Evolution of localized artificial disturbance in 2D and 3D supersonic boundary layer

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Abstract
The evolution of a controlled broadband wave packet in flat-plate and swept-wing supersonic boundary layers was experimentally investigated at Mach number M=2. The wave packet was introduced into the boundary layer by a localized pulse electrical discharge. The structure and evolution downstream of the wave packet were studied by hot-wire measurements. It was found that the wave packet has a symmetric shape in a flat-plate boundary layer, whereas there is asymmetry in case of a swept-wing one. The spectral analysis of the development of different modes of the wave packet was provided.

1. Introduction
The knowledge of the processes leading to the laminar-turbulent transition of a boundary layer is crucial for designing high-speed flight vehicles. The turbulence of a boundary layer affects both skin friction and heat transfer and, hence, performance parameters such as drag and structural heat loads. Despite intensive studies of the transition for more than half a century, the laminar-turbulent transition of a high-speed boundary layer is still poorly understood.

One of the efficient experimental methods of the investigation of the laminar-turbulent transition of a boundary layer is an experiment carried out under controlled conditions. The disturbances with known initial characteristics are artificially introduced into the flow. The study of the evolution of artificial controlled perturbations allows one to find out the wave characteristics of the boundary layer.

For the first time, experiments with the controlled excitation of disturbances were performed by Schubauer and Skramstad [1]. Two-dimensional periodic pulsations were generated due to vibrations of a thin bronze ribbon into a subsonic flat-plate boundary layer. This study for the first time has experimentally confirmed the linear theory of hydrodynamic stability. The experimental data were found to be in compliance with the calculated characteristics of Tollmien-Schlichting waves. At present it is generally recognized that the onset of turbulence is connected with the loss of stability of the initial laminar flow. Achievements in studying the development of plane monochromatic waves in a subsonic boundary layer are substantial and universally recognized. The spatial wave trains excited by local harmonic source are used to study the laminar-turbulent transition of a three-dimensional and modulated boundary layer [2, 3].

The approach for the experiments with the controlled disturbances is also effective for high-speed flows. The first experiments with artificial disturbances at high-speed flow were provided by Kendall [4]. An electrical discharge was used to excite controlled fluctuations in a supersonic boundary layer. Further the development of this approach made it possible to obtain significant results. The electrical discharge ignited at a high frequency was used to obtain experimental data on the linear and weakly non-linear evolution of wave trains. The experimental confirmation of the main results of the linear theory of stability in supersonic zero-pressure-gradient flow on a flat plate was obtained in [5]. The first experimental study of the nonlinear stage of the transition to turbulence in a supersonic boundary layer was performed in [6]. These measurements have shown the existence of a parametric resonance interaction of the asymmetric triad of instability waves. Tumin has numerically revealed the evolution of the most unstable waves in Mach 2 boundary layer [7]. This result and the data of [6] coincided in most parts. The method of an artificial excitation of periodic pulsations in boundary layers is also efficiently used at the hypersonic speeds of oncoming flows [8, 9]. This approach is useful in studies of laminar-turbulent transition in inhomogeneous and 3D high-speed flows.
boundary layers [10, 11]. Also, the excitation of periodical disturbances is used in direct numerical simulation studies (DNS) of mechanisms of laminar-turbulent transition [12-15]. Experiments with the controlled periodic pulsations are very important and play a vital role in validating theoretical results for the linear and the nonlinear stages of transition to turbulence. However, it only provides limited insight into a natural transition scenario where a broad disturbances spectrum is excited by freestream pulsations and where complex wave interactions between all disturbance modes are possible.

The detailed knowledge of mechanisms of disturbances interaction can be obtained by the investigation of evolution of controlled disturbances with broad frequency and wave spectrum (wave packets). The artificial wave packets can be introduced into boundary layer by an action localized in time and space. This approach was proposed for subsonic boundary layers by Gaster and Grant in [16], where the propagation of wave packet in boundary layer was first studied experimentally at low speed of flow. The evolution of wave packet generated by a short-duration pulse through a small hole in the surface of the flat plate was investigated by hot-wire measurements. These measurements compared very well with the results obtained from the theoretical model which represented the wave packet as a superposition of individual disturbances for all frequencies and spanwise wave numbers of the most unstable linear waves [17]. Further development of the approach of wave packets led to significant progress in understanding processes of laminar turbulent transition at subsonic speed. Thus, e.g. in [18] the emergence of wave packets, or forerunners, in the frontal regions of localized structures, and their evolution in subsonic boundary layer, was experimentally examined. Experiments showed that the forerunners were Tollmien-Schlichting wave packets. In [19] the oscillating membrane was used for excitation of A-structures in the boundary layer. Experimental results showed that on a smooth surface of a flat plate the A-structure transforms into a turbulent spot.

At high-speeds of flow the method of controlled wave packets is used too. Broadband disturbances are used in the DNS calculations to study high-speed laminar-turbulent transition [20-22]. These works have shown that the approach of wave packets is efficient and provides unique data on the laminar-turbulent transition. The first experimental studies of the evolution of the broadband artificial disturbances were provided by Ladoon and Schneider [23, 24]. The wave packets were excited by a localized in time discharge in a supersonic cone boundary layer. The growth of the wave packet downstream was studied with the help of hot-wire measurements. The experiments on wave packet evolution in hypersonic boundary layers of a nozzle were provided in [25]. Controlled perturbations were excited by a pulsed glow discharge. Pulsations on the wall of the nozzle were measured by pressure sensors. Linear and nonlinear growth of the wave packets and their breakdown into the turbulent spots were observed in the experiments. The wave packet generated by a pulse discharge was used to study the laminar-turbulent transition of a cone boundary layer at Mach number M=8 [26] and M=6 [27]. Also, the method of artificial wave packets was developed in Khristianovich Institute of Theoretical and Applied Mechanics of SB RAS. The experiments on the evolution of wave packets in a two-dimensional supersonic flat-plate boundary layer at Mach number M=2 were provided in [28-30]. Controlled disturbances were excited by a pulse electric discharge. The evolution downstream was measured by hot-wire anemometry.

This work is a continuation of [28-30]. The article is devoted to the description of experimental data on the evolution of wave packets in two-dimensional flat-plate and three-dimensional swept-wing boundary layers.

2. Experimental setup

The experiments were performed in a T-325 long-duration blowdown low-noise supersonic wind tunnel, at the Khristianovich Institute of Theoretical and Applied Mechanics, Siberian Branch of the Russian Academy of Sciences. The Mach number was M=2 and unit Reynolds number was \(Re_z = \frac{\rho U_z}{\mu}, \) \( (6\pm0.1) \times 10^6 \) m\(^{-1}\).

In the present work the evolution of the wave packet was studied in two-dimensional flat-plate and three-dimensional swept-wing boundary layers. Models (Figure 1) were equipped with a source of controlled disturbances. The plate (Figure 1a) had the following dimensions: width \(200 \) mm, length \(440 \) mm, thickness \(10 \) mm. The bevel angle from the leading edge was \(14.5^\circ\). The leading edge thickness did not exceed \(0.1 \) mm. The wing (Figure 1b) had the following dimensions: width \(200 \) mm, length \(260 \) mm, maximum thickness \(20 \) mm. The swept angle of the leading edge was \(40^\circ\). The plate and the wing were installed with zero angle of attack. The accuracy of models installation was about \(0.06^\circ\).

To initiate controlled wave packets an electric discharge was ignited between two copper electrodes, separated from each other and the model with an insulator. The insulator and electrodes were mounted flush with the models surface. The electrode centers were located parallel to the direction of the flow (the x-axis). High-voltage (~1 kV) pulses were supplied to the electrodes from the ignition schematic.

The main coordinate systems used in this paper are shown in Figure 1. The x-axis is directed downstream and parallel to the oncoming flow in both cases of flat plate and swept wing. The x=0 was set at the leading edge on the line of electrodes center. In the case of the flat-plate model coordinate z is perpendicular to the free stream. In the case of the model of swept-wing the z’ is parallel to the leading edge.
The ignition schematic for pulsed electric discharges is given in Figure 2. It is based on disrupting the current in the primary circuit of the ignition coil with a powerful bipolar transistor. Disrupting the current in the primary circuit of the ignition coil generates high-voltage pulsing. The current in the primary circuit can be regulated by changing the resistor $R'$, which affects energy linked up with the discharge. For the ignition of discharge, the primary circuit was interrupted when the negative-going edge of the control signal hit the transistor base.

A voltage drop on electrode of the pertuber and discharge current were measured by a two-channel oscilloscope Tektronix TDS 1012b to control of the discharge ignition. The voltage was measured with the help of a high-voltage divider. The discharge current was measured on the resistor $20 \Omega$. The example of the oscillogram of the discharge voltage and current in experiment are presented in Figure 3. As it can be seen a single discharge is observed. The discharge breakdown voltage is about 900 V. The duration of the discharge was about 20÷25 mks.

Pulsations in a supersonic flow were measured using constant-temperature hot-wire anemometry (CTA). A hot-wire probe, made of a 10 µm diameter tungsten wire, was used. The constant voltage component “$E$” of the anemometer output was measured with an Agilent 34401A digital voltmeter. To recalculate “$E$”, values of mass flux ($\rho U$) in the supersonic part of the boundary layer, the known calibrating dependency for hot-wire anemometry, were applied.
\[ E^2 = L + K(\rho U)^n, \] where \( L \) and \( K \) are dimensional calibration factors. At the power coefficient \( n = 2 \), \( S_{\text{u}} \) – sensitivity of the hot-wire probe to mass flux pulsations, the equation is slightly simplified: \[ E^2 = K(\rho U)^n. \]

Performing a number of measurements in a free flow, we can evaluate the dimension factor \( K \) and apply it to recalculate the values of mass flux within the boundary layer. Hot-wire probe sensitivity to mass flux pulsations for CTA is constant and equals \( S_{\text{u}} = 0.25 \pm 0.02 \) [31]. The overheat rate of the hot-wire probe was set to 0.7÷0.8, while perturbations measured by CTA were predominantly mass flux pulses (\( m' \)).

The anemometer pulse signal (\( e' \)) was digitalized with a 12-bit analogue-digital convertor (ADC). The ADC sampling frequency was 1250 kHz. Non-dimensional mass flux pulsations were calculated using the following formula:

\[
m'(t_k) = e'(t_k) / (E S_{\text{u}}) \times 100\%
\]

The probe was moved with the help of traversing gears which the wind tunnel is equipped with. The measurements were performed at the layer with the most intense natural pulsations of the boundary layer. A trajectory of the probe was parallel to the leading edge. In case of the flat-plate the probe was moved along \( z \)-coordinate. Five cross sections were measured at \( x = 60, 70, 80, 90 \) and 100 mm. In case of the swept wing the probe was moved along \( z' \)-coordinate. Three cross sections were measured at \( x = 48.6, 58.6 \) and 68.6 mm.

To separate perturbations generated by the discharge from the background natural fluctuations of the boundary layer, the measurements were synchronized with discharge ignition. 320 traces were recorded at each measurement point. Data processing included ensemble-averaging 320 traces, using the following formula:

\[
m'(z_j, t_k) = \frac{1}{320} \sum_{1}^{320} m'(z_j, t_k)
\]

Figure 4 presents the results of ensemble-averaging. In Figure 4a the number of oscillograms are presented. The measurements were taken in synchronization with discharge ignition, and the position of the hot-wire probe remained constant throughout the measurements. In Figure 4b the results of ensemble-averaging are given. We can see that the averaging considerably decreases the amplitude of random natural oscillations in the boundary layer, and it allows the artificial wave packet to be separated.

Figure 4: Ensemble-averaging process: a – a number of oscillograms; b – the results of ensemble-averaging.

The spectral analysis was conducted to obtain the detailed information about the evolution of the wave packet. In this work distributions of the amplitude from the spanwise wave number are examined at different frequencies: \( f = 15, 20, 25 \) and 30 kHz. The frequency-wave spectra of disturbances at a fixed \( x \)-coordinate were determined using the discrete Fourier transform in the form (3).
\[
\tilde{m}(x, \beta, f) = \sum_{j,k} m'(x, z_j, t_k) \exp(i \beta z_j - i 2 \pi f t_k) \Delta t \Delta z_j
\]  

(3)

For clarity, in this work the spectra were normalized on the maximum value in the initial section for each frequency:

\[
A(x, \beta, f) = \frac{1}{\text{Max}_x |\tilde{m}(x_0, \beta, f)|} |\tilde{m}(x, \beta, f)|
\]  

(4)

3. Results

3.1 Flat-plate

The evolution of the wave packet generated by pulse discharge was studied in the supersonic flat-plate boundary layer. Five cross sections were measured at \(x=60, 70, 80, 90\) and \(100\) mm. The measurements were provided in the area of the maximum level of the boundary layer pulsation. For each cross section the hot-wire probe was moved parallel to the leading edge and at a constant distance to the wall. In Figure 5 the contour lines of the mass flux pulsation \(m'\) in plane \((z, t)\) for the cross sections at \(x=60, 80\) and \(100\) mm are presented. In the graphs, the positive values of the pulsations are shown with solid red lines, whereas the negative values are shown with blue dotted lines.

As expected, the disturbances exited by the discharge are symmetric with respect to \(z=0\) mm. The leading edge of the wave packet consists of three regions of instantaneous flow defect. Areas with negative perturbations are observed with the maximum level at \(z \approx 0\) and \(\pm 1.5\) mm. Also, the bifurcated positive structure with the maximum deviations at \(z \approx \pm 0.7\) mm is observed at the trailing front of wave packet.

From the data in Figure 5, it is evident that the wave packet, as it propagates downstream, spreads in cross-flow direction. From the contour lines the estimates of the spreading angle of isolated wave packets were made. The half-angle of spreading of the wave packets across the stream proved to be about \(5^\circ\). Also moments of time of the wave packet registration are varying downstream. Thus the velocity of propagation downstream of the wave packet can be calculated. It is estimated that the velocities of the leading front of the wave packet propagation is about \(0.9 U_\infty\).

The wave packet generated by the pulse discharge is localized in space and time. Therefore the wave and frequency spectra of the controlled disturbances are broadband. Thus the evolution of different modes of the wave packet can be investigated. The spectral analysis of the data measured at \(x=60, 70, 80, 90\) and \(100\) mm was carried out. For each \(z\)-sections the time-frequency and space-wave discreet Fourier transform were provided (Eq. (3)). In Figure 6 the normalized (Eq. (4)) wave spectra are presented for different frequencies.

Figure 5: Contour lines of the mass-flow pulsations of the wave packet in the plane \((z, t)\) at \(x=60\) (a), \(80\) (b) and \(100\) (c) mm.

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Figure 6: Wave spectra at $f=15$ (a), 20 (b), 25 (c) and 30 (d) kHz.

It is clear from the figure that, the selected frequencies modes of the wave packet grow downstream at initial sections. The most unstable waves have spanwise wave numbers $\beta=1\div2$ rad/mm. It means that the fronts of these waves have strong inclination angle to the flow direction. This is consistent with the results of the linear theory of stability of a supersonic boundary layer. On the other hand, these strong values of wave numbers of the most unstable waves are much higher than the results obtained in experiments with the linear wave trains [5]. It should be noted that at the $x=90\div100$ mm the fading of high-frequencies ($f>20$ kHz) modes of the wave packet is observed. At $f=30$ kHz the disturbance fades at $x>70$ mm. The fading of the disturbances seems to be consistent with the upper branch of the neutral stability curve.

3.1 Swept-wing

The contour lines of the mass flux pulsation $m'$ in plane $(z', t)$ for the cross sections at $x=48.6$, 58.6 and 68.6 mm are presented in Figure 7. As in the case of the flat-plate the measurements were provided in the area of the maximum level of the boundary layer pulsation. For each cross section the hot-wire probe was moved parallel to the leading edge and at a constant distance to the wall.

The disturbances excited by the discharge are asymmetric and shifted to negative values of the $z'$-coordinate. This result differs from the case of a two-dimensional supersonic boundary layer. This is due to the crossflow in the boundary layer of the swept-wing. Similar results were obtained in the experiments on the evolution of localized disturbances in an incompressible boundary layer [32].

In the initial section, the perturbation from a pulsed discharge is a flow defect localized in space and time. The spatial scales of the wave packet increase downstream. The structures of an excess and a decrease of the mass flow are formed at the boundaries of the disturbance from the discharge. At $z'= -3\div4$ mm wave packet is represented as a vortex in the plane of the model surface. At $z' < -3$ mm the short-term defects of mean flow are observed.
Moments of time of the wave packet registration are varying downstream. Thus the velocity of propagation downstream of the wave packet can be calculated. It is estimated that the velocities of the wave packet propagation is about $0.5 \div 0.6 U_{\infty}$.

![Figure 7](image1.png)

**Figure 7**: Contour lines of the mass-flow pulsations of the wave packet in the plane $(z', t)$ at $x=48.6$ (a), 58.6 (b) and 68.6 (c) mm.

The evolution of different modes of the wave packet is presented in Figure 8. The spectral analysis of the data shown in Figure 7 was carried out. The normalized (Eq. (4)) wave spectra are presented for different frequencies.

![Figure 8](image2.png)

**Figure 8**: Wave spectra at $f=15$ (a), 20 (b), 25 (c) and 30 (d) kHz.

As can be seen, the selected frequencies modes of the wave packet grow downstream at the measured area. The frequency of the most unstable modes of the wave packet is about 25 kHz. In contrast to the case of the two-dimensional boundary layer, the wave number spectra of the wave packet in the swept-wing boundary layer is asymmetric. Similar spectra were observed in experiments with controlled wave trains [10].
The most unstable waves have values of spanwise wave numbers $\beta^*=-2.5\pm1$ rad/mm. These results agree qualitatively with the linear calculations for this experimental model and close flow parameters [33].

4. Conclusion

The structure and evolution of the wave packet in the supersonic flat-plate and swept-wing boundary layers have been experimentally studied. In case of the flat-plate the wave packet is symmetrical, whereas there is asymmetry in a swept-wing boundary layer. The spanwise spreading downstream of the wave packet has been observed. In case of the flat-plate supersonic boundary layer the spectral analysis has shown that the most unstable modes of the wave packet are strongly inclined. This is consistent with the results of the linear theory of stability of a supersonic boundary layer. The spectral analysis of the experimental data on the evolution of the wave packet in the swept-wing boundary layer has shown that the wave number spectra are asymmetric. The approach of the broadband artificial disturbances allows one to obtain an extensive picture of the evolution downstream of perturbation in boundary layers. In contrast with the method of the narrowband artificial disturbances, the development of waves with different frequencies can be qualitatively investigated with the help of artificial wave packets.

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