Stabilization of Crossflow Instability with Plasma Actuators:
Linearized Navier Stokes Simulations

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Abstract
This paper describes work carried out within the Buterfli project to look at the control of transition causing “target” stationary cross flow vortices, by the use of distributed plasma actuation to generate sub-dominant “killer” modes. The objective is to use the “killer” modes to control the “target” modes through a non-linear stabilising mechanism. The numerical modelling and results are compared to experimental studies performed at the TsAGI T124 tunnel for a swept plate subject to a favourable pressure gradient flow. A mathematical model for the actuator developed at TsAGI was implemented in a linearised Navier Stokes (LNS) code and used to model and hence predict “killer” mode amplitudes at a measurement plane in the experiment. The LNS analysis shows good agreement with experiment, and the results are used as input for non-linear PSE analysis to predict the effect of these modes on crossflow transition. Whilst the numerical model indicates a delay in transition, experimental results indicated an advance rather than delay in transition; this was determined to be due to actuator induced unsteadiness arising in the experiment, resulting in the generation of travelling crossflow disturbances which tended to obscure and thus dominate the plasma stabilised stationary disturbances.

1. Introduction

The laminar-turbulent transition process over a swept transport wing under cruise conditions is dominated near the leading edge by crossflow instability. A common approach to controlling such disturbances in laminar wing design involves modifying the base flow of the boundary layer through suction to make it less susceptible to the growth of these instabilities. The system to achieve this incurs a drag penalty due to weight and actuation energy and raises reliability issues due to the complex network of pipes and pumps required. An alternative approach for controlling laminar-turbulent transition at the leading edge is through the promotion of “killer” crossflow modes that retard the growth of the most unstable “target” transition-causing modes via nonlinear interaction. This can be realised through excitation of the “killer” modes with distributed roughness elements or through an active approach involving span-wise periodic forcing, for example with plasmas, heat spots or blowing. Such active approaches for introducing disturbances require less energy compared to approaches involving base flow modification because they involve only small perturbations to the flow over a much more limited chord-wise extent. Furthermore, such a dynamic method offers the possibility of varying the wavelength of the control perturbations to match the changing stability properties of the boundary layer under variations in cruise conditions that occurs during a typical aircraft flight. Plasma actuators appear to be well suited to this role, offering the benefit of rapid response time and flexibility and are capable of producing the small velocity amplitudes that are required.

The viability of this approach has been explored in the EU–Russia “Buterfli” project. In this project a distributed dielectric barrier discharge (DBD) plasma actuator was designed and built by the Joint Institute for High Temperatures (JIHT) in Russia and incorporated into a swept plate model at TsAGI for testing in their T124 low turbulence tunnel under conditions where transition occurs due to crossflow instability. The effectiveness of the control through plasma actuation has been explored through the tunnel test and also through numerical simulations, the results of which are reported in this paper. This involves implementation of an actuator body force model from TsAGI, which was incorporated as a field source term in a Linearized Navier Stokes (LNS) code. The initial “killer” mode amplitudes are determined with the LNS code, which are subsequently used to force nonlinear parabolized stability equations (PSE), to model the primary “killer” crossflow disturbance growth along with the transition-causing mode to study the nonlinear interaction and transition delay achievable downstream. Although the predicted “killer” modes evolution match experimental measurements at a downstream plane the predicted delay in transition...
from the numerical study does not correspond with experiment due to additional unsteady effects arising from the plasma actuation device.

2. Experiment

The experiment in the TsAGI T124 tunnel has been described in detail in [1]. In summary a flat plate with 35 degree sweep was subjected to a favourable pressure gradient flow on one surface through tunnel wall profiling. This was designed to promote the development of cross flow instabilities. Hot wire velocity profiles measurements were made across three spanwise planes at distances of 250mm, 350mm and 600mm from the leading edge. The last of these was expected to be close to the onset of transition. Extensive surface pressure data was also collected. Preliminary numerical studies [2], [3] had shown that the most amplified stationary cross flow mode was close to 7.5mm in spanwise wavelength and this could most effectively be controlled though introduction of a “killer” mode having 2/3 of the spanwise wavelength i.e. 5mm. Thus a DBD actuator was developed by JIHT to promote this particular mode through a periodic arrangement of electrodes [4] The DBD actuator was embedded in the plate with the downstream edge of the exposed electrode at a distance of 120mm from the leading edge of the plate. Measurement data was collected with and without actuation at a tunnel speed of 25m/s which was equivalent to a virtual freestream of 31.9m/s. The plasma actuation in the experiment, unlike the simulations reported in this paper, was found to advance rather than delay transition in the experiment. From the spectral velocity data this was thought to be due to unsteady forcing by the plasma actuator which generated travelling crossflow modes alongside the stationary modes. The travelling modes were sufficiently unstable to trigger the early transition. Finding a way of eliminating this unsteadiness is one of the major technical challenges to be overcome if plasma actuators are to be used in this way. This will be the subject of future work. This paper describes numerical modelling of the actuators neglecting the unsteady effects and certainly the simulations indicate that the actuators should be able to produce a delay in transition, should it prove possible to eliminate and manufacture plasma actuators exhibiting reduced levels of unsteadiness during operation. Nevertheless, the numerical model of the actuator arising from the Buterfly project, and our simulations have been successful in validating some aspects of the experiment, and indeed we show that plasma actuators can be used to control stationary crossflow vortices.

3. Numerical model

3.1 Equations

The generation of the killer cross flow mode has been modelled through incorporation of a modified form of the TsAGI body force model [5] into a linearized Navier Stokes (LNS) solver developed at Imperial College with support from Airbus Group Innovations. As described in some detail in [6], the LNS equations are solved for spanwise periodic disturbances in a spanwise homogeneous base flow with a computational domain that encloses one period of the disturbance inducing roughness or forcing as indicated in Figure 1. A time harmonic span periodic ansatz is assumed for the disturbance, $q$, i.e.

$$\frac{\partial q}{\partial t} = -i\omega q$$

$$\frac{\partial q}{\partial y} = -i\beta q$$

for a specified angular frequency and wavenumber pair $(\omega, \beta)$. Roughness is represented through a “wall” boundary condition obtained from a Taylor expansion in the total flow (base plus disturbance) in conjunction with the requirement that no slip is satisfied where the surface of the roughness would be (i.e. in the interior of the flow rather than the “wall”). Thus,

$$q(x, y, 0) = -H(x, y)\frac{\partial Q}{\partial z}$$

for a given height function $H$ and base flow $Q$. A linear system of equations of the form

$$Lq = r$$
is solved for a linear operator L derived from the linearized Navier Stokes equations through a fourth-order accurate finite difference discretization in x, a pseudo spectral decomposition in z and a spectral (Fourier) decomposition in y the spanwise domain. The forcing term on the right hand side represents the roughness. A similar set of equations is solved for body forcing in the flow interior (as source terms) and has been implemented here for the plasma model developed by TsAGI.

The body force model described in detail in [5], consists of the following expressions for the longitudinal and spanwise components of the force density (N/m$^2$),

$$F_x = \frac{1}{x_0,y_0,z_0} F_{\Sigma} \theta(x) \frac{x_0}{\sqrt{\pi}} e^{-(x_0^2+y_0^2+z_0^2)/2}$$

$$F_y = \frac{1}{x_0,y_0,z_0} F_{\Sigma} \theta(y) y_0 \frac{x_0}{\sqrt{\pi}} e^{-(x_0^2+y_0^2+z_0^2)/2}$$

in which the non-dimensionalisation of coordinates is with respect to the location of the force maximum i.e. $x = x/x_0$, $y = y/y_0$, $z = z/z_0$ with the coordinates of the force maximum location depending linearly on the ratio of the applied voltage to the discharge voltage as

$$x_0 = F_{x1} \left( \frac{V}{V_0} - 1 \right)$$

$$y_0 = F_{y1} \left( \frac{V}{V_0} - 1 \right)$$

$$z_0 = H_1 V$$

The integrated values (for one electrode intersection) of the force density components depend quadratically on the voltage ratio:

$$F_{\Sigma} = A_x \left( \frac{V}{V_0} - 1 \right)^2$$

$$F_{\Sigma} = A_y \left( \frac{V}{V_0} - 1 \right)^2$$

Figure 1 Computational domain of Linearised Navier Stokes solver for spanwise periodic roughness or body forcing in spanwise homogeneous flow.
The integration is over one side (left or right) for the \( y \) component; \( \theta \) is a Heaviside function to ensure forces are zero for negative \( x \). The various coefficients have been empirically determined to have the following values: \( A_x = 1.82 \times 10^{-5} \text{N} \), \( A_y = 4.37 \times 10^{-6} \text{N} \), \( F_{x1} = 1.43 \text{mm} \), \( F_{y1} = 0.65 \text{mm} \), (changed from \( F_{x1} = 0.95 \text{mm} \) in original model) and \( H_1 = 0.0312 \text{mm/kV} \).

The LNS simulations with the implemented model have been applied to a boundary layer corresponding to the experimental conditions for a tunnel reference velocity of 25m/s as described in [1]. The boundary layer was computed with the non-similar boundary layer solver CoBL (Imperial College), using an equivalent virtual freestream velocity of 31.9m/s together with the corresponding pressure coefficient interpolated from experimental pressure measurements, shown in Figure 2 (left). The interpolated pressure data was then used to compute boundary-layer profiles for the subsequent disturbance control simulations involving either LNS or nonlinear PSE. As a check on the correct implementation of the surface pressure data, the CoBL recomputed boundary-layer edge velocity distribution was found to compare very well with the experimentally measured edge velocities, as may be seen in Figure 2 (right). This is a crucial check since instability analysis is known to be very sensitive to any inaccuracies or inconsistencies in the boundary-layer computations.

The resulting velocity profiles are displayed in dimensional and non-dimensional form in Figure 3 and Figure 4 respectively.

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Figure 2 Pressure coefficient based on virtual free stream conditions and measured data (left) and comparison of computed and measured edge velocity (right).

Figure 3 Dimensional boundary layer profiles: component in directions of inviscid stream (left) and cross-flow component (right).
Stability analysis with a linear PSE solver (CoPSE, Imperial College) for a range of stationary cross-flow modes revealed a most amplified mode of around 8mm, Figure 5 (left); which is in line with the analysis in [2] and [3]. A comparison between linear PSE for the 5mm “killer” mode and the LNS simulation for a 5mm span periodic cylindrical roughness element showed good agreement in the amplification based N-factor, shown in Figure 5 (right).

The LNS solver was then applied with the plasma actuator for a range of applied voltages from 3.0kV to 4.0kV. Disturbance contours for an applied voltage of 4.0kV are shown in Figure 6 and Figure 7. The components of the mean velocity or (0,0) mode of the disturbance in Figure 6, show the fluid to be accelerated in the normal to leading edge direction and also the span-wise direction whilst also being drawn towards the surface at the commencement of the actuation (x=0.1m). The real part of the first wavenumber or (0,1) mode, for the stream-wise component of the velocity (in Figure 7) shows the continuous growth in amplitude of the killer mode.
The downstream development of the peak amplitude in the normal to leading edge component of the velocity disturbance is shown in Figure 8. The linear growth is seen to commence in all cases at about 200mm from the leading edge. The development of the disturbance was compared with measurements made at a plane 250mm from the leading edge as shown in Figure 9. This reveals a linear increase in amplitude with applied voltage for both the experiment and the numerical model with good agreement between the two. The amplitude for the highest voltage was used in subsequent non-linear PSE analysis.

Figure 6 Base flow modification, (0,0) mode due to plasma actuation (applied voltage 4.0kV). Normal to leading edge u streamwise component (top), spanwise v component (middle) and wall normal w component (bottom).

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Figure 7 Fundamental (0,1) stationary cross-flow mode in vicinity of actuator (top) and downstream (bottom) for plasma actuation (applied voltage 3.6kV).

Figure 8 Profile maximum disturbance amplitude of killer mode for different actuation voltages; including downstream region with exponential growth.
Figure 9 Amplitude of killer mode as a function of the actuator voltage on the measurement plane, 250mm from the leading edge.

The killer (0,3) mode amplitude at x=0.25m obtained from the LNS model with actuator voltage of 4.0kV has been used to initialise an appropriate initial killer mode amplitude in a non-linear PSE computation with CoPSE, the results of which are shown in Figure 10. The amplitude of the transition causing target (0,2) mode with spanwise wavelength of 7.5mm has been set to correspond with the experimental measurement data reported in [1]. Comparison of the uncontrolled and controlled growth of the target mode indicates a clear reduction in amplitude and an expected delay in transition of about 0.1m. In reality, as was mentioned previously, there was a forward movement in transition in the experiment as reported in [1], where this is explained in terms of the excitation of transition causing travelling modes by the plasma actuator due to the plasma induced unsteady velocity fluctuations which increase linearly with actuation voltage. This unsteadiness in the actuation is not captured in the actuator model described here and thus it is not possible to capture the generation of the travelling modes.

Figure 10 Non-linear PSE computation showing interaction of (0,3) killer mode generated by plasma actuation with (0,2) target mode for (0,1) fundamental mode of 15mm. Red star shows amplitude from LNS calculation.
4. Conclusions

It has been demonstrated that the generation of “killer” modes by plasma actuation can be modelled through a linear Navier Stokes approach which when used in conjunction with a non-linear PSE solver can make a prediction about the delay in transition arising from the actuation. Although the numerical model has been shown to give good agreement with experiment for the amplitude of the killer modes on a downstream plane the subsequent non-linear interaction leading to suppression of the transition causing “target” mode and delay in transition is not what was observed in experiment. This is due to unsteady effects in the plasma actuation and the generation of transition causing travelling crossflow modes in the experiment. Nevertheless should future refinements in the actuator hardware permit the elimination of the unsteady actuation effects, then the modelling approach outlined provides a good basis from which to predict the effectiveness of such actuators.

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References