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Analysis of aircraft trajectory uncertainty using Ensemble Weather Forecasts

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Abstract

The problem of aircraft trajectory prediction subject to weather uncertainty is addressed. In particular, a probabilistic analysis of aircraft fuel consumption taking into account wind uncertainty is presented. The wind uncertainty is obtained from ensemble weather forecasts. The analysis is focused on the cruise flight, which is composed of several segments. Each segment is subject to both an average constant along-track wind and an average constant crosswind. The resulting ground speed is modeled as a random variable. A probabilistic trajectory predictor is presented, based on the Probabilistic Transformation Method. Results are presented for a given trans-oceanic route and a real ensemble weather forecast.

1. Introduction

The future Air Traffic Management (ATM) system must address the performance challenges posed by today's airspace: the capacity and the efficiency of the system must be increased while preserving or augmenting the safety levels. To accomplish these goals, in this future system the trajectory becomes the fundamental element of a new set of operating procedures, collectively referred to as Trajectory-Based Operations (TBO) [1].

One key factor that affects those challenges is uncertainty, which is an inherent property of real-world socio-technical complex systems, and ATM is clearly not an exception. Uncertainty is critical from different perspectives in air transport: safety, environment and cost. Researchers must accept the fact that uncertainty is unavoidable and must be dealt with, rather than ignored. If the capacity of the ATM system is to be increased while maintaining high safety standards and improving the overall performance, uncertainty levels must be reduced and new strategies to deal with the remaining uncertainty must be found. In particular, procedures to integrate uncertainty information into the ATM planning process must be developed. In Rivas and Vazquez [2] one can find a review of all the uncertainty sources that affect the ATM system. Among those, weather has perhaps the greatest impact. The analysis of weather uncertainty has been addressed by many authors, using different methods. Among many others, Zheng and Zhao [3] develop a statistical model of wind uncertainties and apply it to stochastic trajectory prediction in the case of straight, level flight trajectories.

The general framework for this paper is the development of a methodology to manage weather uncertainty suitable to be integrated into the trajectory planning process.

In this paper a probabilistic analysis of aircraft fuel consumption taking into account wind uncertainty is presented. The study is focused on the cruise phase and considers the wind uncertainty provided by Ensemble Prediction Systems (EPS), which have proved to be an effective way to quantify weather uncertainties. An analysis of wind-optimal cruise trajectories using ensemble probabilistic forecasts together with pseudospectral methods is performed in Gonzalez-Arribas et al. [4]. A conceptual vision of the integration of ensemble-based, probabilistic weather information with ATM decision support tools, focused on convective storms, is presented in Steiner et al. [5]. The importance of weather uncertainty information in probabilistic air traffic flow management is shown in Steiner et al. [6], where the translation of ensemble weather forecasts into probabilistic air traffic capacity impact is described. These papers clearly show the importance of making use of ensemble weather forecasts to generate probabilistic weather information for aviation needs.

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The two main approaches to the analysis of aircraft trajectory uncertainty using ensemble weather forecasts are considered: ensemble trajectory prediction (eTP) and probabilistic trajectory prediction (pTP). The ensemble TP is the approach used by Cheung et al. [7]. The probabilistic TP considered has been presented for a one-segment cruise in Rivas et al. [8]. In this second approach the wind uncertainty is propagated along the aircraft trajectory. The method used for the uncertainty propagation is the Probabilistic Transformation Method (see Kadry [9] and Kadry and Smaily [10]). Both approaches are applied to trajectories composed of a given number of cruise segments, taking into account the wind distributions obtained from a real EPS. The parameters that define the aircraft and the flight Mach number are obtained from Eurocontrol BADA data base.

This study is relevant because wind is one of the main sources of uncertainty in trajectory prediction, and because cruise uncertainties have a large impact on the overall flight since the cruise phase is the largest portion of the flight (at least for long-haul routes). In particular it is expected that this study be relevant for the determination of the contingency fuel, and, hence, for allowing a more effective decision making.

2. Ensemble weather forecasting

To model weather for strategic planning horizons, a probabilistic approach is the appropriate one, so that the inherent weather uncertainty can be taken into account. Today's trend is to use Ensemble Prediction Systems (EPS), which attempt to characterize and quantify the inherent prediction uncertainty based on ensemble modeling. Ensemble forecasting is a prediction technique that consists in running an Ensemble of Weather Forecasts (EWF) by slightly altering the initial conditions and/or the parameters that model the atmospheric physical processes, and/or by considering time-lagged or multi-model approaches. Thus, this technique generates a representative sample of the possible (deterministic) realizations of the potential weather outcome [5].

An ensemble forecast is a collection of typically 10 to 50 weather forecasts (referred to as members). Cheung et al. [11] review various EPSs: PEARP (from Météo France), consisting of 35 members; MOGREPS (from the UK Met Office), with 12 members; the European ECMWF, with 51 members; and a multi-model ensemble (SUPER) constructed by combining the previous three forming a 98-member ensemble, designed so that it is more likely to capture outliers and give a higher degree of confidence in predicting future atmospheric evolution.

Ensemble forecasting has proved to be an effective way to quantify weather prediction uncertainty. The uncertainty information is on the spread of the solutions in the ensemble, and the hope is that this spread bracket the true weather outcome [5]. It is important to notice that for strategic planning the analysis of all the individual ensemble members must be included (rather than an ensemble mean) [6].

3. Trajectory prediction considering ensemble weather uncertainty

In this section, the two approaches for trajectory prediction subject to uncertainty provided by ensemble weather forecasts are described.

1) Ensemble trajectory prediction (see Fig. 1). In this case, for each member of the ensemble, a deterministic trajectory predictor (TP) is used, leading to an ensemble of trajectories from which probability distributions can be derived. This approach is used in [7, 11].

2) Probabilistic trajectory prediction (see Fig. 2). Now, probability distributions of meteorological parameters of interest (such as wind) are evolved along the aircraft trajectory using a probabilistic trajectory predictor (pTP), leading to probability distributions of trajectory parameters of interest (such as fuel consumption). The pTP defined in this paper for multi-segment trajectories follows this probabilistic approach (as described in Section 6).

The required input from the EWF to the trajectory predictors will depend on the ATM problem under consideration. In this work the fuel consumption in cruise flight is studied, subject to wind uncertainty; therefore, w_1, w_2, ... w_n represent the wind fields defined by each ensemble member.

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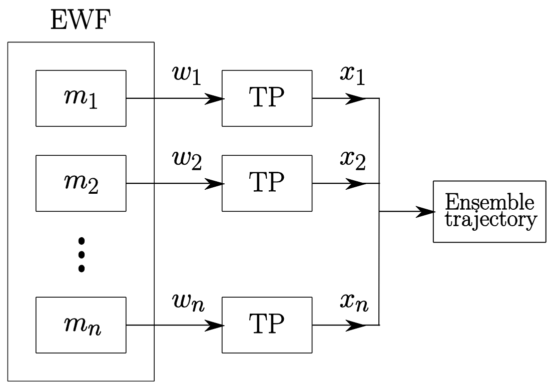

Figure 1: Ensemble trajectory prediction. Legend: m - member, w - weather, x - trajectory

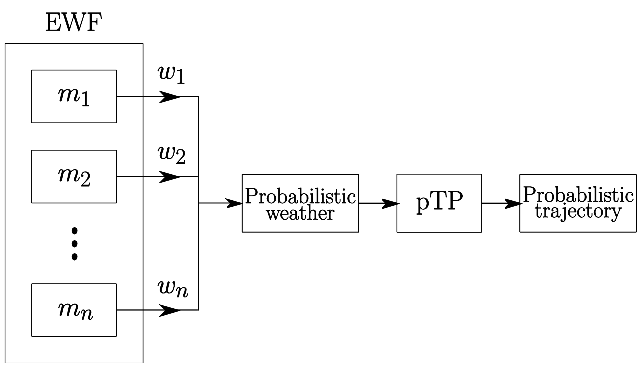

Figure 2: Probabilistic trajectory prediction. Legend: m - member, w - weather

4. Fuel consumption in cruise flight

As already indicated, in this paper the fuel consumption in cruise flight is studied. The cruise is supposed to be formed by p cruise segments, each one of them defined by a constant heading, and flown at constant speed and constant altitude, as usually required by Air Traffic Control (ATC). Sketches of a multi-segment cruise and a generic cruise segment can be found in Fig. 3 and Fig. 4, respectively.

In a cruise segment j, the flight is supposed to be subject to both constant average along-track winds, w_j, and constant average crosswinds, w_{c_j}. These can be different for the different segments, thus modeling the wind variation along the cruise.

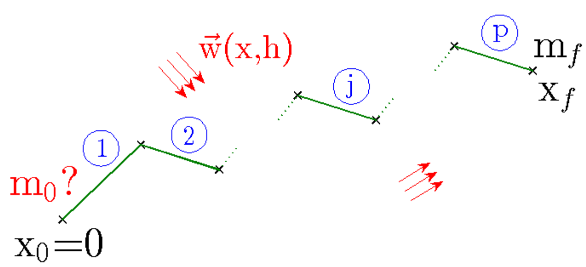

Figure 3: Sketch of a multi-segment cruise (uncertain variables in red)

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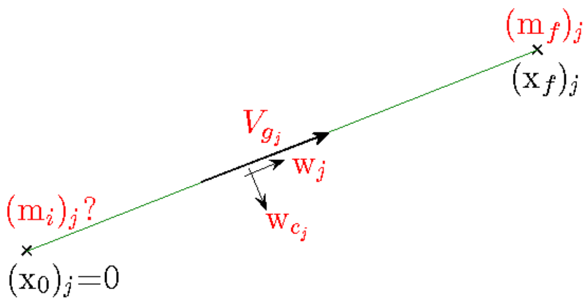

Figure 4: Sketch of a generic cruise segment (uncertain variables in red)

The effects of the crosswinds are analyzed by taking them into account in the kinematic equations, ignoring the lateral dynamics, and translating the crosswind into an equivalent headwind. This leads to a reduced ground speed for a cruise segment j, which is given by

$$V_{g_j} = \sqrt{V^2 - w_{c_j}^2} + w_j , \tag{1}$$

where V_{g_j} is the ground speed and V is the airspeed. Because w_j and w_{c_j} are uncertain, V_{g_j} is uncertain as well.

Assuming symmetric flight and the flat Earth model, the equations of motion for a cruise segment j are

$$\begin{aligned} &\frac{dx}{dt} = V_{g_j}, \quad \frac{dm}{dt} = -cT \\ &T = D, \quad L = mg \end{aligned} , \tag{2}$$

where x is the horizontal distance, t is the time, T is the thrust, D and L are the aerodynamic drag and the lift, m is the aircraft mass, $g = 9.8\ m/s^2$ is the acceleration of gravity, and c is the specific fuel consumption, which can be taken as a function of altitude and speed, and it is therefore constant under the given cruise condition.

The drag can be written as $D = \rho V^2 S C_D / 2$, where ρ is the air density, S is the wing surface area, and the drag coefficient C_D is modeled by a parabolic polar $C_D = C_{D_0} + C_{D_2} C_L^2$, where C_L is the lift coefficient given by $C_L = 2L/(\rho V^2 S)$, and the coefficients C_{D_0} and C_{D_2} are constant under the given cruise condition. Using these definitions and Eq. (2), the following equation is obtained for the aircraft mass

$$\frac{dm}{dx} = -\left(A + Bm^2\right) / V_{g_j} , \tag{3}$$

where the positive constants A and B are defined as

$$A = \frac{c}{2} \rho V^2 S C_{D_0}, \quad B = \frac{2cC_{D_2} g^2}{\rho V^2 S} . \tag{4}$$

Equation (3) is a nonlinear equation describing the evolution of the aircraft mass as a function of distance. Even though this model is quite simple, it is adequate to describe the cruise flight of commercial transport aircraft, since they usually fly segments of constant Mach number (M) and constant altitude (h) following ATC practice, and it is assumed that

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the constant values of the parameters of the aircraft model (C_{D_0} , C_{D_2} , and c) correspond to the values of M and h set for the flight.

In this work, the range of each cruise segment $(x_f)_j$ and the final aircraft mass $(m_f)_p = m_f$ are given. Fixing m_f (instead of the initial aircraft mass) is consistent with having a fixed landing weight. It also allows for a fair comparison for different values of the wind, which lead to different fuel loads and therefore to different values of the initial aircraft mass.

Therefore, the whole cruise flight has to be computed backwards, starting from the last segment ($j = p$) and ending at the first one ($j = 1$). For each segment, Eq. (3) is to be solved backwards, from $(x_f)_j$ to $(x_0)_j = 0$, with the boundary condition

$$m((x_f)_j) = (m_f)_j \tag{5}$$

where $(m_f)_j = (m_i)_{j+1}, \quad j < p$, as dictated by mass continuity.

This problem has the following explicit solution

$$\arctan\left[\sqrt{\frac{B}{A}}\left(m_i\right)_j\right] = \arctan\left[\sqrt{\frac{B}{A}}\left(m_f\right)_j\right] + \sqrt{AB}\left(\Delta t\right)_j , \tag{6}$$

where $\left(\Delta t\right)_j = \left(x_f\right)_j / V_{g_j}$ is the flight time corresponding to the cruise segment j. This solution defines the transformation $\left(\Delta t\right)_j = g_j(V_{g_j})$.

Combining the solutions for all the cruise segments, one can easily obtain the initial aircraft mass $(m_i)_1 = m_i$. Then, the cruise fuel consumption follows from $m_F = m_i - m_f$. Hence, one has

$$m_F = \sqrt{\frac{A}{B}} \tan\left\{\arctan\left(\sqrt{\frac{B}{A}} m_f\right) + \sqrt{AB}\ t_f\right\} - m_f , \tag{7}$$

where $t_f = \sum_{j=1}^{p}\left(\Delta t\right)_j$. This solution defines the transformation $m_F = \hat{g}(t_f)$.

5. Ensemble trajectory predictor

In this section the ensemble TP is described. Suppose that the ensemble has n members, then, the first step is to determine, for each member k of the ensemble and each segment j, the average along-track wind, say $w_{j,k}$, and the average crosswind, say $w_{c_{j,k}}$. Next, the ground speed $V_{g_{j,k}} = \sqrt{V^2 - w_{c_{j,k}}^2} + w_{j,k}$ has to be computed. Then, the corresponding flight time is determined by the transformation $\left(\Delta t\right)_{j,k} = g_j(V_{g_{j,k}})$. Finally, for the member k of the ensemble, the cruise flight time follows from $t_{f_k} = \sum_{j=1}^{p}\left(\Delta t\right)_{j,k}$, and the cruise fuel consumption is given by the transformation $m_{F_k} = \hat{g}(t_{f_k})$. Therefore, the final result is a set of n values of the cruise fuel consumption (m_{F_1}, … m_{F_n}), data that needs some postprocessing to help the decision making process.

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6. Probabilistic trajectory predictor

The pTP is described in this section. The input is the pdf of the aircraft ground speed at each segment and the output is the pdf of the fuel consumption, see sketch in Fig. 5. As already indicated, the pTP is based on the Probability Transformation Method (PTM). The basis of this method is the following theorem (see Canavos [12]): Given a random variable y with probability density function $f_y(y)$, if one defines another random variable z using a transformation g such that $z = g(y)$, then the probability density function of z is given by

$$f_z(z) = \frac{f_y(g^{-1}(z))}{|\,g'(g^{-1}(z))\,|}, \tag{8}$$

expression that is valid only if the function $g(y)$ is invertible on the domain of y.

The pTP needed in this work for multi-segment trajectories relies on the well-known result in statistics that the pdf of the sum of independent variables is the convolution of the pdfs of the addends

$$\Theta = X + Y \;\rightarrow\; f_\theta(\theta) = f_x * f_y, \tag{9}$$

where

$$f_x * f_y = \int_{-\infty}^{\infty} f_x(x) * f_y(\theta - x)dx = \int_{-\infty}^{\infty} f_x(\theta - y) * f_y(y)dy. \tag{10}$$

In this paper, the following procedure is proposed, see sketch in Fig. 5:

1) In each cruise segment, the ground speed is transformed into the flight time according to the transformation $(\Delta t)_j = g_j(V_{g_j})$. Let $f_{V_{g_j}}(V_{g_j})$ be the pdf of the ground speed of the cruise segment j (to be defined in Section 7); then, the pdf of the corresponding flight time is obtained by applying Eq. (8)

$$f_{(\Delta t)_j}((\Delta t)_j) = \frac{(x_f)_j}{(\Delta t)_j^{\,2}}\, f_{V_{g_j}}\left(\frac{(x_f)_j}{(\Delta t)_j}\right). \tag{11}$$

2) Afterwards, the pdf of the cruise flight time, namely $f_{t_f}(t_f)$, is obtained from the pdfs of the flight times corresponding to the cruise segments. For that purpose, the ground speed in the cruise segments (and, therefore, also the flight times) are considered to be independent of one another.

Assuming the independence of the flight times, Eq. (9) can be applied and the pdf of the cruise flight is given by

$$f_{t_f}(t_f) = f_{(\Delta t)_1} * f_{(\Delta t)_2} * \ldots * f_{(\Delta t)_p}. \tag{12}$$

3) Finally, the pdf of the fuel consumption follows from Eq. (8)

$$f_{m_F}(m_F) = \frac{f_{t_f}(\hat{g}^{-1}(m_F))}{A + B(m_f + m_F)^2}, \tag{13}$$

where $\hat{g}^{-1}(m_F)$ is easily obtained from Eq. (7).

This analysis is valid because all the transformation functions are invertible on their respective domains. Once the pdf is known, one can compute the mean and the standard deviation, as follows

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$$\begin{aligned} E[m_F] &= \int_{-\infty}^{\infty} m_F f_{m_F}(m_F) dm_F \\ \sigma[m_F] &= \left[\int_{-\infty}^{\infty} m_F^2 f_{m_F}(m_F) dm_F - \left(E[m_F(w)]\right)^2\right]^{1/2} \end{aligned} . \tag{14}$$

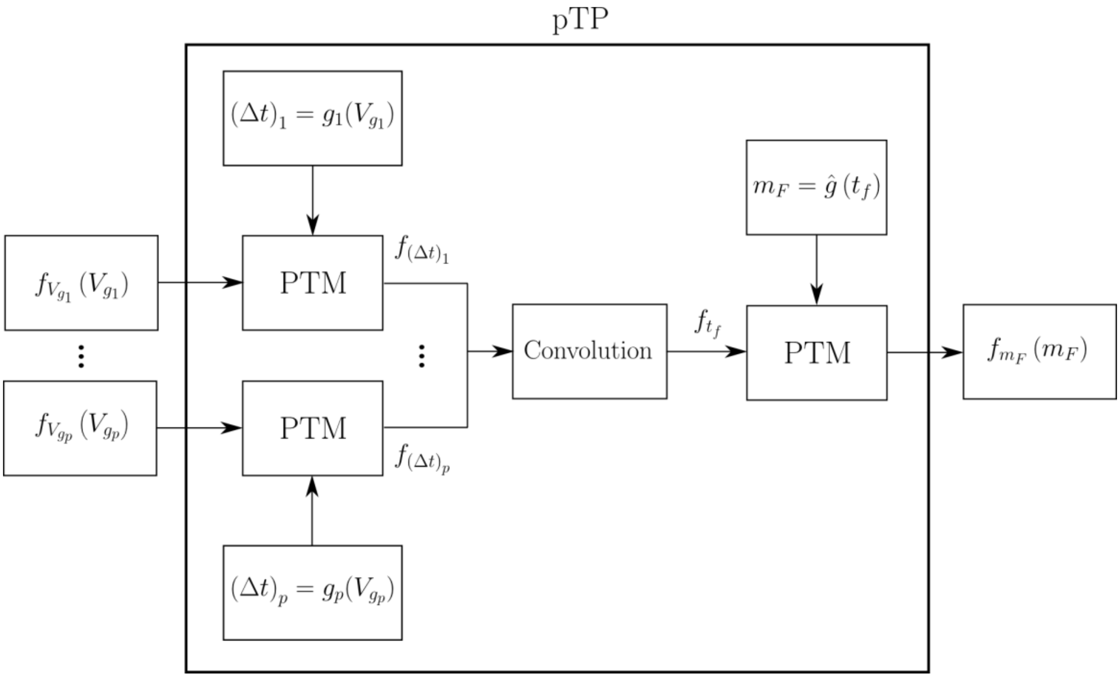

Figure 5: Probabilistic trajectory predictor (pTP). Trajectory with p segments

7. Probabilistic wind model

In this section the input to the pTP is defined (see Fig. 5), that is, the probabilistic ground speed that affects the aircraft trajectory. In the following, the approach to obtain the pdf of the ground speed in each cruise segment is described.

For an ensemble that has n members, let $\{V_{g_{j,1}}, \ldots, V_{g_{j,n}}\}$ be the ground speeds for segment j, as obtained in Section 5. Now one must assume that they follow a particular distribution. This is not a minor point, and in fact is one of the open challenges in this problem. In this paper, to obtain the pdf of the ground speed in each segment, it is assumed that the ground speed is distributed as a uniform continuous variable in the interval $[V_{g_{j,m}}, V_{g_{j,M}}]$, where $V_{g_{j,m}}$ and $V_{g_{j,M}}$ are estimated from the sample by using the method of moments. Therefore, the ground speed along the cruise segment j has the following pdf

$$f_{V_{g_j}}(V_{g_j}) = \begin{cases} 1/(V_{g_{j,M}} - V_{g_{j,m}}), & V_{g_j} \in [V_{g_{j,m}}, V_{g_{j,M}}] \\ 0, & V_{g_j} \notin [V_{g_{j,m}}, V_{g_{j,M}}] \end{cases} . \tag{15}$$

The mean and the standard deviation of V_{g_j} are given as follows

$$\begin{aligned} E[V_{g_j}] &= (V_{g_{j,M}} + V_{g_{j,m}})/2 \\ \sigma[V_{g_j}] &= (V_{g_{j,M}} - V_{g_{j,m}})/(2\sqrt{3}) \end{aligned} \tag{16}$$

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8. Analysis of fuel consumption uncertainty

In the following, results are presented for a given aircraft and a given cruise flight with $p = 9$ segments defined by the following parameters: $\rho = 0.3216$ kg/m^3 ($h \approx 11784\,\text{m}$), $V = 236\,\text{m/s}$ ($M \approx 0.8$), $C_{D_0} = 0.01744$, $C_{D_2} = 0.04823$, $c = 1.49 \cdot 10^{-5}$ s/m, $S = 283.5\,\text{m}^2$, and $m_f = 110000\,\text{kg}$. The selected route (westbound) is described in Tables 1 and 2, where the waypoints that define it and the horizontal distances for each segment are given (note that for the eastbound trajectory the cruise starts with segment 9 and ends with segment 1).

Table 1: Route waypoints (westbound numbering)

	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
Latitude	43º 37.5’ N	46º N	48º N	49º N	49º N	49º N	48º N	46º N	42º N	40º 38.4’ N
Longitude	6º 9.84’ E	0º	10º W	20º W	30º W	40º W	50º W	60º W	70º W	73º 46.74’ W

Table 2: Horizontal distance travelled in each segment (westbound numbering)

Horizontal distance (km)	**Cruise segment**								
	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*
$(x_f)_j$	551.999	788.396	743.446	727.875	727.875	743.446	788.396	912.765	349.151

Results are presented for the aircraft travelling the route both westbound and eastbound. Thus, one can analyse the difference in trajectory uncertainty between the cases of being in the presence of tailwinds and headwinds.

The EPS chosen has been PEARPS, from Météo France, and winds have been retrieved from the ECMWF database, corresponding to 5 May 2016, a release time of 6:00, a time step of 0 hours, and for pressure level 200 HPa ($h \approx 11784\,\text{m}$). Raw wind data have been processed to give a constant wind per segment and per ensemble member. The resulting averaged winds are a set of 35×9×2=630 values of along-track and cross-track winds, not included for brevity.

From this weather input, 35×9=315 values of ground speed are obtained for the westbound route, which are not listed here, but they are represented for the different cruise segments in Fig. 6, in the form of relative frequency histograms, along with the corresponding pdfs (assuming uniform distributions). One can see that empirical data do not clearly follow any common statistical distribution; however, the uniform distribution turns out to be a good proposal because it is simple, yet roughly matches the data. Since the same scale for the abscissa is selected in all subfigures, it is easy to see that the spread of the ground speed is different for the different segments, with the sample standard deviation ranging from 0.489 m/s for segment 6 to 0.925 m/s for segment 1.

Ground speed distributions have also been obtained for the eastbound route; however, they are not represented for the sake of brevity, because they are quite similar to the ones presented. It is important to note that there are not substantial differences in the spread of the distributions between both cases. For the eastbound trajectory, the average along-track and cross-track winds are the same as for the westbound trajectory in magnitude, just with the opposite signs.

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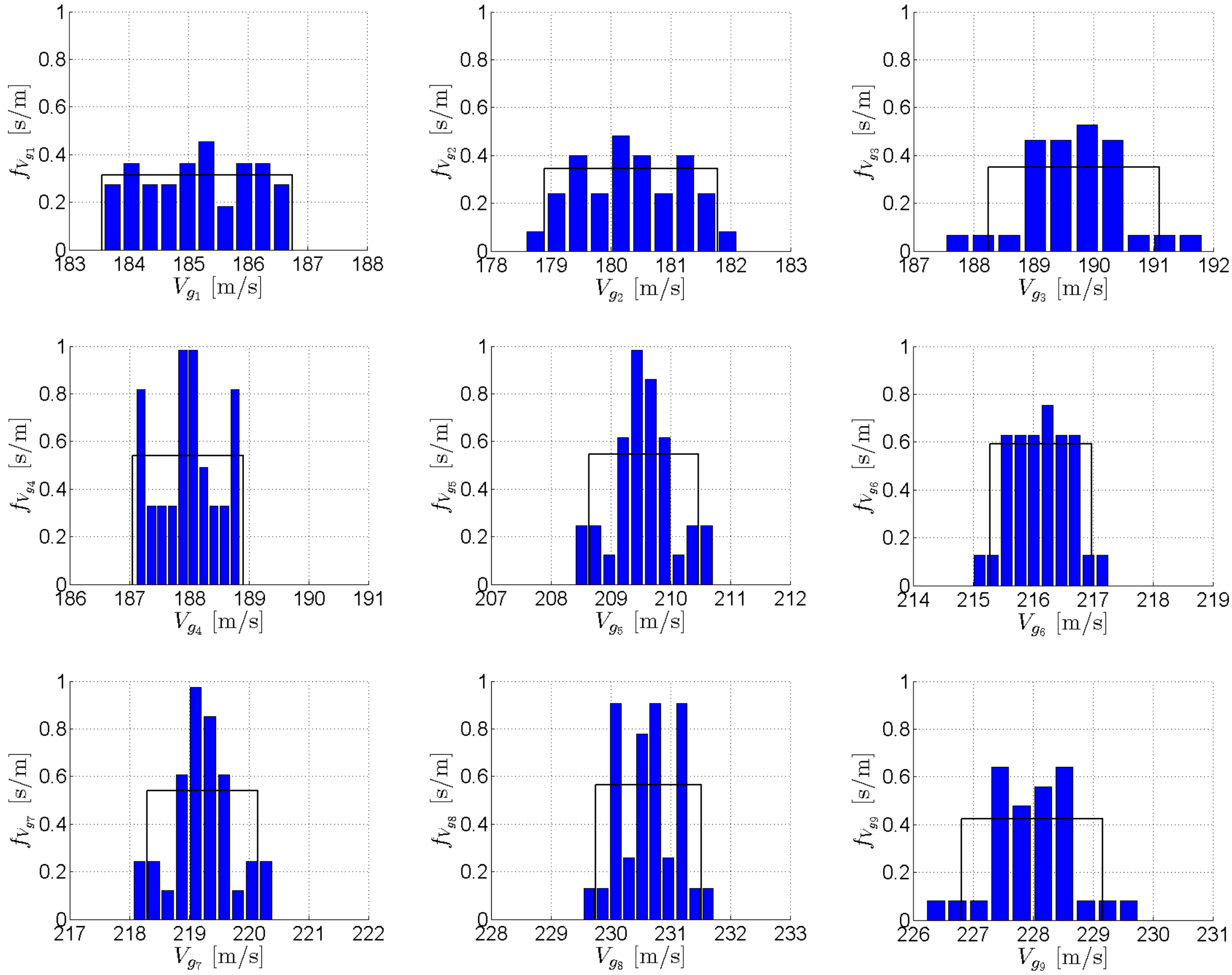

Figure 6: Ground speed distributions corresponding to the cruise segments (westbound route). Blue bars: relative frequency histograms. Black curves: uniform distributions for the pTP approach. (Gaps between bars are introduced only for aesthetical purposes.)

8.1 Results from ensemble TP

As already known, the result from the ensemble TP is a set of n values of the cruise fuel consumption (m_{F_1} , … m_{F_n}). They are listed in Table 3 and represented in the form of relative frequency histograms in Fig. 7. The same width has been considered for all the histogram bins, so that one can compare the spread in the results. Values of the mean and the standard deviation are presented in Table 4.

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Table 3: Fuel consumption (in kg) for each EPS member (westbound/eastbound)

1/34186 /25526	**8**/34157 /25554	**15**/34131 /25568	**22**/34235 /25520	**29**/34195 /25541
2/34126 /25579	**9**/34181 /25535	**16**/34181 /25538	**23**/34131 /25567	**30**/34117 /25564
3/34162 /25525	**10**/34130 /25570	**17**/34169 /25567	**24**/34181 /25538	**31**/34156 /25544
4/34150 /25581	**11**/34212 /25526	**18**/34144 /25539	**25**/34222 /25546	**32**/34155 /25561
5/34137 /25574	**12**/34099 /25580	**19**/34150 /25570	**26**/34092 /25559	**33**/34172 /25546
6/34175 /25531	**13**/34113 /25563	**20**/34162 /25535	**27**/34123 /25571	**34**/34140 /25560
7/34155 /25551	**14**/34199 /25542	**21**/34077 /25585	**28**/34188 /25534	**35**/34155 /25552

8.2 Results from probabilistic TP

Once the input to the pTP is defined (given by Eq. (15)), the pdf of the fuel consumption $f_{m_F}(m_F)$ is computed following the procedure defined in Section 6 (and outlined in Fig. 5). The pdfs of the aircraft fuel consumption obtained with the pTP approach are shown in Fig. 7 for both westbound and eastbound cruises. Values of the mean and the standard deviation are presented in Table 4.

These results show that for the westbound cruise one has larger values of the mean (as expected, because in this case one has predominant headwinds), and also larger values of the standard deviation, implying that the trajectory uncertainty is larger in the presence of headwinds. This trend also holds for the relative uncertainty $\sigma[m_F]/E[m_F]$, increasing from 0.881×10^{-3} to 1.344×10^{-3} (it roughly increases 50%). Therefore, more extra fuel needs to be loaded in the presence of headwinds, for a given standard for safe operation.

The results also show that $\sigma[m_F]/E[m_F]$ is approximately constant for different aircraft (medium and heavy), which is in line with the common practice of loading the same percentage of the trip fuel for contingencies. That is, our analysis shows that the same percentage of contingency fuel should be loaded due to wind uncertainty for all aircraft (for the given route and EPS), percentage that can be quantified for the particular wind forecast under consideration.

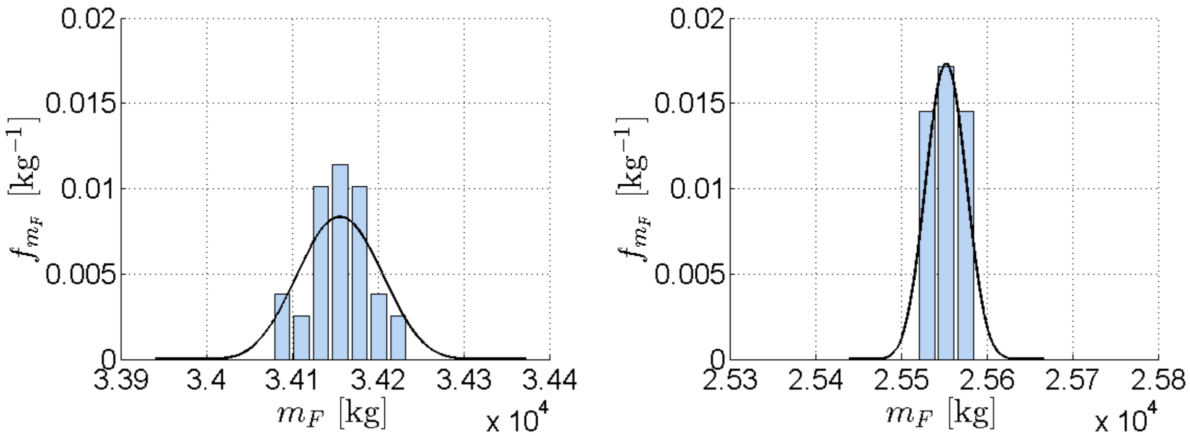

Figure 7: Fuel mass distributions for westbound cruise (left) and eastbound (right). Blue bars: relative frequency histograms (ensemble TP). Black curves: pdfs (probabilistic TP)

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Table 4: Mean and standard deviation of fuel consumption

	Westbound		**Eastbound**	
	$E[m_F]$ (kg)	$\sigma[m_F]$ (kg)	$E[m_F]$ (kg)	$\sigma[m_F]$ (kg)
eTP	34 156	35.7	25 553	18.3
pTP	34 156	45.9	25 553	22.5

9. Final remarks

This work has provided an assessment of the impact of wind uncertainty on aircraft trajectory, and in particular on the cruise fuel load. It is expected that by considering the weather uncertainty in the trajectory prediction process, one could adjust the contingency fuel depending on the uncertainty obtained for the fuel consumption.

Note that the larger the values of the horizontal distance travelled in each segment, the more realistic the assumption that the ground speeds in the cruise segments (and, therefore, also the flight times) are independent of one another, but the less appropriate the consideration of constant average winds. Therefore, a trade-off between these two effects has to be considered when selecting the segment lengths (or, equivalently, the location of the waypoints).

The consideration of temperature uncertainty, also provided by EWF, is left for future work. As cruise segments are usually flown at constant Mach number and constant pressure altitude, the main effect of the temperature distribution is a change in true airspeed (due to the change in the speed of sound), which leads to changes in ground speed and specific fuel consumption.

The probabilistic trajectory predictor presented in this paper is capable of taking as input any type of ground speed distribution. In this work, simple uniform distributions have been considered, although other types of distributions could be considered as well. It is clear that the determination of the ground speed pdf from the uncertainty information contained in the EPSs is an open challenge in this problem. This issue poses a multidisciplinary task to be addressed jointly by meteorologists, statisticians and ATM experts.

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