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Motor mass optimization for the maximization of solar-powered aircraft performance

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Abstract
The problem of energy saving through the maximal power restriction for the solar-powered airplane is investigated. Based on the previous results the model of this phenomenon was formed. Numerical and analytical investigations of maximal energy saved were conducted. Numerical results for artificial plane were obtained. Analytical results were compared with numerical ones and have shown acceptable accuracy.

Nomenclature

A_0 – derivative of power obtained from the solar cells with respect to altitude
C_{D0} – drag coefficient at zero lift
C_L – lift coefficient
g – acceleration of gravity
h – flight altitude with respect to the reference altitude
I_0 – power obtained from the solar cells at the reference altitude
J – power obtained from the unit area of solar cell
m – aircraft mass
S – wing area
V – airplane velocity
V_Y – vertical component of velocity
W – power of powerplant
W_A – aerodynamic power
W_{SC} – power available from solar cells
β – motor power/mass ratio, corresponding to the maximal motor efficiency
λ – aspect ratio of the wing
ρ – air density
ρ_0 – air density at the reference level where h=0
η – powerplant efficiency
Ω – angular velocity of the Earth's rotation

1. Introduction

The idea of solar power flight is old enough. Up to now a set of solar-powered aircrafts was designed and build, a set of records was registered for such the aircraft, among them are the altitude record and flight duration record for electrical-powered aircraft. The flight of Solar Impulse plane has demonstrated the possibility of flights with the man onboard.
But at the same time the design and utilizing of such an aircraft up to now is a great challenge because of the moderate value of solar radiation, low value of solar cells efficiency and some other reasons. This makes the investigations for the performance improvement necessary enough. The investigations in this way can be in the area

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of design and in the area of optimal control during the flight. But some improvements need the analysis in both areas simultaneously.
A set of investigations concerning the design and control was conducted previously [1]–[4] and a set of useful results was obtained. Among the results are: optimal mass distribution between the parts of aircraft [2], optimal flight trajectory in the absence of limitations [1], [3] and with restrictions on trajectory [1], [3] and accumulator mass [4].
Another question must be investigated. While moving along the optimal trajectory the aircraft needs the power for the motor that changes with time (and altitude). If one restricts the power by some value, the flight trajectory becomes "less optimal" and some extra energy consumption (for all the flight) occurs. On the other hand, less power consumption imply less powerful motor with less mass. This leads to the less total weight of aircraft and less power consumption. These two effects in total can lead to some optimal value of maximal power which gives the best result in energy consumption.

2. Model and assumptions

The models of aircraft, solar radiation and atmosphere used in this investigation are the same as for the previous ones done by authors [1], [3].
So, the solutions for the optimal control are also the same:

- during the night time the aircraft must fly at the lowest altitude available,
- during the time of enough solar radiation three types of control are available:

a) maximal power available
b) minimal power available
c) singular control (SC) (the altitude dependence on the time of day was obtained in [1], [3]).

If the dependence of air density and solar radiation on the altitude h are defined, then the values of ρ and h at the mode of SC can be found from [1], [3]

$$\rho^3 = 8\left(\eta\frac{\partial J}{\partial h}\right)^{-2}\left(\frac{G}{S}\right)^3\sqrt{\frac{A^3 C_{D0}}{27}}\left(\frac{\partial \rho}{\partial h}\right)^2, \qquad (1)$$

where efficiency η is assumed to be constant.
The equation for the isothermal atmosphere in which the density depends on the altitude by the formula defined in [1] as

$$\rho=\rho_0\exp(-h/h_0) \qquad (2)$$

(h_0=6 374 m, ρ_0 is the air density at the reference level where h=0), gives

$$\rho = 8\left(\eta h_0\frac{\partial J}{\partial h}\right)^{-2}\left(\frac{G}{S}\right)^3\sqrt{\frac{A^3 C_{D0}}{27}}. \qquad (1a)$$

The corresponding velocity V is [1], [3]

$$V = -\sqrt{\frac{3}{AC_{D0}}}\frac{\eta\rho}{2(G/S)\frac{\partial \rho}{\partial h}}\frac{\partial J}{\partial h}, \qquad (3)$$

that for the isothermal atmosphere gives

$$V = \sqrt{\frac{3}{AC_{D0}}}\frac{\eta h_0}{2(G/S)}\frac{\partial J}{\partial h}. \qquad (3a)$$

The corresponding value of lift coefficient C_L is [1], [3]

$$C_L = \sqrt{3\pi\lambda C_{D0}} = \sqrt{\frac{3C_{D0}}{\pi\lambda}}. \qquad (4)$$

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It was decided to restrict this investigation only by the case of isothermal atmosphere.
Assuming A_0 as constant, the model of solar radiation for the power W_{SC} available from solar cells is

$$W_{SC} = JS = \left(I_0 + A_0 h\right)\sin\left(\Omega t\right), \quad (5)$$

So, the value of the aerodynamic power W_A can be obtained from [1]–[3] as

$$W_A = C_D \rho \frac{V^3}{2} S = 2\eta h_0 A_0 \sin\left(\Omega t\right).$$

The example of the trajectory is shown in Figure 1.

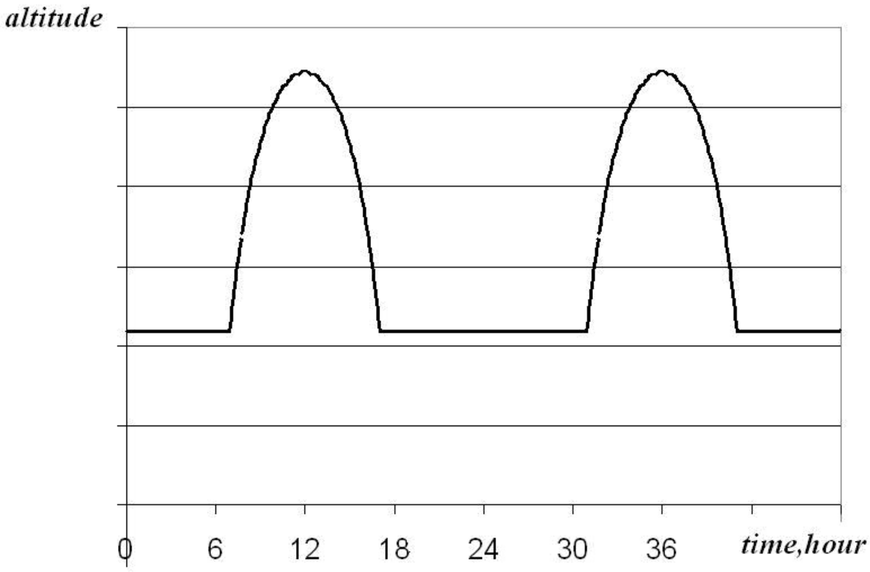

Figure 1: Example of optimal trajectory [1], [3]

It should be mentioned that the cases of maximal and minimal power were not investigated previously as it was assumed that powerplant has enough power and also can work as energy generator so the cases of maximal and minimal power are not realized.
In the present investigation the main goal is to investigate the cases of power restriction. On the other hand there is no limitation on the vertical component of airplane velocity V_Y and its derivative, so for the case investigated the dependences obtained for vertical velocity singular control are valid.

3. Preliminary analysis

First of all, some preliminary investigations were conducted to understand the value of energy losses due to the restrictions of power.
It is necessary to define the dependence of motor power required as function of day time. This power is given by formula

$$W = \frac{1}{\eta}\left(\frac{m}{2}\frac{d}{dt}\left(V^2\right) + 4C_{D0}\rho\frac{V^3}{2}S + mgV_Y\right).$$

Substituting dependencies (1)–(4) one can obtain

$$W = \frac{1}{\eta}\left(\frac{3}{AC_{D0}m}\left(\frac{\eta h_0}{2g}\right)^2 A_0^{\,2}\Omega\cos\left(\Omega t\right)\sin\left(\Omega t\right) + 2h_0 A_0 \sin\left(\Omega t\right) + mg2h_0\frac{\Omega\cos\left(\Omega t\right)}{\sin\left(\Omega t\right)}\right). \quad (6)$$

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The example of power for some aircraft performance is shown in Figure 2. One can see that the highest value of power corresponds to the beginning of SC mode. At the second part of this mode (after the midday) the negative power realizes. This means that the motor must work as generator. If this function of motor is unavailable, the aircraft must fly at the lowest power available (for example with zero power that corresponds to the "glider" flight). This case is not investigated in the present research.

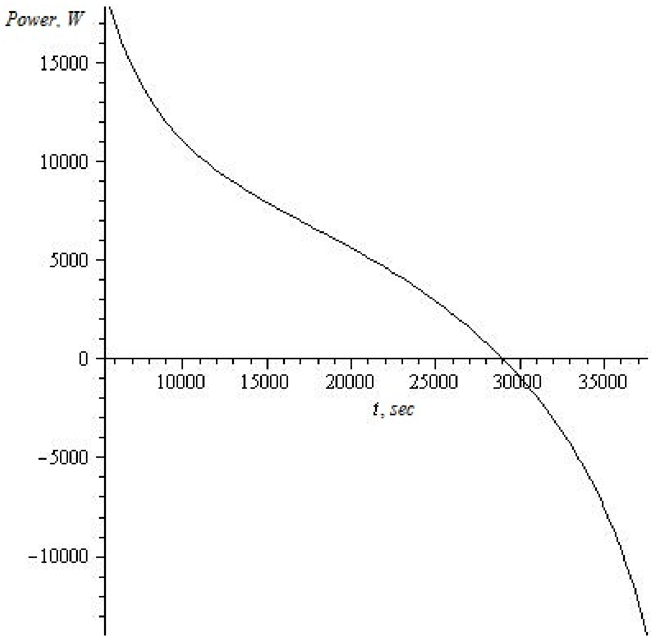

Figure 2: Power as function of time at the SC mode

The method of preliminary investigation was as follows. The optimal trajectory with the restriction on lowest altitude was used as the basis (see Figure 1). Then the limitation on the power was implied and the trajectory was changed. According to the rules and requirements of optimal control for this case [1], [3] the new trajectory must have the shape shown in Figure 3. So, one can see that the difference between the optimal and "restricted" trajectories is between the points 1 and 2. This means that the stored energy difference between the cases with and without restriction is only the energy difference between points 1 and 2.

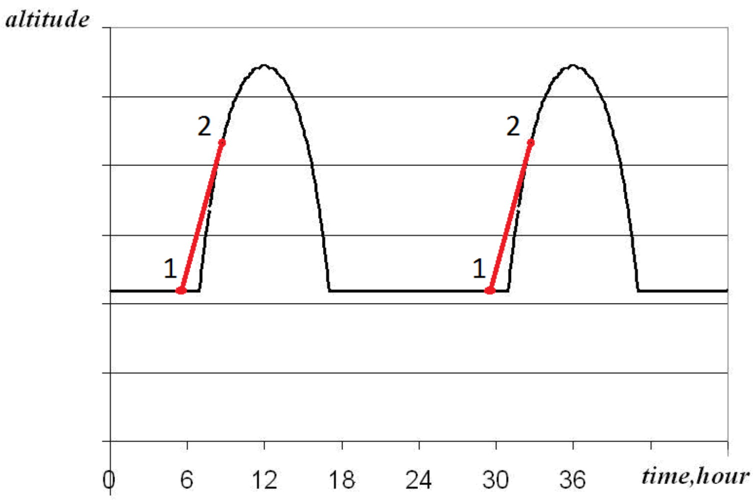

Figure 3: Example of the trajectory with the restriction on the power. Red line – mode of power restriction

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Using the equations obtained previously [1]–[3] one can find the difference between the energy from the solar cells and the energy consumed by the motor.
It should be mentioned that the dependence for lift coefficient during the mode of maximal power is not so simple as for the singular control (SC). In the case of SC it was as (4). In the case of power restriction this dependence can be derived from the equations [1], [3]:

$$C_L \rho \frac{V^2}{2} S = mg \,,$$

$$m\dot{V}V = m\dot{Z} = W_{max}\eta - \left(C_{D0} + AC_L{}^2\right)\rho \frac{V^3}{2} S - mgV_Y \,,$$

$$\dot{h} = V_Y \,,$$

$$\dot{P}_Z = \frac{2P_Z}{m}\left(\frac{3}{2}C_{D0}\rho \frac{Z^{1/2}}{2} S - \frac{1}{2}\frac{A(2mg)^2}{2Z^{3/2}\rho S}\right),$$

$$-2g\frac{2P_Z}{m}\left(\frac{3}{2}C_{D0}\rho \frac{Z^{1/2}}{2} S - \frac{1}{2}\frac{A(2mg)^2}{2Z^{3/2}\rho S}\right) + \left(\frac{2P_Z}{m}\left(C_{D0}\frac{Z^{3/2}}{2} S\frac{\partial \rho}{\partial h} - \frac{A(2mg)^2}{2Z^{1/2}\rho^2 S}\frac{\partial \rho}{\partial h}\right) + \left(\frac{\partial I}{\partial h}\right)\right) = 0 \,,$$

$$\dot{E} = W_{max} - I\left(h(\rho), t\right).$$

where Z and P_Z – some additional variables, see [1]–[3].
But it should be mentioned that the difference in results between these two cases is very small. So, for the simplicity of calculations the dependence for C_L is here assumed to be as at SC mode.
The artificial aircraft tested has the following characteristics:

- total mass – 599 kg
- wing area – 200 m^2
- powerplant efficiency – 80%
- solar cells efficiency – 20%
- lowest altitude – 12 200 m (ρ_0=0.3 kg/m^3)

For these data the maximal power is 17 829 Watt, power for the night time at lowest altitude is 1 941 Watt. One can see that the difference between these values is more than nine times.
Numerical calculations were conducted and the results are shown in Table 1 and in Figure 4. One can see that the difference is small enough comparing to the energy consumed by the motor for night time (~$8.4 \cdot 10^7$ Joule). This dependence is non-linear.

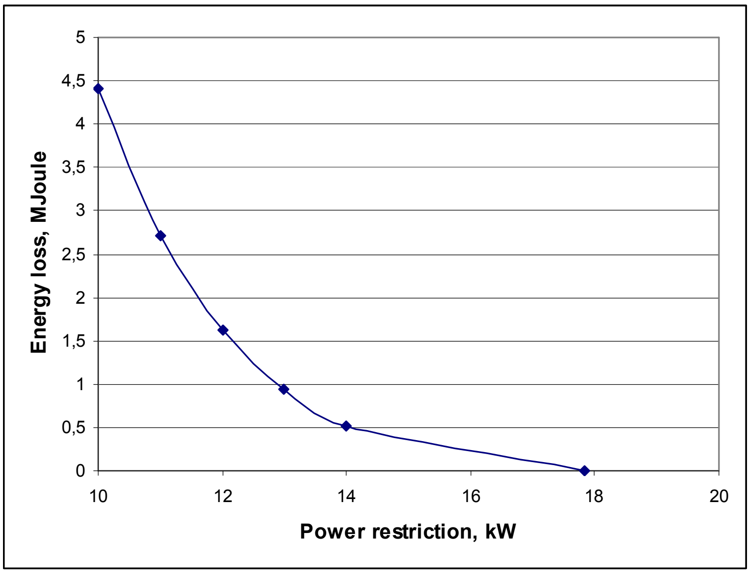

Figure 5: Energy loss for different power restrictions for different power restrictions for m=599 kg.

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Table 1: Energy loss for different power restrictions for different power restrictions for *m*=599 kg

	14 kW	13 kW	12 kW	11 kW	10 kW
Energy difference, Joule	$5.08 \cdot 10^5$	$9.44 \cdot 10^5$	$1.63 \cdot 10^6$	$2.72 \cdot 10^6$	$4.48 \cdot 10^6$

4. Mass difference investigation

Let's make the investigation of characteristics changes for total mass change of 4 kg (for total mass *m*=595 kg) and 9 kg (for total mass *m*=590 kg).
For *m*=595 kg, the power at the beginning of SC mode is 17 893 Watt (so it has become little higher) and power consumption during the nigh time is 1 897 Watt (it has become little lower). This can be explained from previous results [3], [4]. The difference in stored energy comparing to the case of *m*=599 kg is $2.28 \cdot 10^6$ Joule for 24 hours. So, one can see that the lower mass is really leads to lower amount of energy.
For *m*=590 kg, the power at the beginning of SC mode is 17 973 Watt and power consumption during the night time is 1 897 Watt. The difference in stored energy comparing to the case of *m*=599 kg is $5.15 \cdot 10^6$ Joule for 24 hours.
The dependence of energy changes as function of aircraft mass in the range of 590–599 kg is shown in Figure 5. One can see that the dependence is linear, so it can be possible to make the linearization of the dependencies to obtain analytical result.

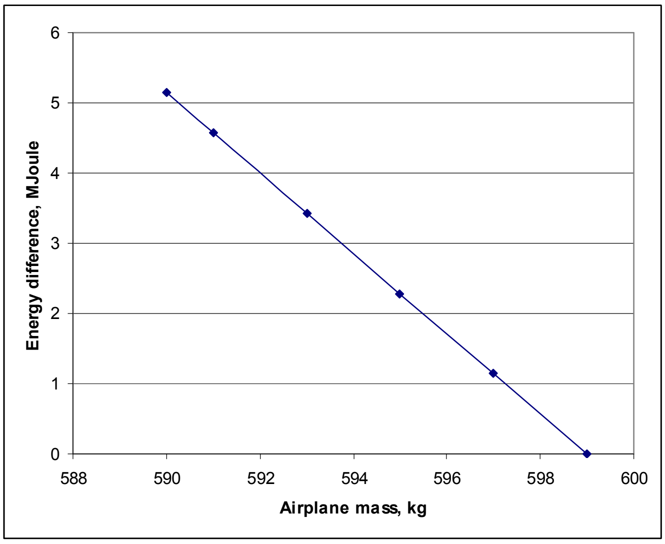

Figure 5: Energy difference as the function of airplane mass

The energy losses due to the power restrictions for the cases investigated are shown in Tables 2, 3. Comparing data from Tables 1–3 one can conclude that the energy loss for the power restriction analysed is practically independent of the airplane mass in the range taken into account.

Table 2: Energy loss for different power restrictions, value and in % of the energy consumed by the motor for 24 hours for different power restrictions for *m*=595 kg

	14 kW	12 kW	10 kW
Energy difference, Joule	$5.13 \cdot 10^5$	$1.63 \cdot 10^6$	$4.45 \cdot 10^6$
Difference, %	0.32	0.99	2.69

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Table 3: Energy loss for different power restrictions, value and in % of the energy consumed by the motor for 24 hours for different power restrictions for m=590 kg

	14 kW	**12 kW**	**10 kW**
Energy difference, Joule	$5.19 \cdot 10^5$	$1.63 \cdot 10^6$	$4.41 \cdot 10^6$
Difference, %	0.32	0.99	2.69

From Figures 2, 3 one can see that energy loss grows higher than energy saving due to the mass decrease. So, it is profitable to use the restriction in power only till some margin of power. After this margin the decreasing the motor mass will not give the energy saving.
The main question now is: to what amount of power this amount of mass decrease corresponds. For modern electrical motors the power/mass ratio β corresponding to the maximal motor efficiency is about 1–1.5 kW/kg.
For example, in the case of aircraft considered for 1 kW/kg the mass decrease make effect up to approximately 591 kg of total mass, and the maximal advantage corresponds to about 594 kg and has value of 1.9 MJoule. For 1.5 kW/kg the effect is up to 594 kg, maximal advantage is for 595.8 kg and the value is 1.2 MJoule.
But we must take into account the following. The lower mass implies less power consumption during the night time. This means that less amount of accumulator is required. The consumed energy difference during the night time ΔE_N for the mass difference Δm can be estimated as

$$\Delta E_N = 1.5 E_N \frac{\Delta m}{m}$$

where E_N is the energy consumed during night time. So, for the aircraft investigated for the energy density of LiPo accumulators 0.05 kg/(W·h) one can obtain extra energy saving of $\Delta E_N \approx 1.84$ kg.
One thing must be emphasized. Even the motor with lower mass is chosen, all the investigations were made for the case of motor working on the regime of maximal efficiency. This means that the motor is assumed to work not at the maximal power.
This can lead to the another conclusion. It is well known that the motor working not at the most efficient power has higher efficiency and the more is the difference in power with respect to the optimal the less is the efficiency. So, in the reality the motor that enables the flight without the restrictions on power will work in more wide range of power. So, it's «mean efficiency» for 24 hours will be less than for the motor with restrictions. So, in reality it is profitable to use the restriction in power.

4. Analytical approach

The abovementioned results can be estimated using analytical method. Moreover, analytical approach can give some additional results.
The energy consumption for the singular control mode without restriction can be obtained using (5)-(6) as

$$E = \frac{1}{\eta}\left(\frac{m}{2}V^2 + mgh\right) + \frac{1}{\Omega}\left(I_0\cos(\Omega t) - 2A_0h_0\left(2\cos(\Omega t) + \ln\left(\mathrm{tg}\left(\frac{\Omega t}{2}\right)\right) - \cos(\Omega t)\ln\left(\frac{\sin(\Omega t)}{\sin(\Omega t_0)}\right)\right)\right) + \mathrm{const}$$

where t_0 is some reference time corresponding to some reference altitude.
If the first and last points of the trajectory are at the same altitude then

$$\Delta E_1 = \frac{2}{\Omega}\left(I_0\cos(\Omega t) - 2A_0h_0\left(2\cos(\Omega t) + \ln\left(\mathrm{tg}\left(\frac{\Omega t}{2}\right)\right)\right)\right).$$

Assuming t as the initial moment of SC mode one can obtain the energy consumed from the accumulator. The value of t can be obtained from (1), (2) if ρ is the air density at the altitude corresponding to the beginning of SC mode (lowest altitude restriction).

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For the other part of the trajectory (at the restriction) the energy consumption can be calculated using the formula

$$\Delta E_2 = \frac{2I_0}{\Omega}\left(1-\cos\left(\Omega t\right)\right)+2\eta h_0 A_0 \sin\left(\Omega t\right)\left(\frac{\pi}{\Omega}+2t\right)$$

where t is the initial moment of SC mode as

$$t = \frac{1}{\Omega}\arcsin\left(\frac{1}{\eta h_0}\sqrt{8\frac{\left(mg\right)^3}{\rho S}\sqrt{\frac{A^3 C_{D0}}{27}}}\right)$$

Assume that the value of t is known and corresponds to some aircraft mass m_0. Let's try to find the difference in the energy consumed if the mass become $m_1=m_0+\Delta m$.
First of all, the difference in time of SC beginning, considering (1), will be

$$\Delta t = \frac{1.5}{\Omega}\mathrm{tg}\left(\Omega t\right)\frac{\Delta m}{m}.$$

As the part of the energy consumption corresponding to the I_0 for the 24 hours is the same for the different masses we do not take it into account.
Difference in the energy consumption during the SC mode is

$$\Delta\left(\Delta E_1\right) = -4A_0 h_0\left(2\sin\left(\Omega t\right)-\frac{1}{\sin\left(\Omega t\right)}\right)\Delta t = -\frac{6}{\Omega}A_0 h_0\left(2\sin\left(\Omega t\right)-\frac{1}{\sin\left(\Omega t\right)}\right)\mathrm{tg}\left(\Omega t\right)\frac{\Delta m}{m}$$

For the trajectory part at the restriction

$$\Delta\left(\Delta E_2\right) = 2h_0 A_0 \sin\left(\Omega t\right)\left(\frac{\pi}{\Omega}+2t\right)\frac{3\Delta m}{2m}+2h_0 A_0 \sin\left(\Omega t\right)2\Delta t = 3h_0 A_0 \sin\left(\Omega t\right)\left(\frac{\pi+\mathrm{tg}\left(\Omega t\right)}{\Omega}+2t\right)\frac{\Delta m}{m}$$

Totally, the consumed energy difference is

$$\Delta\left(\Delta E\right) = \frac{3h_0 A_0}{\Omega}\left(\sin\left(\Omega t\right)\left(\pi-3\mathrm{tg}\left(\Omega t\right)+2\Omega t\right)+\frac{2}{\cos\left(\Omega t\right)}\right)\frac{\Delta m}{m}$$

The comparison of this formula for m=599 kg and Δm=9 kg gives about 6% of error comparing to the result obtained above.
If one denotes ΔW as the difference between the maximal power without the restriction and the restricted value then, after some derivation (that are skipped for the simplicity) the approximate formula for the energy difference $\Delta(\Delta E)_R$ in the case of restriction can be obtained

$$\Delta\left(\Delta E\right)_R = \left(\frac{\left(\mathrm{tg}\left(\Omega t\right)\right)^2}{2mgh_0\Omega^2}\Delta W\right)^3 \frac{\Omega^2 2h_0 A_0}{3\sin\left(\Omega t\right)}$$

Assuming $\Delta W=\beta\Delta m$ and making two last formulas equal one can find the maximal value of Δm that can give any advantage in energy saving in the form of

$$\frac{\Delta m_{max}}{m} = \frac{6}{\left(\mathrm{tg}\left(\Omega t\right)\right)^3}\left(\frac{\Omega g h_0}{\beta}\right)^{1.5}\sqrt{\sin\left(\Omega t\right)\left(\sin\left(\Omega t\right)\left(\pi-3\mathrm{tg}\left(\Omega t\right)+2\Omega t\right)+\frac{2}{\cos\left(\Omega t\right)}\right)} \tag{7}$$

It should be mentioned that for the value of Δm_{max} found the energy saving is zero. In this approximate model the maximal advantage corresponds to the value of

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$$\Delta m_{adv} = \frac{\Delta m_{max}}{\sqrt{2}}. \tag{8}$$

Even formulas (7) – (8) are rather approximate they give nearly the same results as obtained above in Chapter 4. Also they can be used for the analysis of the factors influenced on the value of energy saved. Practically all the parameters in this formula (g, h_0, Ω, t) are predefined. The only parameter for the investigation is β. The higher β the lower is advantage. On the other hand, higher β makes motor mass and construction lighter. It is preferable to use higher β and this fact makes the variation of power restriction not to useful. Another thing mentioned above is that high difference in the power during the flight makes the motor work at not highest values of efficiency. But the analysis of this effect needs more accurate model of powerplant efficiency.

Conclusion

The influence of power restriction on the energy saving of the solar-powered aircraft was investigated. The mathematical model of this phenomenon was proposed and numerical and analytical analyses of the problem were conducted.
Numerical investigations were conducted for some artificial airplane with the characteristics close to the existing ones. The values of energy difference due to the mass change and power restriction were obtained. It was shown that the optimal mass difference is of order of 1% of total mass and the advantage is of the same order of the energy consumed by the motor during the night time.
Analytical investigation of the linearised formulas was conducted. The value of the optimal mass difference coincide with the numerical one.
The influence of power/mass motor parameter was analyzed using the formulas obtained. It was shown that the motors with higher performance makes the construction lighter but enable lower energy saving. More thorough analysis of this effect requires more precise powerplant model.

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