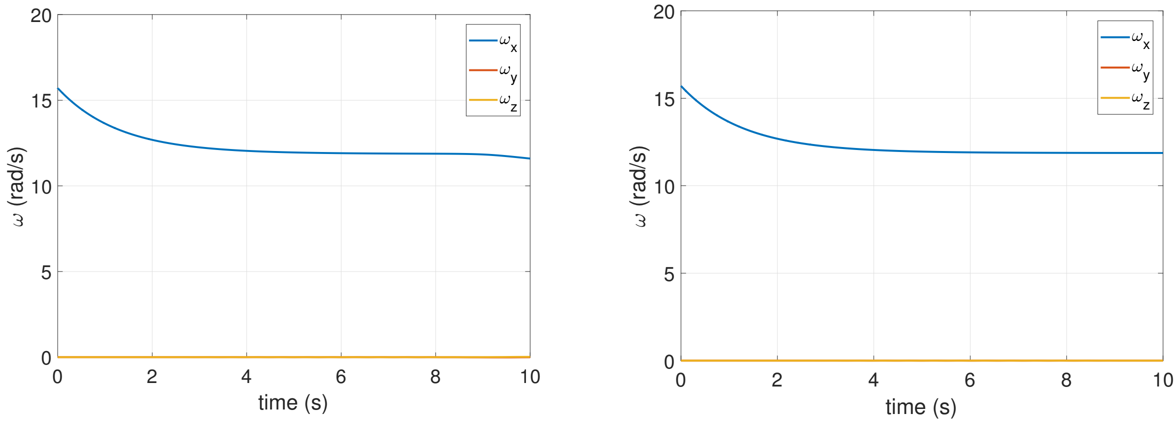
Figure 3 portrays four cases, which underline the very different behaviours of the fluid mass according to the sloshing frequency. In this phase, the rocket spins at an angular rate of 2.5 Hz; the parametric sloshing analysis considered frequencies up to 10 Hz, but above the 4 Hz no significant differences were encountered in the dynamical response.

The swirling mode is dominant; if its frequency is sufficiently low, and goes below the spin frequency, a significant coupling is encountered, where the transversal oscillations of the fluid starts diverging. On the contrary, the longitudinal mode seems to have little effect on the overall stability of the system, probably due to the beneficial effect of the spinning motion. Figures 3b and 3d show a similar behaviour, where after a brief transient the swirling mode is no longer excited; Figure 3c portrays the case where the swirling frequency is lower than the spin rate, triggering an unstable nutation of the fluid; Figure 3a is the most unstable case, where both modes have a low frequency and result in a coupled unstable motion. Note that the sign of the nutation angle is not significant, since it is intended to describe a rotation with respect to the spin axis, indifferent to the sign.



(a) Longitudinal = 2 Hz, swirling = 2 Hz

(b) Longitudinal = 2 Hz, swirling = 4 Hz



(c) Longitudinal = 4 Hz, swirling = 2 Hz

(d) Longitudinal = 4 Hz, swirling = 4 Hz

Figure 4: Rocket angular velocities in transient phase, spin rate = 2.5 Hz

Figure 4 portrays the angular velocity of the rocket, in the four different cases. The exponential decay is correlated with the energy exchange, due to the viscous interaction at the wall, between the fluid mass and the spinning vehicle. The strong approximations employed for the viscous model make it difficult to establish a correlation, and to validate the behaviour itself; it is nevertheless correct to deduce a spin rate decay, and to expect a loss of gyroscopic stability if too much energy is transferred to the rotating fluid. The equilibrium condition will be reached when both the vehicle and the fluid possess the same angular rate, which may be computed as

$$\omega_{eq} = \frac{I_{rocket}}{I_{rocket} + I_{fluid}} \Omega(0) \tag{6}$$

where $\Omega(0)$ is the initial spin rate, and the symbol I denotes the moment of inertia about the longitudinal axis. Note that eq. 6 approximates the equilibrium angular velocity, but yields no information on the time instant when the value is

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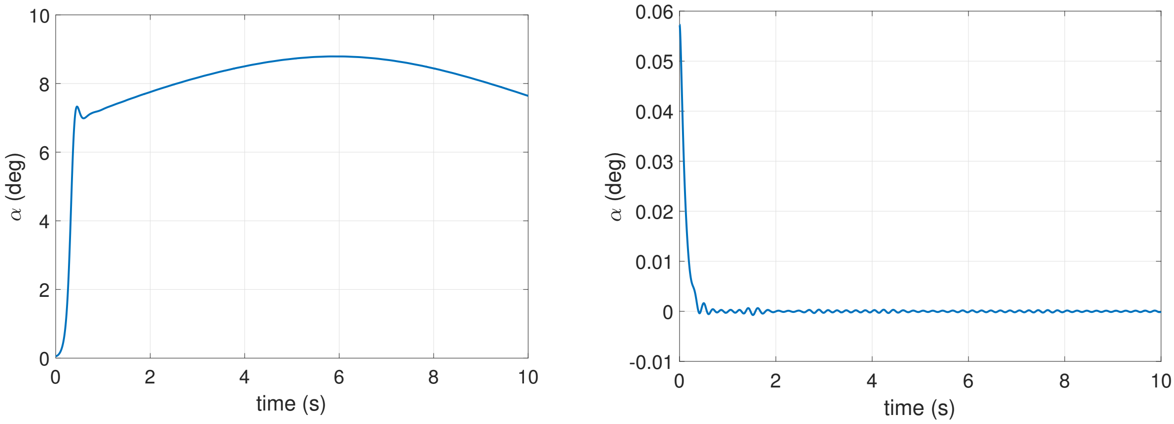
reached; the time constant on the system depends in fact on the viscous stress model, which may vary with the angular rate itself and thus be hard to predict analytically. Nevertheless, the estimated equilibrium angular velocity may be employed to have a first grasp on how much the rocket motion is influenced by the energy exchange, and if such value is high enough to guarantee the gyroscopic stability sought for the spinning vehicle.

4.2 Steady state dynamics

The second set of analysis investigated the steady state dynamics of the rocket, assuming all transient phenomena to have ended. Namely, the fluid mass is assumed to possess the same spin velocity as the rocket body, thus investigating the small vibrations around such condition. It is reasonable to assume that such dynamical state is verified mid-flight and at the end of the ascent phase.

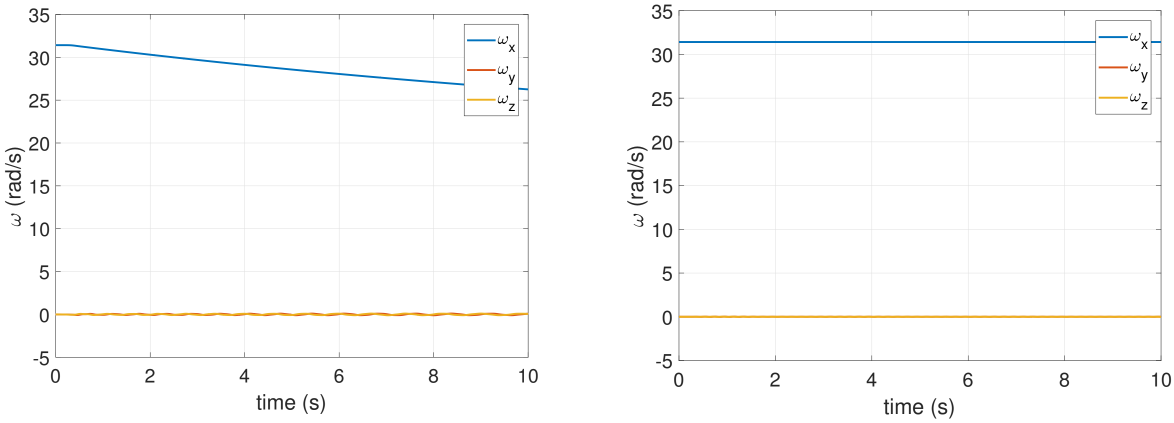
Section 4.1 underlined the primary role of the swirling sloshing mode, which dominates in triggering instabilities; the following results will thus present the analysis in respect to this mode, whereas the longitudinal mode was observed to be less significant. In steady state, the energy transfer between rocket body and fluid mass is no longer observed; the viscous stress at the tank wall maintains the relative angular velocity close to zero, thus preserving the spin of the vehicle. If the unstable swirling mode is triggered, it is noted, nevertheless, that the vehicle loses spin rate indeed, as expected since the fluid starts gaining rotational energy. It is noted, again, how the difference between stable and unstable behaviours lies in the value of the swirling frequency, with respect to the rocket spin frequency; when the latter is higher than the sloshing mode's, instabilities are triggered, in analogy with what was observed in the transient case.



(a) Swirling mode frequency = 3 Hz

(b) Swirling mode frequency = 6 Hz

Figure 5: Fuel "nutation", half-empty tank, spin rate = 5 Hz



(a) Swirling mode frequency = 3 Hz

(b) Swirling mode frequency = 6 Hz

Figure 6: Rocket angular velocities, half-empty tank, spin rate = 5 Hz

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Figure 5 portrays the nutation coordinate of the fluid mass, for two different values of sloshing swirling frequency. An unstable, diverging rotation is observed for the swirling frequency of 3 Hz (lower than the 5 Hz spin frequency), while stable oscillations are triggered for higher sloshing frequency. The coupling between spin and sloshing dynamics is again noted to be crucial in the overall stability of the system, which is dominated by the swirling mode. The same instability is observed in the angular velocity profiles, portrayed in Figure 6; when unstable swirling mode is triggered, the vehicle starts losing angular velocity, since rotational energy is transferred to the fluid which starts swirling, possibly leading to a loss in overall stability.

4.3 Remarks

Both the transient and steady state simulations remarked the prominent role of the swirling mode; the first natural frequency of such mode might be low, and thus trigger unstable couplings with the spinning motion of the rocket. The first longitudinal mode, although non negligible, proved to be less significant than the swirling mode, being unable to destabilise the spinning vehicle.

It is thus envisaged, for this kind of mission, the possibility of introducing transversal baffles in the fluid tank. A twofold advantage would result:

- In the transient phase, transversal baffles would help in transferring the spin motion to the fluid, an reach faster the steady state. The energy transfer, from the vehicle to the fluid mass, would take place with considerable less energy dissipation, since the relative motion at the wall would be reduced, together with the viscous stress.
- The swirling mode frequency would increase in value, since the swirling motion would be constrained between the baffles, increasing the frequency of such oscillations.

It is nevertheless remarked how the baffle analysis was outside the scope of the current study; a proper baffle design should be coupled with more refined fluid dynamics analyses, in order to identify their effect on the swirling modes and the possible drawback of the additional components.

5. Conclusions

The paper presented a preliminary investigation, and its results, on the sloshing dynamics inside a fast spinning vehicle. The study is based on a multi-body software, which allows for a parametric modelling of the problem and for good operational flexibility. The fluid mass is modelled as a set of rigid bodies and visco-elastic connection, with the purpose of representing its inertia properties together with its frequency behaviour; within the current investigation, a single body is employed, but the model allows for a higher degree of complexity.

The scope of the investigation was to analyse the coupling between the sloshing modes and the spinning motion of the rocket; such coupling was observed to be significant, if the frequency of the first swirling sloshing mode is below the frequency of the spinning motion. This result is consistent with the expectations from modal analysis theory, since the lowest frequency mode dominates the system dynamics; it is interesting to observe that such expectation is indeed confirmed fo the swirling mode, whereas the longitudinal sloshing mode was observed to be less effective in destabilizing the vehicle's dynamics. A preliminary conclusion seems to be linked to the gyroscopic stiffness yielded by the spinning motion, which contributed to the stability about the longitudinal axis and is not hindered by longitudinal vibrations.

The current study employed linear visco-elastic laws, having in mind that they could be strongly different from the real behaviour of sloshing fluids; the model was nevertheless proved to be effective, thus leaving room for future improvements, both in the number of bodies, visco-elastic models and overall fluid modelling. The main objective, successfully reached, was to proved the effectiveness of the multi-body tool for sloshing analyses, without the need of coupling dynamics simulations with complex and time-consuming CFD studies.

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