DOI: 10.13009/EUCASS2017-452

Synthesis of a control system for relative movement of closely spaced satellites

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Abstract

In this paper, we consider an option for controlling the relative motion of picosatellites (CubeSat). The purpose of the control is to create an ordered group of satellites (Formation Flying), working together to perform the same task. In such groups, control is carried out by taking into account the mutual position of satellites moving in very close orbits. The purpose of forming and rebuilding groups is to solve various practical tasks, such as monitoring the Earth's surface and creating stereo images, building large-diameter telescopes in space and a number of other tasks. Particular attention is paid to the control of the approaching satellites. It is proposed to use an optical system of relative orientation and navigation. The relative orientation and navigation parameters (6 parameters) are calculated from the coordinates of the images of the points of the neighboring satellite on the camera matrix. The measured parameters are used to form the optimal terminal control by the motion of the center of mass of the active satellite. The control forces are created by miniature engines. The system is implemented on a low-cost element base. The results of the simulation are presented.

1. Introduction

In recent years, there has been increased interest towards Formation Flying - group flights of satellites - formations from several satellites flying at a relatively close mutual distance from several tens of meters to hundreds of kilometers. At the end of the last century and the beginning of this century, the latest advances in electronics and other fields of science and technology led to the possibility of miniaturization of almost all spacecraft (SC) service systems without compromising their functional qualities. In this connection, it became possible to use spacecraft of small overall dimensions for solving rather complex scientific and applied problems. This is one of the factors that drives the increased interest of the industry and "non-profit" research institutions to replace existing large space systems with small satellites and opens up new opportunities for these small systems. This trend towards the use of small satellite systems is also confirmed in the annual "SpaceWorks" report [1], which shows the "exponential" growth in applications for small satellite systems.

Typically, these satellites work together to perform the same task. Group flights allow to partially reduce the project costs and to bring qualitatively new opportunities to scientific and technological flight programs.

In group flights of the Formation Flying type, control is performed by taking into account the relative position of the satellites moving in very close orbits, that is, the relative motion control is implemented. In the formation of group flights of the Constellation type, individual control of each object is carried out. Structure Formation Flying is a much more flexible system than a single satellite, because if one of them fails, the group mission can continue its operation, unlike a single mission. The formation of satellites is more easily upgraded and is capable of solving problems that can not be solved by a single spacecraft. When using the reconfiguration property, the constellation of satellites can be used in a single mission to solve several problems. In addition, this configuration is capable of providing a higher positioning accuracy of satellites in a group.

There are a number of problems in the implementation of such projects, including from the point of view of the dynamics of its constituent satellites. However, the ability to work together in the formation of satellites is determined by the ability to determine the relative position of each system object and the ability to control their relative position. The management of a group flight must be based on direct or indirect measurement of the relative position of the satellites in a group. Direct measurement refers to the use of radar, laser rangefinders, and the like. A frequently encountered indirect measurement is the calculation of the distance based on the GPS or GLONASS data:

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the difference of the absolute position vectors of the apparatus is taken as the relative position vector. In this case, the necessary attribute, as a rule, is the radio frequency channel, which is a means of data exchange between satellites in the formation.

Determination of relative positions of satellites is one of the main problems of group flight, and it is one upon solving which the success of the mission often depends. To determine the relative phase state of the apparatus in a group, the processing of video images obtained by shooting one apparatus using a video camera installed on another device is often used. There are several possible options for using this approach to determine the relative position. The relative navigation system based on image processing is implemented in PRISMA [2, 3]. After processing the frame, the program determines the mutual distance and orientation of the devices. In the same way, navigation of the transport ships approaching the International Space Station (ISS) takes place. Up to a distance of several hundred meters, the GPS data from the receivers installed on the ship and on the ISS are successfully used. Information about the position of the station is transmitted to the ship through the radio channel. At the small distances, automatic visual monitoring of the progress of the approach and docking of Progress spacecraft with the ISS is used [4]. The initial information is a video signal coming from the camera on board the ship. The resulting sequence of frames is processed in real time. In each frame, the details of a special target are allocated, the dimensions of which and the geometrical parameters of the relative positioning are used as primary measurements. According to these measurements, the ship's movement is restored relative to the station.

The greatest accuracy is provided by installing light-emitting diodes of different colors on one of the devices at some locations. This option seems to be the cheapest for navigation of small satellites and is considered in [5, 6] and in this article.

The purpose of this work is to determine the parameters of the relative orientation and navigation of satellites (6 parameters), as well as the synthesis of the laws governing the motion of the centre of mass of the controlled satellite when moving it from an arbitrary position to a given position relative to the target uncontrolled satellite. A basic requirement to the control system is the simplicity of its elements.

2. Optical-electronic system of relative orientation and navigation of locally located satellites in orbit

Consider a simplified system of optoelectronic relative orientation and navigation, consisting of two satellites. The first satellite has eight infrared LEDs. The LEDs are located at the vertices of the transparent cube and are numbered from 1 to 8. The second satellite is equipped with a video camera.

On the sensitive matrix of the camera there are images of infrared LEDs. The coordinates of these LEDs are measured by a special program and are transmitted to calculate the relative orientation angles and calculate the relative coordinates x, y, z. When solving problems of relative orientation and navigation, it is assumed that all LEDs or arbitrary sets of 3 to 8 LEDs can be "visible" to the video camera. Euler angles are measured by means of a sequential rotation of the initial coordinate system in succession to the angles ψ , γ and Θ around the y, z and x axes. In this case, the matrix of the direction cosines will be written as [7]

$$\begin{bmatrix} \cos\Theta\cos\psi & \sin\Theta & -\cos\Theta\sin\psi \\ \cos\gamma\cos\psi\sin\Theta+\sin\gamma\sin\psi & \cos\gamma\cos\Theta & \cos\gamma\sin\psi\sin\Theta+\sin\gamma\cos\psi \\ \sin\gamma\cos\psi\sin\Theta+\cos\gamma\sin\psi & -\sin\gamma\cos\Theta & \sin\gamma\sin\psi\sin\Theta+\cos\gamma\cos\psi \end{bmatrix}$$

The coordinates of the vertices y and z of the cube after 3 turns in the scalar form can be written in the following lines:

```
y2f_ = sin(teta);
z2f_= - cos(teta)*sin(psi);
y3f_ =  sin(teta) +  cos(gama)*cos(teta);
z3f_= -cos(teta)*sin(psi) +  cos(psi)*sin(gama) + cos(gama)*sin(psi)*sin(teta);
y4f_= cos(gama)*cos(teta);
z4f_ =cos(psi)*sin(gama) + cos(gama)*sin(psi)*sin(teta);
y5f_ =   cos(teta)*sin(gama);
z5f_ = -( cos(gama)*cos(psi) - sin(gama)*sin(psi)*sin(teta));
y6f_ =  sin(teta) +cos(teta)*sin(gama);
z6f_ =  -cos(teta)*sin(psi) -(cos(gama)*cos(psi) - sin(gama)*sin(psi)*sin(teta));
y7f_ = sin(teta) + cos(gama)*cos(teta) + cos(teta)*sin(gama);
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z7f_ = -cos(teta)*sin(psi) + cos(psi)*sin(gama) + cos(gama)*sin(psi)*sin(teta) - (cos(gama)*cos(psi) - sin(gama)*sin(psi)*sin(teta));
y8f_ = cos(gama)*cos(teta) + cos(teta)*sin(gama);
z8f_ = cos(psi)*sin(gama) + cos(gama)*sin(psi)*sin(teta) - (cos(gama)*cos(psi) - sin(gama)*sin(psi)*sin(teta)).

Here yif_ and zif_ are the final positions of the cube vertex in the 0yz plane normalized to the length of the edge of the cube.

3. Analysis of the accuracy of the orientation system

The coordinates of the vertices of the cube have images on the sensitive matrix of the camera and their coordinates are measured by a special program. Measurements of each vertex of the cube except the first allow the formation of two equations. As a result, we obtained an overdetermined system of 14 equations with three unknowns. Often part of the infrared LEDs hidden by the design of the satellite is unavailable for observation. In this case, the number of equations decreases. In any case, redefinition of the system of equations makes it possible to improve the accuracy of its solution in the presence of errors in measuring the coordinates of points on the matrix of the video camera.
Usually, the method of successive approximations is used to solve systems of transcendental equations, requiring the specification of initial conditions. In practice, as such conditions, it is proposed to use estimates of the orientation parameters on the previous tracking cycle.

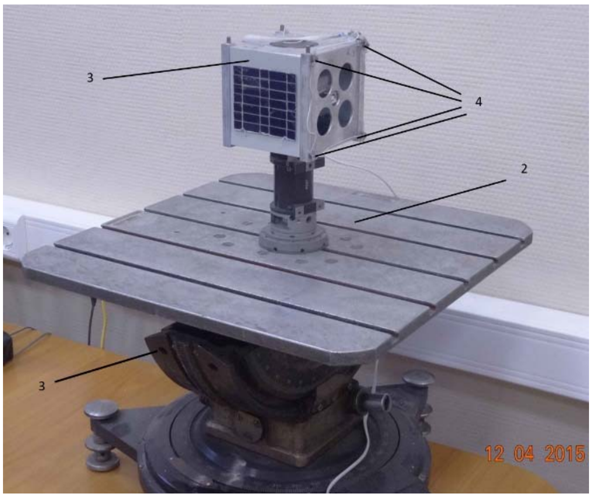

Figure 1: Stand for the experimental investigation of the optical system relative orientation picosatellites (CubeSat)

This figure shows:
1. Precision triaxial rotary mechanism
2. Rotating platform
3. Small satellite (CubeSat)
4. Infrared LEDs

Table 1 shows the statistical properties of satellite orientation measurement errors for one satellite orientation. In Figure2 is a graph of the change in the measurement error of the angle ψ for 1000 measurements. The actual value is 22.2 degrees.

Table 1: Statistical properties of satellite orientation measurement

Parameters	**Values, deg**		
	ψ	υ	γ
Actual rotation angles	22.20	44.00	61.10
Standard deviation	0.332	0.465	0.362
Estimates of the mathematical expectation	22.19	44.01	61.10

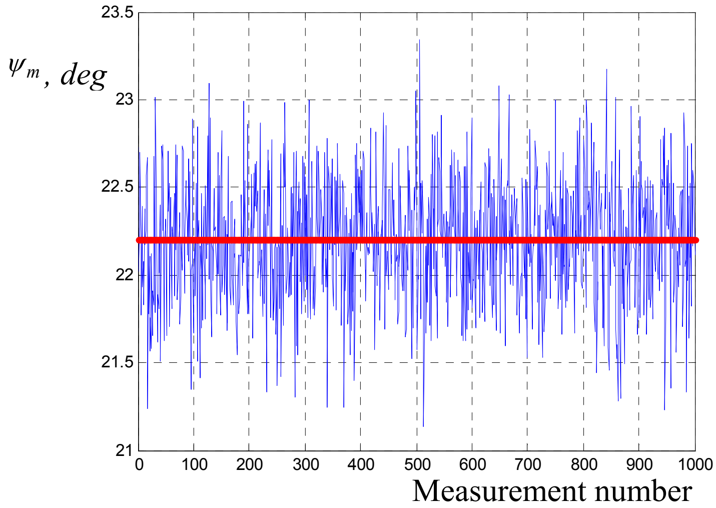

Figure 2. Measurement error. The actual value is 22.2 degrees.

4. Synthesis of the control system of the relative movement of low-located satellites

From the point of view of control theory, the output of a satellite to a given point is a typical task of terminal control. This means that, at a fixed or arbitrary time, the state vector of the control object must be close to the specified value. At the same time, acceptable system behavior must be ensured throughout the entire management process. Usually this problem is formulated as an optimization problem and solved by the methods of the calculus of variations. In the general case, the solution of such problems leads to a two-point boundary value problem.

The boundary conditions for the equations of the object are given at the initial instant of time, and the boundary conditions for the conjugate equations are given at a finite time. The solution of such a problem requires a large amount of computation, the algorithm obtained is of a rather theoretical value and is not suitable for realization in real-time systems. To simplify the implementation, the process of controlling the output of a small satellite into a finite satellite is divided into three stages. At the first stage, the satellite is sent to the finish line. At the second stage, the rotation around the center of mass is performed and the satellite is set in the given orientation relative to the target. At the same time, its center of mass at the finish line is stabilized. At the third stage, the satellites move closer together as they move along the finish line.

At all three stages, the control is carried out by similar algorithms, which facilitates the implementation of a multi-mode control system. The most complex control algorithm corresponds to the first stage of control. For this reason, the study of this stage of traffic management will focus on.

The equation of the unperturbed relative motion of satellites by the Hill-Clohessy-Wiltshire equation, which in vector form looks like this:

$$\frac{d^2\mathbf{s}}{dt^2}[\varepsilon,\mathbf{s}]+[\boldsymbol{\omega},\boldsymbol{\omega},\mathbf{s}]+2\left[\boldsymbol{\omega},\frac{d\mathbf{s}}{dt}\right]=-\frac{\mu\mathbf{s}}{r_p^3}+\frac{3\mu\mathbf{r}_p(\mathbf{r}_p,\mathbf{s})}{\mathbf{r}_p^5}, \qquad (1)$$

Here is the angular velocity of the satellite's rotation around the planet, is the radius vector directed from one satellite to another, and is the radius of the orbit of the main satellite. Then, in the projections on the axis of the orbital coordinate system, in the absence of perturbations, we obtain the following system of linear differential equations:

$$\begin{aligned}&\ddot{x}+2\cdot\omega\cdot\dot{z}=a_x(t);\\&\ddot{y}+2\cdot\omega^2\cdot y=a_y(t);\\&\ddot{z}-3\cdot\omega^2\cdot z-2\cdot\omega\cdot\dot{x}=a_z(t),\end{aligned} \qquad (2)$$

where a_x, a_x, a_x are accelerations from external forces and engines:

$$a_x(t) = \frac{F_x(t)}{m}, \; a_y(t) = \frac{F_x(t)}{m}, \; a_z(t) = \frac{F_x(t)}{m}. \qquad (3)$$

Taking into account the smallness of the satellite's angular velocity around planet $\boldsymbol{\omega}$, at the stage of synthesis of an optimal control system, a simpler model can be used.

$$\ddot{x}(t) = a_x(t), \quad \ddot{y}(t) = a_y(t), \quad \ddot{z}(t) = a_z(t), \qquad (4)$$

In the future, this simplification will be removed. The origin of coordinates of the orbital system O is placed in the center of mass of the main satellite, the axis Oz is directed along the radius vector connecting the centers of mass of the Earth and the satellite; The axis Ox is perpendicular to the Oz axis and lies in the plane passing through the radius vector and the velocity vector of the satellite's center of mass, the Oy axis complements the coordinate system to the right orthogonal system. The system, rotating around the Oy axis, moves along with the satellite in orbit. Such a coordinate system is convenient in that the motion of objects relative to it is represented as if we were observing objects while on the main satellite.

Equations of the relative motion of any two closely spaced satellites (4) in an elliptical orbit under various perturbations, such as the solar wind and other stochastic forces in the state space, can be written in the following form:

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{G}\mathbf{w}(t), \qquad (5)$$

where w (t) is a white noise with a unit intensity.

We synthesize the control law, which minimizes the following functional:

$$J = \frac{1}{2}(\mathbf{x}^T(t_f)\mathbf{S}_f\mathbf{x}(t_f)) + \frac{1}{2}\int_{t_0}^{t_f}(\mathbf{x}^T\mathbf{Q}\mathbf{x} + \mathbf{u}^T\mathbf{R}\mathbf{u})dt, \qquad (6)$$

The solution of the problem is known and can be written in the following form:

$$\mathbf{u}(t) = -\mathbf{K}_c(t)\mathbf{x}(t), \qquad (7)$$

where

$$\mathbf{K}_c(t) = [\mathbf{R}(t)]^{-1}\mathbf{B}^T(t)\mathbf{S}(t), \qquad (8)$$

and

$$\dot{\mathbf{S}}(t) = -\mathbf{S}\mathbf{A} - \mathbf{A}^{\mathrm{T}}\mathbf{S} + \mathbf{S}\mathbf{B}[\mathbf{R}]^{-1}\mathbf{B}^T\mathbf{S} - \mathbf{Q}, \qquad (9)$$

The boundary condition for the last equation:

$$\mathbf{S}(t_f) = \mathbf{S}_f \qquad (10)$$

The system of equations (5) consists of three simple independent equations of the second order. Each of these equations can be rewritten in the following form:

$$\begin{aligned} \dot{x}_1(t) &= 0, \\ \dot{x}_2(t) &= a(t). \end{aligned} \qquad (11)$$

Functional (6) in this case takes the form

$$J = \frac{1}{2}\left([x_1, x_2]\begin{bmatrix} s_{1f} & 0 \\ 0 & s_{2f} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) + \frac{1}{2}\int_{t_0}^{t_f}(a^2(t))dt. \qquad (12)$$

В соответствии с общими формулами (8), (9) и (10) оптимальное управление может быть записано следующим образом:

$$a(t) = -K_x x_{x1}(t) - K_v x_{x2}(t); \qquad (13)$$

DOI: 10.13009/EUCASS2017-452

where $K_x = \dfrac{{}^{1}\!/_{s_{2f}}(t_f - t) + {}^{1}\!/_{2}(t_f - t)^2}{D(t_f - t)}$, $\quad K_v = \dfrac{{}^{1}\!/_{s_{1f}} + {}^{1}\!/_{s_{2f}}(t_f - t)^2 + {}^{1}\!/_{3}(t_f - t)^3}{D(t_f - t)}$, (14)

and

$$D(t_f - t) = \left[\frac{1}{s_{1f}} + {}^{1}\!/_{3}(t_f - t)^3\right]\left[\frac{1}{s_{2f}} + t_f - t\right] - {}^{1}\!/_{4}(t_f - t).^4$$

Structure of the program for the study of the motion of the satellite control processes at a given point is shown in Figure3.

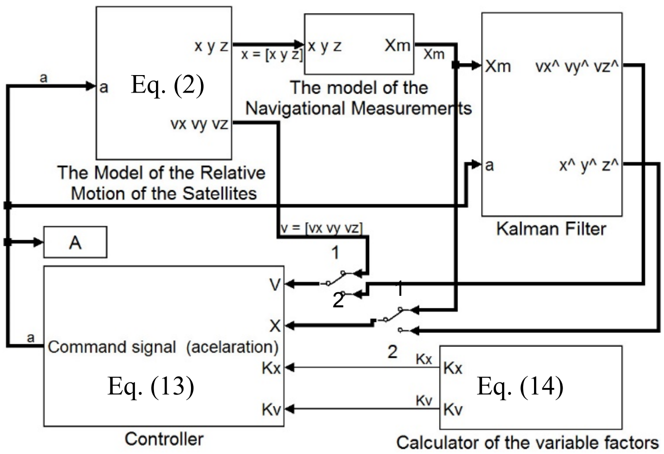

Figure3.Structure of the program for the research of the motion of the satellite control processes

Changing the distance of the satellite with respect to the projections endpoint control process for tf = 360 s shown in Figure4. Changing the projection of the velocity and acceleration of the satellite with respect to is shown in Figure5 and 6.
Changing a satellite acceleration projection when approaching the endpoint to tf = 360 s for the standard deviation of the relative velocity and distance measurement errors σv = 0.3 m/s and σd = 0.3 m shown in Figure 8.

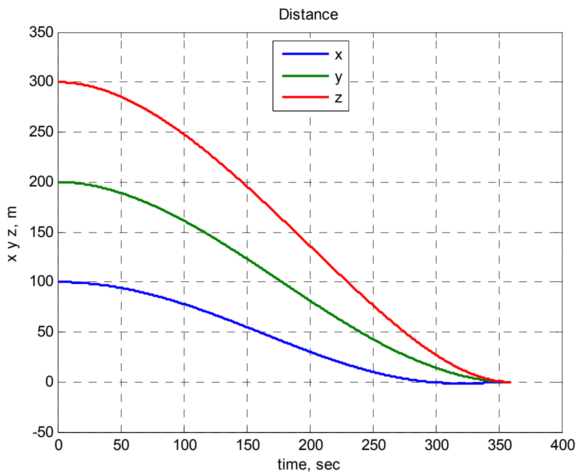

Figure 4: Changing the projections of the satellite distance relative to the end point for tf = 360s..

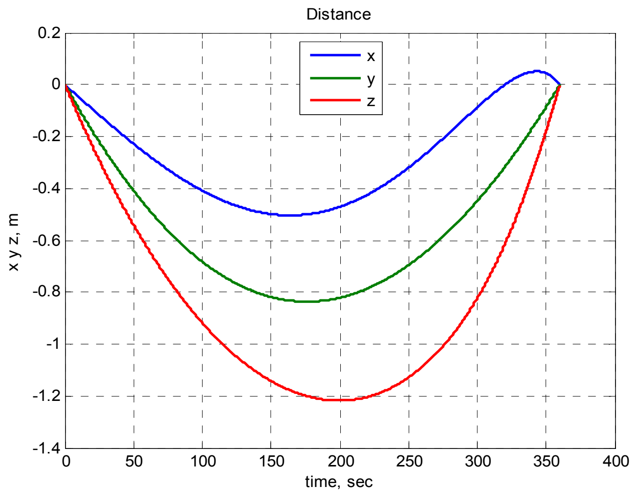

Figure 5: Changing of the projections relative satellite velocity endpoint.

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The simulation showed that the synthesized algorithm provides satellite motion from an arbitrary point in space at a specified point. Random measurement errors component has little effect on the final position, but has a significant effect on the uncertainty of the satellite acceleration at end time (compare Figure 6 and 7).

This effect is explained by the significant increase in controller gain in the vicinity of the endpoint. In Figure9 shows the variation of the regulator's gain factors for the optimal control law (14). This is the reason that the closer to the final control point increases rigidity of control law and increases the effect of random measurement errors in the speed and acceleration of the satellite as it approaches the final point.

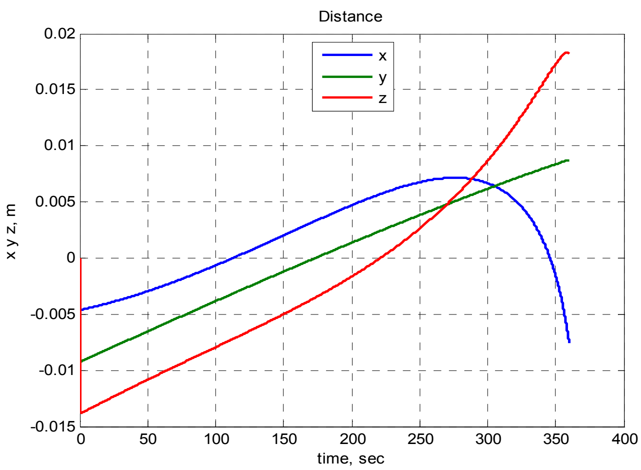

Figure 6: Changing the projections of satellite accelerations when approaching the end point in the absence of measurement errors.

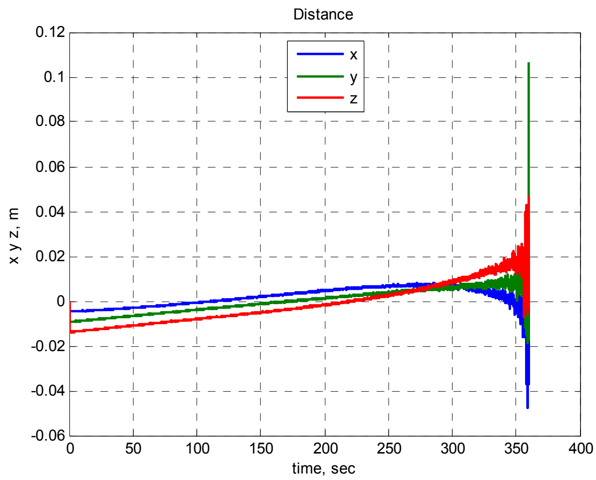

Fig. 7. Changing the projections of satellite accelerations as we approach the end point, taking into account measurement errors.

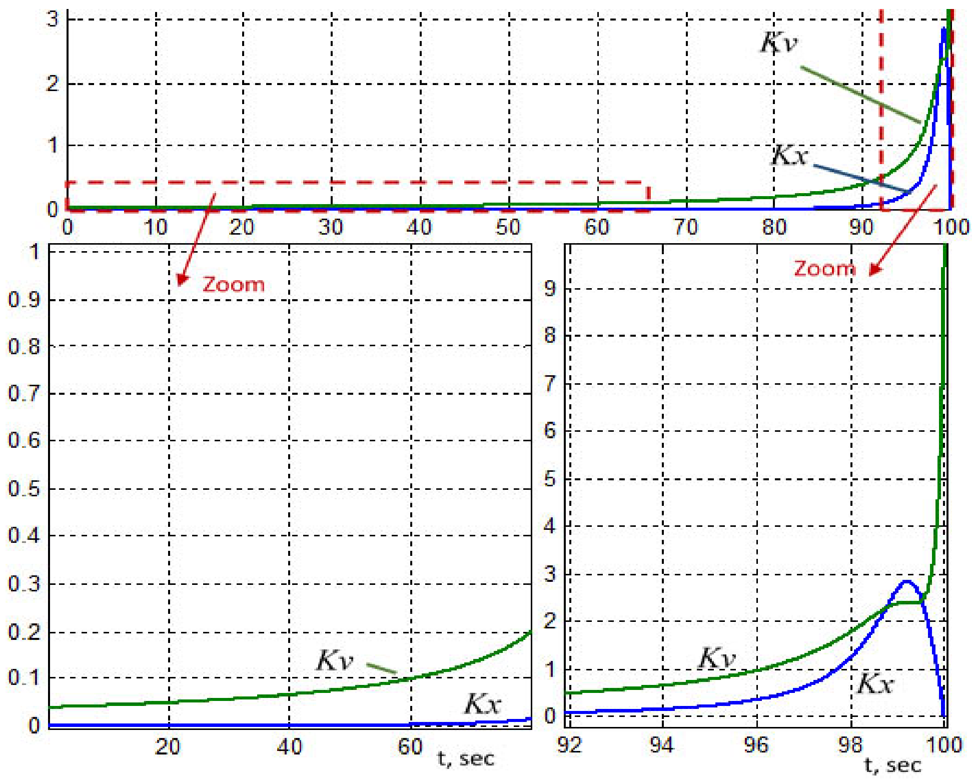

Figure 9: Changing the gain of controller Kx and Kv in time

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5. Filtration of measurements of the optical navigation system

Modeling of the spacecraft control system has shown a strong influence of errors in relative range measurements on the motion of space vehicles, especially at the most important final stage of approach.

For this reason, it is necessary to develop a special algorithm for filtering the measurement information from the optical navigation system. The most effective approach to solving this problem is to use the Kalman filter.

To use the Kalman filter, it is necessary to convert the mathematical model of the satellite motion to the following form:

$$\dot{x}(t) = Ax(t) + Bu(t) + G(t)w(t),$$
$$y(t) = Cx(t) + Dv(t),$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2\omega^2 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -\omega^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 2\omega & 0 & 0 & 3\omega^2 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

Optical navigation system for measuring the 3 projections of the active satellite position relative to the passive satellite. For this reason, we can write

$$D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}.$$

The Kalman filter equation can be written in the following form:

$$\dot{\hat{X}}(t) = A\hat{X}(t) + K_{kf}(Y(t) - C\hat{X}(t)),$$

$$K_{kf} = P(t)C^T(t)R^{-1}(t)$$

$$\dot{P}(t) = AP + PA^T + BQB^T - PC^TR^{-1}CP,$$

$$\hat{X}(0) = M[X(0)], \quad P(0) = P_0.$$

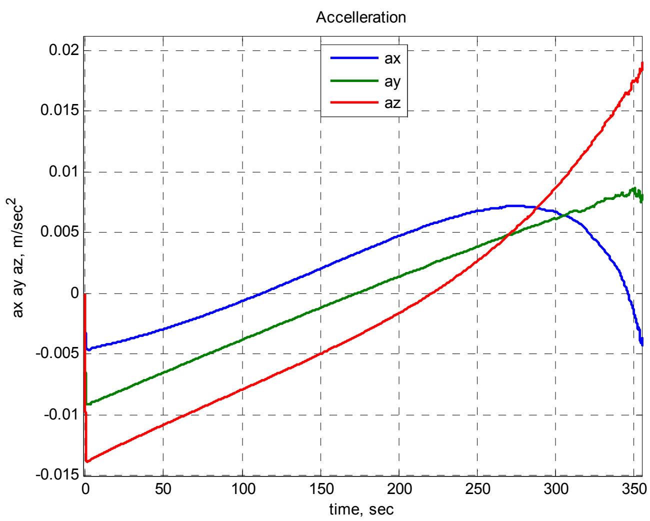

Figure 10 Changing the projection of the acceleration vector of the satellite in the process of approaching the endpoint considering measurement errors.

Simulation of the motion control system for a satellite with a Kalman filter is performed by a program, the structure of which is shown in Figure 4. Switches in this case are set to position 2. The result of the change in the projections of the satellite accelerations is shown in Figure10. The result of comparison with a similar simulation without the filter in Figure8 shows the high efficiency of the Kalman filter application.

Conclusions

The algorithms of relative orientation of the CubeSat based on the optical system were synthesized. Their potential accuracy was investigated. Studies have shown the principle capacity for operation and effectiveness of the proposed control algorithms by the final state. The synthesized control law provides the movement of the satellite from an arbitrary point in space to a given point. It is shown that for control the motion of a satellite it is sufficient to use an optical system, augmented the Kalman filter to reduce the influence of measurement errors on the vector estimation of relative position and the relative velocity vector.

Acknowledgement

The research was supported by the Ministry of Education and Science of the Russian Federation according to the project part of the state funding assignment No 9.1559.2017/PP, 2017-2019. The models of sensors were developed under the support of the Russian Foundation for Basic Research under the project 15-08-00423-a.

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