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Validation of Natural Frequencies Formulas of Flight Modes for Small UAVs

El-Salamony Mostafa⋆ *and Serokhvostov Sergey*⋆
⋆*Moscow Institute of Physics and Technology (MIPT)*
16 Gagarina street, Zhukovsky, Russia.140180
elsalamony.mostafa@phystech.edu · serokhvostov@phystech.edu
⋆Corresponding author

Abstract

Formulae of the Natural Frequencies of the different flight modes for Small UAVs are validated. Comparison between the formulae of the large aircraft applied on small UAVs scale, Vortex Lattice Method code and experimental data obtained from flight tests is made. Method to validate natural frequency for UAVs is presented. It is found that for the long mode, methods of Roskam (exact), Ostoslavsky, and XFLR5 estimate the frequency within range of the experimental results while the methods of Roskam (approximation) and Hull overestimate it and should not be applied in case of small UAVs. Considering the short mode, all methods predict the actual frequency with acceptable accuracy. Considering the dutch roll mode, estimation methods of Roskam (exact), Ostoslavsky and XFLR5 give good results in agreement with the experiment, while the approximate methods of Roskam underestimate the frequency.

Nomenclature

$\bar{r_z^2}$ dimensionless UAV radius of inertia along the wing

μ dimensionless aircraft density

ϕ roll angle

ψ yaw angle

ρ air density

θ pitch angle

b wing span

c aircraft chord

C_L airplane lift coefficient in steady state condition

g gravitational acceleration

I_{xx} moment of inertia around fuselage axis

I_{yy} moment of inertia around wing axis

I_{zz} moment of inertia around normal to fuselage axis

m aircraft mass

M_{T_α} pitch angular acceleration per unit angle of attack due to thrust

M_u pitch angular acceleration per unit change in speed

N_{T_β} yaw angular acceleration per unit sideslip angle (due to thrust)

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S wing reference area

U_1 cruise velocity

X_α forward acceleration per unit angle of attack

Z_{T_u} forward acceleration per unit change in speed due to thrust

1. Introduction

Making a mathematical model of flight mechanics for the airplane is the first and main step to design a control system for an aircraft. The accuracy of the mathematical model with respect to the physical model is critical for controller's accuracy. Even a simple PID control can be used to make the required response if the mathematical model is accurate enough.
It is well known in flight mechanics that if any disturbance influences the aircraft (as control surface deflection or gust wind) the stable aircraft will damp these disturbances and return to its steady state. Such behaviour is crucial for the non-manoeuvrable aircraft and the knowledge about the stability is highly required for the aircraft and autopilot design. For the conventional aircraft shapes it is possible to separate the disturbed motion to longitudinal and lateral ones.
Longitudinal motion can be modelled as the fourth-order equation that describes two second-order modes: short mode which is a function of angle of attack and characterized by its relatively high frequency, and the long (phugoid) mode which is a function of aircraft speed and is characterized by its relatively low frequency. Lateral motion can be modelled as the fourth-order equation that describes three modes: spiral, roll and dutch roll. The first two modes are first-order (corresponding to the exponential decrease or increase) and the dutch roll is second-order (corresponding to the decaying or amplifying oscillations).
In case of damped vibrations, there are two definitions for the frequency: the damped frequency and the undamped / natural frequency. In this paper, the natural frequencies are considered.
Since UAVs (including micro and mini) are usually designed according to the procedure used for the large aircrafts, the stability calculations also commonly follow the same formulas derived for the large aircraft although the geometrical scale of UAVs is much smaller. Obviously, the forces acting on the UAV change their order of power nonlinearly and some assumptions for the manned aircrafts may be not valid in this case, and new assumptions could be introduced. One of the main questions investigated here is how the formulas for these frequencies change with the scale and Reynolds number.
First attempts of these investigations were conducted previously in.[13] One of the main finding was the possibility of separating the equations of UAVs disturbed motion into the longitudinal and lateral motions. A test case was studied for an UAV of wing span of 85 cm and mass of 150 gram in.[12] The analysis based on the procedure of[9] found that the natural frequency of the short mode of longitudinal direction is big enough compared to the long mode. Recently, longitudinal flight modes were investigated in more detail in.[2] It was found that the natural frequency of the long mode can be predicted accurately by the exact methods used in[11] and[9] but the short mode was not captured due to the high damping ratio of the short mode and the testing conditions. A method was recommended to overcome the high damping ratio in[3] by shifting the center of gravity (CG).
Goal of the current investigation is to understand the accuracy of estimting the natural frequencies of the various flight modes by the calculations based on the "traditional" formulas with respect to the experimental values. In this research, an investigation is carried out on the formulas and assumptions of calculating the natural frequency of the dutch roll mentioned by J. Roskam, D. Hull, and I. Ostoslavsky and compare their results with the numerical VLM calculations from XFLR5 –which is mainly designed for small UAVs– then these results are compared to real measurements of the natural frequencies obtained from UAV "Sonic 185" at flight.
For big aircraft commonly a mathematical model is created to simulate its response to follow predefined motion in case of small air disturbance and the response is recorded in the form of flight path angles then compared to the oscillations from flight log of flight test to validate the mathematical model and in particular the frequencies. Such a method is not appropriate for small UAVs since the disturbances are relatively high compared to the forces applying on the UAV, that's why another method is proposed. Instead of comparing the data of flight angles obtained from the experiment and mathematical model, the flight path angles are processed to obtain the main modelling parameters (natural frequencies) and to compare them to the theoretical results. This method seems to be more suitable for small UAVs flying in disturbed air.

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2. Procedure

For an airplane, one can describe its longitudinal motion as fourth order equation which –under some conditions – can be divided into two second order equations, short and long period modes. Roskam and Ostoslavsky considered this task using different techniques. Roskam's procedure[11] –and same for Hull[6]– starts from the aerodynamic coefficients, then calculating the forces acting on the aircraft then combine them together to construct the coefficients of the main characteristic equation. Through deep understanding of the physics behind the nature of aircraft forces and moments and their order of magnitude and vibrations, Roskam and Hull made an approximate solutions to obtain the natural frequencies by linking the forces directly to the natural frequency and damping ratio instead of solving the main fourth order equation.

Ostoslavsky derived the characteristic equation by another technique. Instead of calculating the forces, he used the non-dimensional aerodynamic coefficients directly and cast them together to get the coefficients of the characteristic equation. By deep understanding of the mathematics behind the main characteristic equation, he decomposed the fourth order equation into two second order ones, each one describe a single flight mode under the fact that the natural frequency of the short mode is much bigger than for the long mode.

In numerical methods as VLM – as used in XFLR5 – the aircraft is divided into small panels. For each panel, a normal vector is set to be perpendicular to the camber. Also a combination of source, doublet, and vortex are added in one quarter of the panel, and a control point is added after three quarters of the panel to achieve the no-penetration condition.[4] By solving N equations obtained from the N panels, the total vortex strength is determined then the normal and tangential forces acting on the aircraft are obtained then converting them into non-dimensional coefficients. The next step is to import these values –which depend on the angle of attack – into the state space matrix and obtain the eigen values of the matrix which are a combination of natural frequency and damping ratio and they can be separated easily. XFLR5 is used because it is open source and used widely for UAV design process and also has the ability to obtain the natural frequencies and damping ratios.

2.1 Investigated UAVs

Two UAV are used in this research which are Sonic 185 of DYNAM[7] and Cirrus of Reichard.[8] Aircraft parameters and geometry are measured and listed in Tables 1 – 3. Based on the cruise speed and chord length, the Reynolds number here is about 10^5. Airfoil of Sonic is estimated to be Eppler E231 while Cirrus airfoil is determined to be NH F3J.

Table 1: Aircraft parameters

Property	**Sonic 185**	**Cirrus**
Mass (m) [kg]	1.18	1.97
Span [m]	1.85	3.4
Wing area (S) [m^2]	0.33	0.698
Cruise velocity (U_1) [m/s]	8	6
I_{xx} [$kg{\cdot}m^2$]	0.108	0.979
I_{yy} [$kg{\cdot}m^2$]	0.065	0.149
I_{zz} [$kg{\cdot}m^2$]	0.122	0.753
Airfoil	E231	NH F3J
CG from leading edge of root section [m]	0.07	0.11

Table 2: Aircraft Sonic 185 geometry

Property	**Wing**	**Horizontal Tail**	**Vertical Tail**
Aspect ratio	10.295	4.92	2.03
Mean chord (c) [m]	0.189	0.1	0.16
Span [m]	1.85	0.48	0.16
Root chord [m]	0.205	0.125	0.2
Tip chord [m]	0.06	0.02	0.115
Sweep angle from leading edge [$degree$]	6.71	18.17	23.25

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Table 3: Aircraft Cirrus geometry

Property	**Wing**	**Horizontal Tail**	**Vertical Tail**
spect ratio	16.556	6.03	4
Mean chord (c) [m]	0.216	0.1	0.11
Span [m]	3.4	0.3	0.21
Root chord [m]	0.245	0.115	0.15
Tip chord [m]	0.06	0.06	0.05
Sweep angle from leading edge [$degree$]	4.67	23.57	7.83

3. Analytical Approach

Roskam and Ostoslavsky considered this task by two different procedures using dimensional and nondimensional stability parameters as shown below.

3.1 Roskam Procedure

Roskam's procedure[11] starts from estimating the aerodynamic coefficients, calculating the forces and moments acting on the aircraft, then calculates the main characteristic equation. Based on the big amount of experimental data, the coefficients are estimated taking into account many details which may increase the accumulative errors during calculations. The aerodynamic and stability coefficients are calculated according to Roskam procedure[10] and the results are shown in Tables 6 and 7. It is noticed that C_{mq} in the formula used in Roskam procedure is twice the one in Ostoslavsky procedure due to mismatching in definition.

Table 4: Aerodynamic longitudinal derivatives of Sonic 185 based on Roskam method

Derivative	**Angle of attack** (α)	**Pitch rate** (q)	**Rate change of angle of attack** ($\dot{\alpha}$)
Lift coeff.(C_L)	5.253	7.312	4.655
Drag coeff.(C_D)	0.377	0	0
Moment coeff.(C_m)	-0.47	Roskam:-8.59 Ostoslavsky: -4.295	-4.1337

Table 5: Aerodynamic lateral derivatives of Sonic 185 based on Roskam method

Derivative	**Sideslip Angle** (β)	**Roll Rate** (p)	**Yaw Rate** (r)
Side Force Coeff. (C_y)	-0.17	0	0.14
Roll Moment Coeff. (C_l)	-0.113	-0.873	0.204
Yaw Moment Coeff. (C_n)	0.064	-0.127	0.124

Table 6: Aerodynamic longitudinal derivatives of Cirrus based on Roskam method

Derivative	**Angle of attack** (α)	**Pitch rate** (q)	**Rate change of angle of attack** ($\dot{\alpha}$)
Lift coeff.(C_L)	5.841	7.92	4.468
Drag coeff.(C_D)	0.532	0	0
Moment coeff.(C_m)	-0.743	Roskam:-17.584 Ostoslavsky: -8.792	-5.245

3.1.1 Longitudinal Motion

In this section the procedures of Roskam (exact and approximate) and Hull are considered together because they have quite similar formulas and results as a sequence.

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Table 7: Aerodynamic lateral derivatives of Cirrus based on Roskam method

Derivative	**Sideslip Angle** (β)	**Roll Rate** (p)	**Yaw Rate** (r)
Side Force Coeff. (C_y)	−0.2236	0	0.115
Roll Moment Coeff. (C_l)	−0.3216	−1.415	0.3353
Yaw Moment Coeff. (C_n)	0.0584	−0.6437	0.2155

The longitudinal characteristic equation of Roskam[11] is

$$Ax^4 + Bx^3 + Cx^2 + Dx + E = 0 \tag{1}$$

where

$$A = U_1 - Z_{\dot\alpha} \tag{2}$$

$$B = (-(U_1 - Z_{\dot\alpha})(X_u + X_{Tu} + M_q) - Z_\alpha M_{\dot\alpha}(U_1 + Z_q) \tag{3}$$

$$C = (X_u + X_{Tu})[M_q(U_1 - Z_{\dot\alpha}) + Z_\alpha + M_{\dot\alpha}(U_1 + Z_q)] + M_q Z_\alpha - Z_u X_\alpha + M_{\dot\alpha} g \sin\theta_1 - (M_\alpha + M_{T\alpha})(U_1 + Z_q) \tag{4}$$

$$\begin{aligned} D = g\sin\theta_1[M_\alpha + M_{T\alpha} - M_{\dot\alpha}(X_u + X_{Tu})] + g\cos\theta_1[Z_u M_{\dot\alpha} + (Mu + M_{Tu})(U_1 - Z_{\dot\alpha})] \\ + (M_u + M_{Tu})[-X_{\dot\alpha}(U_1 + Zq)] + Z_u X_\alpha M_q + (X_u + X_{Tu})[(M_\alpha + M_{T\alpha})(U_1 + Z_q) - M_q Z_\alpha] \end{aligned} \tag{5}$$

$$E = g\cos\theta_1[(M_\alpha + M_{T\alpha})Z_u - Z_\alpha(M_u + M_{Tu})] + g\sin\theta_1[(M_u + M_{Tu})X_\alpha - (X_u + X_{Tu})(M_\alpha + M_{T\alpha})] \tag{6}$$

After analysis, the results showed that for Sonic ω_n=1.056 Hz for the short mode, and 0.178 Hz for the long mode. As for Cirrus ω_n=1.205 Hz for the short mode, and 0.187 Hz for the long mode.
The approximate solution for the natural frequency of the short and long modes according to Roskam are:

$$\omega_{n,sp} = \sqrt{\frac{Z_\alpha M_q}{U_1} - M_\alpha} \tag{7}$$

$$\omega_{n,lp} = \sqrt{\frac{-Z_u g}{U_1}} \tag{8}$$

The results are: ω_n=1.096 Hz for the short mode, and 0.2835 Hz for the long mode for Sonic and ω_n=1.307 Hz for the short mode, and 0.3744 Hz for the long mode for Cirrus.
According to Hull method,[6] the approximate formulae for short mode and long mode are

$$\omega_{n,sp} = \sqrt{\frac{Z_\alpha M_q - M_\alpha(U_1 + Z_q)}{U_1 + Z_{\dot\alpha}}} \tag{9}$$

$$\omega_{n,lp} = \sqrt{\frac{-Z_u g}{U_1 + Z_q}} \tag{10}$$

The results are quite similar to the approximate Roskam formulae as shown: for short mode ω_n=1.137 Hz and for long mode ω_n=0.302 Hz for Sonic nd for Cirrus ω_n=1.216 Hz and for long mode ω_n=0.415 Hz.

3.1.2 Lateral Motion

The characteristic equation of the lateral motion[11] is:

$$Ax^4 + Bx^3 + Cx^2 + Dx + E = 0 \tag{11}$$

where

$$A = U_1(1 - \bar{A}\bar{B}) \tag{12}$$

$$B = -Y_\beta(1 - \bar{A}\bar{B}) - U_1(L_p + N_r + \bar{A}N_p + \bar{B}L_r) \tag{13}$$

$$\begin{aligned} C = U_1(L_pN_r - L_rN_p) + Y_\beta(N_r + L_p + \bar{A}N_p + \bar{B}L_r) - Y_p(L_\beta + N_\beta\bar{A} + N_{T\beta}\bar{A}) \\ +U_1(L_\beta\bar{B} + N_\beta + N_{T\beta}) - Y_r(L_\beta\bar{B} + N_\beta + N_{T\beta}) \end{aligned} \tag{14}$$

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$$D = -Y_\beta(L_pN_r - L_rN_p) + Y_p(L_\beta N_r - N_\beta L_r - N_{T\beta}L_r) - g\cos\theta_1(L_\beta + N_\beta\bar{A} + N_{T\beta}\bar{A}) \\ +U_1(L_\beta N_p - N_\beta L_p - N_{T\beta}L_p) - Y_r(L_\beta N_p - N_\beta L_p - N_{T\beta}L_p) \tag{15}$$

$$E = g\cos\theta_1(L_\beta N_r - N_\beta L_r - N_{T\beta}L_r) \tag{16}$$

where

$$\bar{A} = I_{xz}/I_{xx} \tag{17}$$

$$\bar{B} = I_{xz}/I_{zz} \tag{18}$$

After analysis, the results showed that ω_n for dutch roll is 0.59 Hz for Sonic 185 and 0.8911 Hz for Cirrus.
In order to simplify the decomposition of modes Roskam made an approximate solution for obtaining the natural frequency by linking the forces directly to the natural frequency instead of solving the main fourth order equation as follows:

$$\omega_{n1} = \sqrt{N_\beta + \frac{Y_\beta N_r - N_\beta Y_r}{U_1}} \tag{19}$$

Under the assumption that $(Y_\beta N_r - N_\beta Y_r)/U_1$ is significantly less in magnitude than N_β, the former equation can be written as:

$$\omega_{n2} = \sqrt{N_\beta} \tag{20}$$

The natural frequencies for these cases are ω_{n1}=0.5 Hz and ω_{n2}=0.49 Hz for Sonic 185. As for Cirrus ω_{n1}=0.71 Hz and ω_{n2}=0.632 Hz .

3.2 Ostoslavsky Procedure

3.2.1 Longitudinal Motion

Ostoslavsky[9] has derived the longitudinal characteristic equation by a different method. Instead of calculating the forces, the non-dimensional aerodynamic coefficients are used directly then the coefficients of the lateral characteristic equation are obtained. He introduced the characteristic equation of the longitudinal system as fourth order function in its eigen values as:

$$F = \lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + a_4 = 0 \tag{21}$$

where the coefficients of this equations under the steady state condition are:

$$a_1 = C_{L\alpha} - \frac{C_{m\dot{\alpha}} + C_{mq}}{\bar{r_z^2}} \tag{22}$$

$$a_2 = \frac{C_{m\alpha}\mu + C_{L\alpha}C_{mq}}{\bar{r_z^2}} \tag{23}$$

$$a_3 = \frac{-2C_L[(C_L - C_{D\alpha})C_{mq} + C_LC_{m\dot{\alpha}}]}{\bar{r_z^2}} \tag{24}$$

$$a_4 = \frac{-2C_L^2\mu C_{m\alpha}}{\bar{r_z^2}} \tag{25}$$

and

$$\bar{r_z^2} = \frac{I_{yy}}{mc^2} \tag{26}$$

$$\mu = \frac{2m}{\rho S c} \tag{27}$$

By solving Equation (21), one can obtain the eigen values of the system which resemble the natural frequencies and damping ratios.
To simplify the decomposition of the longitudinal characteristic equation (21), Ostoslavsky supposed that the eigen values of the long mode are negligible with respect to the short mode, and finally the natural frequencies of the short and long modes can respectively obtained from:

$$\lambda^2 + a_1\lambda + a_2 = 0 \tag{28}$$

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$$\lambda^2 + \frac{a_2 a_3 - a_1 a_4}{a_2^2}\lambda + \frac{a_4}{a_2} = 0 \tag{29}$$

The results of Sonic 185 for the exact equation are ω_n= 1.119 Hz and 0.1787 Hz for the short and long modes respectively. The results of the simplified equations are ω_n=0.1758 Hz for the long mode and ω_n=1.122 Hz for the short mode for Sonic. For Cirrus results, the exact method shows overdamped behaviour for the short mode but the long mode frequency is 0.183 Hz. Based on the approximate method, frequencies of short and long modes are 1.211 Hz and 0.174 Hz.

3.2.2 Lateral Motion

Using basic geometrical parameters, the aerodynamic coefficients can be estimated simply. Results are listed in Table 8. Ostoslavsky introduced the characteristic equation of the system as fourth order function in its eigen values as:

Table 8: Aerodynamic lateral derivatives of Sonic 185 based on Ostoslavsky method

Derivative	**Sideslip Angle** (β)	**Roll Rate** (p)	**Yaw Rate** (r)
Side Force Coeff. (C_y)	-1.494	0	0
Roll Moment Coeff. (C_l)	−0.011	−1.038	−0.089
Yaw Moment Coeff. (C_n)	0.036	0.073	0.032

$$F = \lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + a_4 = 0 \tag{30}$$

where the coefficients of this equation for the steady state condition are:

$$a_1 = -(0.5c_{L\beta} + \bar{c_{lp}} + \bar{c_{nr}}) \tag{31}$$

$$a_2 = c_{L\beta}/2(\bar{c_{lp}} + \bar{c_{nr}}) + (\bar{c_{lp}}\bar{c_{nr}} - \bar{c_{lr}}\bar{c_{np}}) - \mu(\bar{c_{n\beta}} + \alpha\bar{c_{l\beta}}) \tag{32}$$

$$a_3 = -\mu(\bar{c_{l\beta}}\bar{c_{np}} - \bar{c_{n\beta}}\bar{c_{lp}}) \tag{33}$$

$$a_4 = \mu c_L(\bar{c_{l\beta}}\bar{c_{nr}} - \bar{c_{n\beta}}\bar{c_{lr}} + \tan\theta_0)(\bar{c_{l\beta}}\bar{c_{np}} - \bar{c_{n\beta}}\bar{c_{lp}})/2 \tag{34}$$

where

$$\overline{c_{lp,lr,l\beta}} = \frac{c_{lp,lr,l\beta}mb^2}{4I_{xx}} \tag{35}$$

$$\overline{c_{np,nr,n\beta}} = \frac{c_{np,nr,n\beta}mb^2}{4I_{yy}} \tag{36}$$

$$\mu = \frac{2m}{\rho S c} \tag{37}$$

In order to simplify the analysis, the following approximate equation can be used:

$$\omega_{n1} = -\frac{a_3}{\bar{c_{lp}}2\pi\tau} \tag{38}$$

where

$$\tau = \frac{2m}{\rho S U_1} \tag{39}$$

The Sonic 185 results of the exact method show over-damped situation while the approximate method showed that ω_{n1}=0.556 Hz. Cirrus has frequencies of 0.8314 Hz and 0.7839 Hz for the exact and approximate methods respectively.

4. Numerical Approach

The UAVs are drawn on XFLR5 using the measurements from Tables 1 – 3 as shown in Figures 1 and 2. It should be noted that there is no fuselage in the XFLR5 model.

Calculations done shows that ω_n=1.171 Hz for the short mode, and 0.178 Hz for the long mode for Sonic 185 and 1.548 Hz for the short mode and 0.168 Hz for the long mode for Cirrus. As for the dutch roll, frequencies are 0.582 Hz and 0.955 Hz for Sonic 185 and Cirrus respectively.

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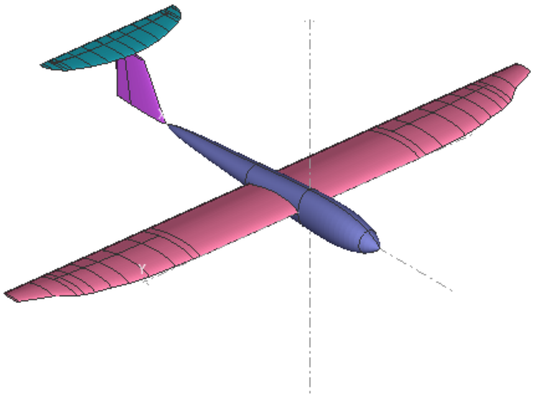

Figure 1: Sonic 185 drawing in XFLR5

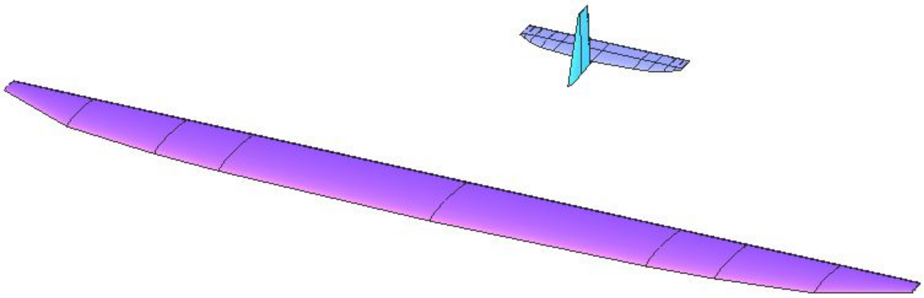

Figure 2: Cirrus drawing in XFLR5

5. Experimental Approach

For big aircraft a mathematical model is created to simulate its response in case of small disturbance and the response is recorded in the form of flight path angles and compared to the oscillations from flight log of flight test. This method is not valid for UAVs because the force scale is close to the scale of air disturbances, which will affect adversely on the measured oscillations.
Since the air disturbances are relatively high compared to the forces acting on the UAV, the ordinary method of validating the stability is not practical for UAVs. Another method is proposed to overcome this problem by processing the flight angles and obtain the main modelling parameters (as natural frequencies) and compare them to the theoretical results. Procedure of this method is as following: during the UAV flight the flight path angles were recorded. After landing these data are retrieved and analysed by Fourier transform and the natural frequencies are obtained then compared to the ones calculated from the calculated ones.
The experiment is conducted for the steady cruise flight with some excitation for the different flight modes. Flight was controlled form the ground station of Ardupilot by means of telemetry link using "manual" mode without autopilot stabilization to prevent additional disturbances. During flight the aircraft was balanced so as to move straight with constant altitude and velocity. Flight data are obtained from the Inertial Measuring Unit (IMU) of the "ArduPilot Mega" autopilot which measure pitch, yaw, and roll angles of the aircraft and has sampling frequency of 3.7 Hz.
The retrieved data are processed by Fast Fourier Transform[5] once without filter and another time with filter using MATLAB. While examining the signal without filter, it is taken into account the signal first and last points have the same values to prevent aliasing.
Data are filtered by Hanning filter[1,14] to prevent leakage in the transform.[15] Such filter is chosen for this case because:

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- the exact amplitude of the frequency is not as important as the value of the frequency itself,
- the investigated signal is random and have unknown frequency,
- the vibrations are within narrow band,
- overcome the noise and obtain the frequency mean value.

For these reasons, the most suitable filter is Hanning filter.[15] Samples of measured pitch and yaw angles are shown in Figures 3 and 4.

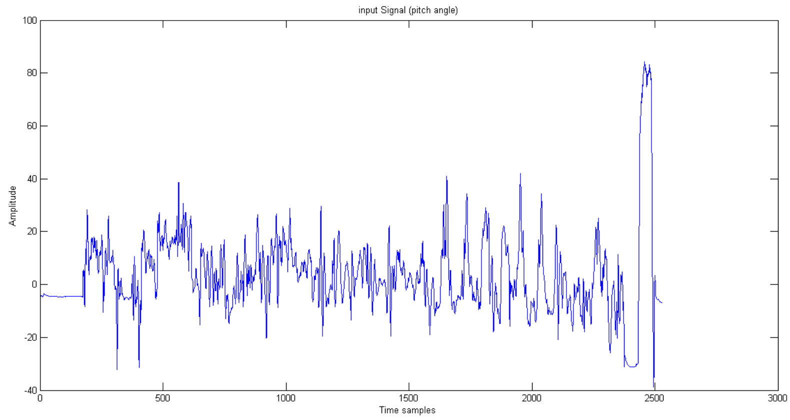

Figure 3: Sample of pitch angle recorded during the flight

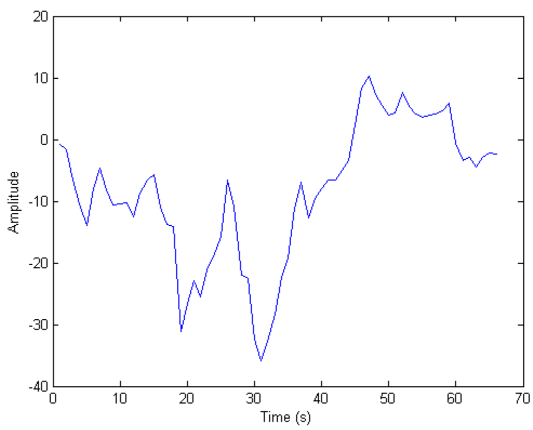

Figure 4: Sample of yaw angle recorded during the flight

Fast Fourier Transform (FFT) is used to convert the discrete time samples from time to frequency domain. After processing and filtering, some frequencies appeared close to each other with comparable amplitudes. Frequencies are weighted based on their amplitudes to obtain their mean value and the deviation. After averaging, the Sonic 185 results show a frequency of 0.18±0.01 Hz for the long mode, 1.12±0.04 Hz for the short mode and 0.61±0.06 Hz for dutch roll. Cirrus results show a frequency of 0.2±0.03 Hz for the long mode, 1.31±0.12 Hz for the short mode and 0.99±0.04 Hz for dutch roll. Figures 5 – 7 show samples of the signals in frequency domain at different periods of time which capture the long, short, and dutch roll modes respectively.
Fast Fourier Transform (FFT) is used to convert discrete time samples from time to frequency domain. After processing and filtering, some frequencies appeared close to each other with comparable amplitudes. After some averaging, the results show a frequency of 0.18±0.01 Hz and 0.2±0.03 Hz for the long mode of Sonic and Cirrus respectively. Figures 5 and 6 show samples of the signals in frequency domain at different periods of time which capture the long mode.

Some peaks are noticed in the frequency spectrum, probably they are caused by motor vibrations or elasticity effect. Identification of these peaks can be a subject of further investigations.

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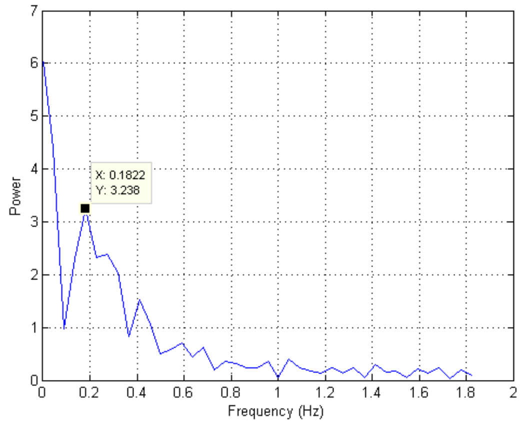

Figure 5: Sample of long mode frequency for Cirrus

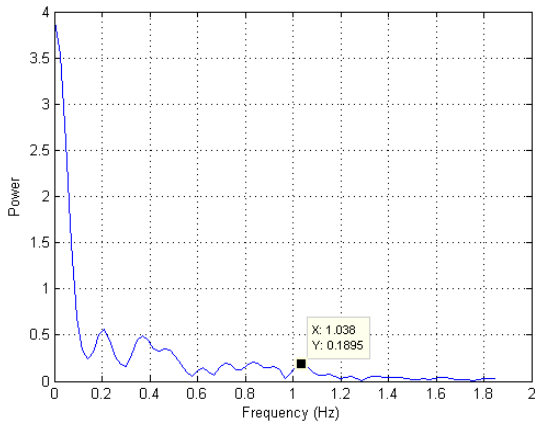

Figure 6: Sample of short mode frequency for Cirrus

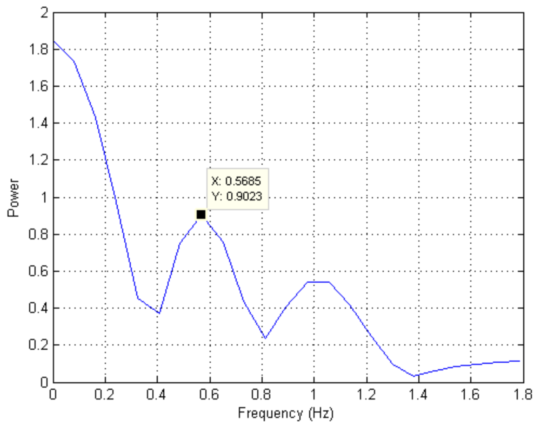

Figure 7: Sample of dutch roll mode frequency for Sonic 185

6. Analysis and Discussion

It is noticed that the aerodynamic coefficients estimated from the procedures of Roskam and Ostoslavsky are not matched together as illustrated from Tables 3 – 5. For example the mismatch of C_{mq} definition between Roskam and Ostoslavsky and the parameter $c_{y\beta}$ in Ostoslavsky method is higher than Roskam by ten times. Though, there is good

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agreement in the natural frequencies. From here it is concluded that each method must use its own aerodynamic and stability coefficients.
By comparing the calculations with the experimental results (see Table 9), it is obvious that the long mode frequencies obtained from Roskam exact formula, Ostoslavsky formulae and XFLR5 are within the allowable region of the experimental result. It is noticed that approximate methods of Roskam and Hull overestimate the long mode frequency. This is because they neglected some terms under the assumption that μ –which indicates the relative mass of the aircraft – is rather big compared to $C_{L\alpha}C_{mq}$ but this condition is not applicable in the UAVs. For the short mode frequency, all of the methods give acceptable results with respect to the one found in the experiment.
Considering the dutch roll mode, the exact method of Ostoslavsky shows high damping ratio while the approximate solutions give results in accordance with the experimental one, which means that the assumptions used are still valid for UAVs. On the other hand, approximate methods of Roskam don't achieve the required accuracy, consequently the assumptions he did are invalid for the case of small UAVs. By comparing the results of the exact method of Roskam, approximate method of Ostoslavsky and XFLR with the experiment, it is notable that these methods calculate result in the same order of the actual one as shown in Tables 9 and 10 while the approximate methods of Roskam are not accurate enough. This means that the exact methods valid for the large aircrafts can be implemented to the smaller ones (for lower *Re* numbers and small geometric scales).

Table 9: Results of natural frequencies of Sonic 185 (in Hz)

Method	**Short**	**Long**	**Dutch Roll**
Roskam – exact	1.056	0.178	0.59
Roskam – approx	1.096	0.284	0.50
Hull (approx)	1.139	0.302	0.49
Ostoslavsky – exact	1.119	0.179	–
Ostoslavsky – approx	1.122	0.1758	0.56
XFLR5	1.168	0.188	0.582
experiment	1.117 ± 0.04	0.18 ± 0.01	0.61± 0.06

Table 10: Results of natural frequencies of Cirrus (in Hz)

Method	**Short**	**Long**	**Dutch Roll**
Roskam – exact	1.205	0.187	0.891
Roskam – approx	1.307	0.374	0.71
Hull (approx)	1.216	0.415	0.632
Ostoslavsky – exact	–	0.183	0.8314
Ostoslavsky – approx	1.211	0.174	0.7839
XFLR5	1.548	0.168	0.955
experiment	1.31 ± 0.12	0.2 ± 0.03	0.99± 0.04

It is noticed that there is a mismatch between the theoretical and experimental results in dutch roll for Cirrus. This is because the wing has dihederal angle in the middle of the wing. This geometry is not covered in the used procedures.
Formulas and experimental data of the dutch roll show that the value of damping ratio is very close to the one corresponding to aperiodical motion. In this case to define the mode realizing (periodic or aperiodic) rather precise values of necessary parameters are required. As we can't grantee the absolute precision of parameters' values used in the calculations this can explain the fact that exact Otoslavsky formula predict overdamped (aperiodic motion) while approximate Ostoslavsky formula gives periodic damped motion.
Regarding the assumption that the long mode frequency has much lower magnitude than the short mode frequency, these two frequencies obtained from the results are compared to each other. as found in the experiment, the long mode frequency is six times less than the short mode. Hence one can conclude that the assumption of short mode is much bigger than the long mode is valid for small UAVs of the class analysed.

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7. Conclusion

This research compares the natural frequencies of longitudinal and lateral motions calculated by the conventional methods of estimation for the manned aircraft provided by the references of Roskam, Hull, and Ostoslavsky, and the numerical VLM program XFLR5 with experimental values of real flight. A special procedure of comparing theoretical and experimental results is proposed for the case of small UAVs.
The assumption of separating the disturbed aircraft motion to longitudinal and lateral motions is valid for small UAVs. Separation of longitudinal disturbed motion to long and short modes is possible for at least the UAVs having conventional configuration (with horizontal and vertical tails) as the short mode frequency is much higher than the one of the long mode.
For the long mode, exact Roskam, Ostoslavsky, and XFLR5 estimate the frequency within the range of the experimental results while the methods of Roskam approximation and Hull are not correct in case of small UAVs. Considering the short mode, all methods can predict the frequency with good accuracy. As for the dutch roll natural frequency, the approximate method of Ostoslavsky, exact method of Roskam, and XFLR5 estimate the frequency within the range of the experimental results while the approximate methods of Roskam underestimates the frequency and hence the assumptions used are not valid in case of small UAVs.
It is noted that results from analytical methods are valid only for the aerodynamic coefficients defined in the same procedure.
The methods for the large aircraft dynamics of the disturbed motion can be implemented to smaller aircraft geometric scales and lower *Re* numbers while the assumptions must considered carefully taking into consideration that some stability coefficients cannot be neglected as in the case of large aircraft.

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