DOI: 10.13009/EUCASS2017-205

Features of the nanosatellite dynamics in the gravitational and magnetic field of the Earth

[1]Suimenbayev B.T., [2]Alekseyeva L.A.,[1]Suimenbayeva Zh.B.,[1]Gusseinov S.R., [1]Yermoldina G.T.
[1]Kazakh National Research Technical University named after K.I.Satpayev,
22a Satpayev Street, 050013, Almaty, Kazakhstan
[2]Institute of mathematics and mathematical modeling,
125 Pushkin Street, 050010, Almaty, Kazakhstan
bts49@mail.ru, alexeeva47@mail.ru, zbs115@mail.ru, samir.gusseinov@gmail.com, gulerm@mail.ru

Abstract
The object of the research is computer modeling of a spacecraft movement and its movement control on the stand base of the flights control center of Kazakh National Research Technical University named after K.I. Satpayev. Nano-satellite ‘Polytech-1’ developed in the flights control center is a scientific educational spacecraft intended for operational measurement of the Earth’s magnetic field at the height of approximately 400 km and transmission of measuring information to the reception means of the university flights control center. Here is a mathematical model of nano-satellite ‘Polytech-1’ movement in the Earth’s gravitational (Spherical Harmonical Gravity Model) and magnetic (The US/UK World Magnetic Model for 2015-2020) fields developed in Matlab-Simulink system and realized in the software.

1. Introduction

For the analysis of technological processes of operation of orbital systems, for the development of technologies of the semi-natural research of the on-board equipment systems as well as for educational purposes, it is necessary to develop mathematical models of movement, control and stabilization of a nano-satellite in the neighborhood of the programmed movement taking into account its constructional features.

To ensure the programmed movement of a spacecraft in the orbit connected with its purpose, it is necessary to take into account and estimate all acting forces. Besides the Earth’s gravitation, there are various forces which act on a spacecraft such as gravitation forces of celestial bodies (Moon, Sun, planets and others), light pressure force, electromagnetic forces, aero-dynamical forces and others which influence its movement to a different extent. The effect of this influence depends on a type of a spacecraft and conditions of its movement such as the type of the orbit, its inclination, the position in the orbit, etc.

Magnetic effects can be used to control and stabilize the satellite position in the orbit. Passive magnetic stabilization uses a set of constant magnets to orientate the satellite in relation to the Earth’s magnetic field. Besides, spacecraft magnetization occurs due to the on-board electronics operation, its radio complex. The spacecraft shield is also magnetized by the Earth’s magnetic field and consequently there is a hysteresis pattern which creates an effect of attenuation of the satellite rotation angular velocity.

In the present work there is modeling of dynamics of the nano-satellite in the Earth’s gravimagnetic field with the use of Matlab-Simulink software system on the basis of a rigid body, which is divided into two basic stages: modeling of mechanics of the translational movement of the nano-satellite center of mass and its own rotational movement around its center of mass [1-3].

1.1 Development of Orbital Environment Simulator

The orbital environment simulator, which is shown in figure 1, is realized to develop the control system of the nano-satellite. It consists of six interconnected blocks as follows:

1. Input and calculation of initial data: coordinates, speed of the center of mass, angular coordinates and the spacecraft speed (Spherical Harmonical Gravity Model, 6DOF equations of motion);
2. Calculation of the spacecraft orbit, speed and position of its center of mass (Integration of the motion of the center of mass);
3. Calculation of angular velocity of the spacecraft rotation and Euler angles (6DOF equations of motion);
4. Calculation of the acting gravitational torque (Gravity Gradient Model)
5. Calculation of the acting magnetic torque (Magnetic Torque acting on satellite)
6. Calculation of the magnetic (hysteresis) torque (Model of Magnetic Torque due to the Hysteresis material)

DOI: 10.13009/EUCASS2017-205

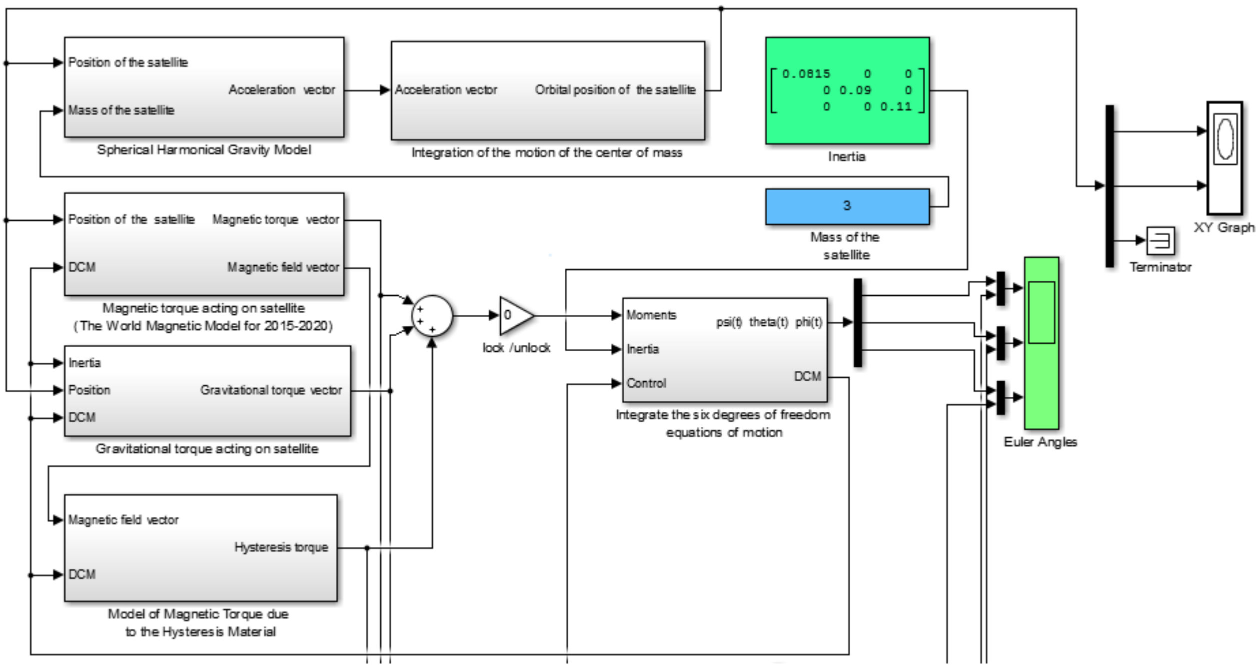


Figure 1: Block-scheme of the Orbital Environment Simulator (Matlab R2015b)

1. Calculating scheme of block 1. Input data. For description of the nano-satellite movement, conditionally the 'static' system is chosen as a basic system, i.e. *inertial coordinates system* (ICS - *XYZ*), intended for modeling of the nano-satellite movement along its orbit, and 'moving' coordinates system rigidly bound to the satellite body and participating in all its movements, so-called *board coordinates system* (BCS – $X_1Y_1Z_1$). For the convenience of calculations the origin of BCS is combined with the spacecraft center of mass, and axes are put along its main inertia axes (axes of Koenig), which are defined by geometry of mass distribution in the spacecraft body. At the initial moment of time t_0 the coordinates of the spacecraft center of mass and its speed are set in the ICS (R_0, V_0).

For the description of the nano-satellite rotation around the center of mass, Euler angles ψ, θ, φ are used: precession angle ψ is the angle between axis X and the node line (line of intersection of planes XY and X_1Y_1), nutation angle θ is the angle between axes Z and Z_1 ,the angle of its self rotation φ is the angle between the node line and axis X_1. Projections of the angular velocity of the spacecraft rotation in BCS (p, q, r) are defined through Euler angles and their velocities $\dot{\theta}$, $\dot{\psi}$, $\dot{\varphi}$ by Poisson formulae. At the initial moment of time Euler angles $\psi(t_0)$, $\theta(t_0)$, $\varphi(t_0)$ and the coordinates of the angular velocity in BCS $p(t_0)$, $q(t_0)$, $r(t_0)$ are set and initials $\dot{\theta}(t_0)$, $\dot{\psi}(t_0)$, $\dot{\varphi}(t_0)$ are calculated.

2. Calculating scheme of block 2. Movement of the spacecraft center of mass. The Earth's gravitational field is defined by its gravitational potential $U(X,Y,Z)$, which can be different depending on the choice of the Earth's standard model. There are several models. Here it is modeled by its gravitational potential (1) through the spherical functions.

All planets of the Solar system have a form close to a spherical one. Therefore, the gravitational field of a sphere can be considered as the first approximation to the gravitational field of a planet. In the second approximation, we can consider the fact that some planets including the Earth could be better presented as a rotation ellipsoid than a sphere. In the third approximation, we can take into account some features of mass distribution inside a planet, etc. The gravitational field of a planet is usually introduced according to its spherical function series. Due to the solving task, there are different requirements to the details of the initial data, to the number of the decomposition members and to the number of the initial parameters.

In the process of modeling the exact theory of the near-earth satellites movement, it should not be limited to an approximate expression for the Earth's potential. The Earth's gravitational potential for the external point which is located at r distance from the Earth's center can be expressed in the general view through the spherical functions in the following way:

$$U(r,\psi,\lambda) = \frac{fM}{r}\left\{1 + \sum_{n=1}^{n}\sum_{m=0}^{n}\left(\frac{a_0}{r}\right)^n (C_{nm}\cos(m\lambda) + S_{nm}\sin(m\lambda))P_{nm}(\sin(\psi))\right\} \tag{1}$$

In this formula fM is the Earth's mass multiplied by the gravitational constant; (r, φ, λ) are the spacecraft spherical coordinates in which the potential U is determined; a_0 is the equatorial radius of the planet (a_0 = 6378.1 km); C_{nm}, S_{nm} are the gravitational torques (they depend on the body shape and the mass distribution inside it). The axis

DOI: 10.13009/EUCASS2017-205

coordinates are chosen as follows: the origin of the coordinates O is located in the Earth's center of mass, the angle $\psi \le \pi/2$ is counted from the Earth's equatorial plane, the angle $0 \le \lambda \le 2\pi$ is counted from the basic meridian.

Function P_{nm} is Legendre associated function of power n and order m. Legendre polynomials are determined by so-called Rodrigo formula:

$$P_{n0}(z) = \frac{1}{2^n n!}\frac{d^n}{dz^n}(z^2-1)^n, \tag{2}$$

Legendre associated functions are determined by the following formula:

$$P_{nm}(z) = (1-z^2)^{\frac{m}{2}}\frac{d^m P_{n0}(z)}{dz^m}, \tag{3}$$

To determine the force acting on a particle (or a spacecraft) located in the potential field (1), it is necessary to take a gradient from potential U as shown below:

$$\vec{F} = -m \times grad(U) = -m\left(\frac{\partial U}{\partial r}\vec{e_r} + \frac{1}{r}\frac{\partial U}{\partial \psi}\vec{e_\psi} + \frac{1}{r\cos\psi}\frac{\partial U}{\partial \lambda}\vec{e_\lambda}\right) \tag{4}$$

where m is mass of a particle (a spacecraft).

The force vector mentioned in formula (4) is written in the spherical coordinates system. To transfer to Cartesian coordinates system it is necessary to do the following:

$$\begin{cases} \vec{e_r} = \cos\psi\cos\lambda\,\vec{i} + \cos\psi\sin\lambda\,\vec{j} + \sin\psi\vec{k} \\ \vec{e_\psi} = \sin\psi\cos\lambda\,\vec{i} + \sin\psi\sin\lambda\,\vec{j} - \cos\psi\vec{k} \\ \vec{e_\lambda} = -\sin\lambda\,\vec{i} + \cos\lambda\,\vec{j} \end{cases} \tag{5}$$

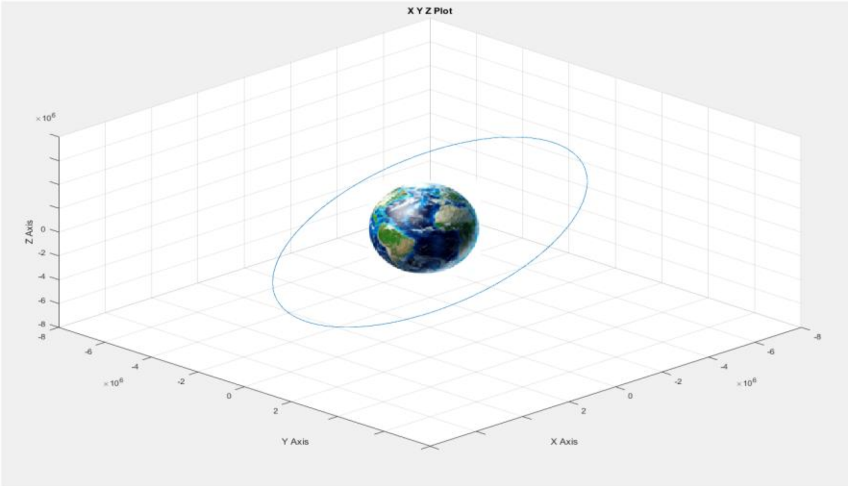
Knowing the force vector, it is not difficult to determine equations of the movement of the spacecraft center of mass by using Newton dynamic equations. Coefficients C_{nm} and S_{nm} in equation (1) are determined experimentally and were first obtained and published by Smithson astrophysical observatory of the USA in 1970.

The subsequent elements, the coefficients of which are lower by a factor of a thousand or more than C_{20}, reflect the details of the Earth's shape and structure. Due to the absence of the exact data concerning the proper mass distribution inside the Earth and its shape, it is impossible to determine coefficients C_{nm} и S_{nm} directly. That is why they are determined indirectly on the basis of measuring the gravity on the surface of the Earth and observation of pertubance in the movement of the near-earth artificial satellites.

Considering all above-mentioned, the model of the Earth's gravitational field has been realized in Matlab–Simulink with the help of spherical harmonics. The given model allows modeling the Earth's gravitational field in any orbit up to a geostationary one with a high degree of accuracy using numerical data values of C_{nm} and S_{nm} (which are dimensionless ratio depending on the Earth's shape and inside mass distribution in it). However, the accuracy of the used model can be changed by adding or removing coefficients C_{nm} and S_{nm} in equation (1) for the gravitational potential.

The coordinates of the position of the spacecraft center of mass in the orbit are given to the Spherical Harmonical Gravity Model block input, and at the output we receive the information concerning the resultant of gravitational forces acting on the spacecraft. Then the information about the acting force goes to the block of integrators where the orbit speed and the spacecraft position in the orbit are counted.

DOI: 10.13009/EUCASS2017-205



(h = 400 km, $T \approx 1.5$ hr, $i = 90^0$, $\varepsilon \approx 0.0304$, a = 6990.6 km)

Figure 2: Animation of the movement path of the spacecraft center of mass in the Earth's gravitational field for the polar orbit made in Matlab-Simulink software

The spacecraft movement has been modeled in the polar orbit. In figure 2 there is animation of the movement path of the spacecraft center of mass in the polar orbit with eccentricity $\varepsilon \approx 0.0304$, inclination i - 90^0 and semi-major axis a – 6990.6 km.

3. Calculating scheme of block 3. The spacecraft rotation. The equations of the spacecraft movement around the center of mass are described by Euler dynamic equations in the coordinates system which is connected with the major axes of the inertia [1,2]:

$$\begin{aligned} A\frac{dp}{dt} &= q \times r \times (B-C) + M_1 \\ B\frac{dq}{dt} &= p \times r \times (C-A) + M_2 \\ C\frac{dr}{dt} &= p \times q \times (A-B) + M_3 \end{aligned} \tag{6}$$

where A, B, C are the main moments of the spacecraft inertia, M_1, M_2, M_3 are the projections of the summarized moment of forces acting on the spacecraft p, q, r are the projections of the vector of the instantaneous angular velocity of rotation on the axis of Koenig.

In block 3 (6DOF equations of motions) there is integration of the Euler equations system with consideration of the moments of the acting forces on the center of mass. They can be divided into external ones, connected with the Earth's gravimagnetic field impact or aerodynamic forces, and internal ones, which depend on the magnetization of the spacecraft itself and the acting driving moment which provides the spacecraft programmed rotation in the orbit connected with its purpose.

4. Calculating scheme of block 4. Gravitational torque model. Gravitational torque M_g is a significant source of the angular moment for high-orbital spacecrafts.

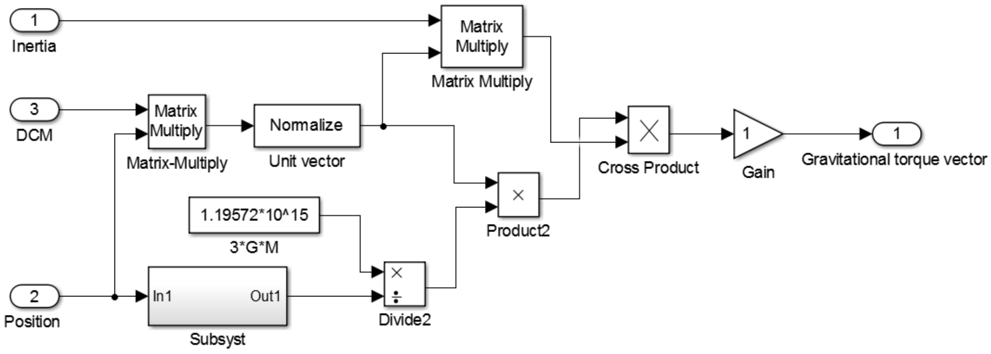


Figure 3: Block-scheme of calculation of the gravitational torque acting on the spacecraft (Gravity gradient model)

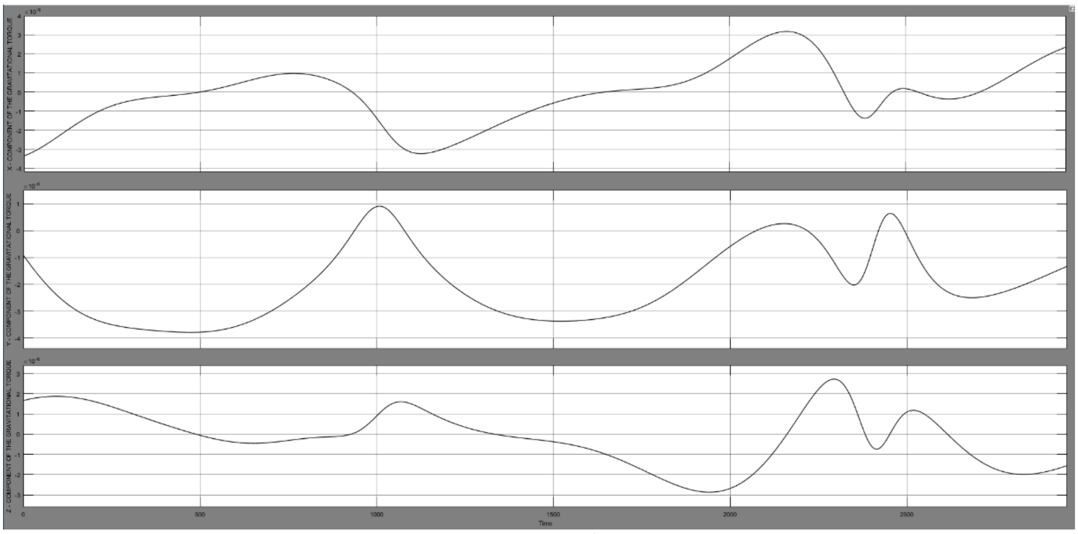
DOI: 10.13009/EUCASS2017-205

To calculate the gravitational torque acting on the satellite at the current moment of time it is necessary to know the spacecraft position in the orbit, the height and the mass characteristics of the satellite. For the accepted model of the Earth's gravitational field here it is counted by V.V. Beletsky formula [1,2]:

$$\overrightarrow{M_g} = \frac{3\mu}{R_0^3} \times \left[\overrightarrow{e_R}, J \times \overrightarrow{e_R} \right] \tag{7}$$

where e_R is a single vector guiding the spacecraft center of mass: $e_R = r/R_0$, R_0 is the distance from the Earth's center of mass to the satellite center of mass, J is the spacecraft inertia tensor (in Figure 1 it is shown as Inertia block).

In figure 3 there is the block-scheme of gravitational torque calculation realized in MatlabR2015b system.



(h = 400 km, $T \approx 0.83$ hours, $i = 90^0$, $\varepsilon \approx 0.0304$, a = 6990.6 km)

Figure 4: Components of the vector of the gravitational torque $\vec{M}_g$ for the polar orbit

In figure 4 there are graphs of changing the gravitational torque components in BCS on the above-shown calculating trajectory for the half-period of movement. As it is seen from the graph, at the height of approximately 400 km from the Earth's surface the components of the gravitational torque vector fluctuate at the range from -4×10^{-6} to 4×10^{-6} N·m. For different heights, the gravitational torque acting on a satellite has different value reducing with increasing height.

Modeling of the Earth's magnetic field. The interaction of the Earth's magnetic field with the spacecraft self magnetic field also influences the spacecraft rotation in the near-earth orbit and it depends on its electric current systems, constant magnets as well as magnetization of its shield material.

The Earth is like a giant magnet. At every location on or above the Earth, its magnetic field has a more or less well-known direction. At some places on the globe the horizontal direction of the magnetic field coincides with the direction of geographic north ("true" north), but in general this is not the case. The main utility of the World Magnetic Model (WMM) is to provide magnetic declination for any desired location on the globe. In addition to the magnetic declination, the WMM also provides the complete geometry of the field from 1 km below the Earth's surface to 850 km above the surface.

The Earth's magnetism has several sources. All sources affect a scientific or navigational instrument but only some of them are represented in the WMM. The strongest contribution, by far, is the magnetic field produced by the Earth's liquid-iron outer core, called the "core field". Magnetic minerals in the crust and upper mantle make a further contribution that can be locally significant. Electric currents induced by the flow of conducting sea water through the ambient magnetic field make a further, albeit weak, contribution to the observed magnetic field. All of these are of "internal" origin and included in the WMM. The mathematical method of the WMM is an expansion of the magnetic potential into spherical harmonic functions to degree and order 12.

The geomagnetic field vector, B_m, is described by seven elements. These are the northerly intensity X, the easterly intensity Y, the vertical intensity Z (positive downwards) and the following quantities derived from X, Y and Z: the horizontal intensity H, the summarised intensity F, the inclination angle I, (also called the dip angle and measured from the horizontal plane to the field vector, positive downwards) and the declination angle D (see Figure 5).

DOI: 10.13009/EUCASS2017-205

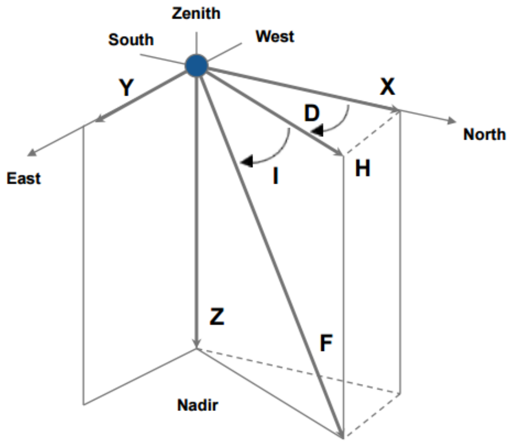


Figure 5: The seven elements of the geomagnetic field vector B_m associated with an arbitrary point in space

The quantities X, Y and Z are the sizes of perpendicular vectors that add vectorially to B_m. Conversely, X, Y and Z can be determined from the quantities F, I and D (i.e., the quantities that specify the size and direction of B_m)

The main magnetic field B_m is a potential field and therefore can be written in geocentric spherical coordinates (longitude λ, latitude ϕ', radius r) as the negative spatial gradient of a scalar potential

$$\vec{B}_m\left(\lambda,\varphi',r,t\right)=-\nabla V\left(\lambda,\varphi',r,t\right) \tag{8}$$

This potential can be expanded in terms of spherical harmonics [4]:

$$V\left(\lambda,\varphi',r,t\right)=a\sum_{n=1}^{N}\left(\frac{a}{r}\right)^{n+1}\sum_{m=0}^{n}\left(g_n^m(t)\cos\left(m\lambda\right)+h_n^m(t)\sin\left(m\lambda\right)\right)P_n^m(\sin(\varphi')) \tag{9}$$

where N=12 is the degree of the expansion of the WMM, a is the geomagnetic reference radius, (λ, ϕ', r) are the longitude, latitude and radius in a spherical geocentric reference frame, and $g_n^m(t)$ and $h_n^m(t)$ are the time-dependent Gauss coefficients of degree n and order m describing the Earth's main magnetic field. For any real number μ, $P_n^m(\mu)$ are the Schmidt semi-normalized associated Legendre functions defined as:

$$\begin{gathered} P_n^m\left(\mu\right)=\sqrt{2\frac{(n-m)!}{(n+m)!}}\times P_{n,m}\left(\mu\right)\quad if\quad m>0 \\ P_n^m\left(\mu\right)=P_{n,m}\left(\mu\right)\quad if\quad m=0 \end{gathered} \tag{10}$$

5. Calculating scheme of block 5. (Magnetic torque acting on satellite). Here the magnetic torque acting on the spacecraft conditioned by the geomagnetic field is modeled by interaction of dipolar magnetic torques according to the following formula [2,5-7]:

$$\vec{M}_{mgn}=\left[\vec{m},\vec{B}\left(\vec{X}_{mc}\right)\right] \tag{11}$$

where m is the magnetic dipolar torque of the spacecraft $A\cdot m^2$, $B(X_{mc})$ is the vector of the geomagnetic field intensity in the spacecraft center of mass.

Below in figure 6 there is realization of the block-scheme of calculating the geomagnetic torque acting on the spacecraft.

DOI: 10.13009/EUCASS2017-205

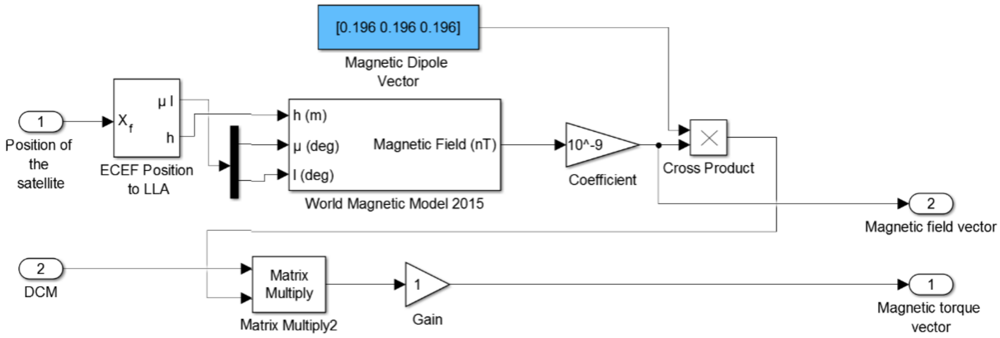
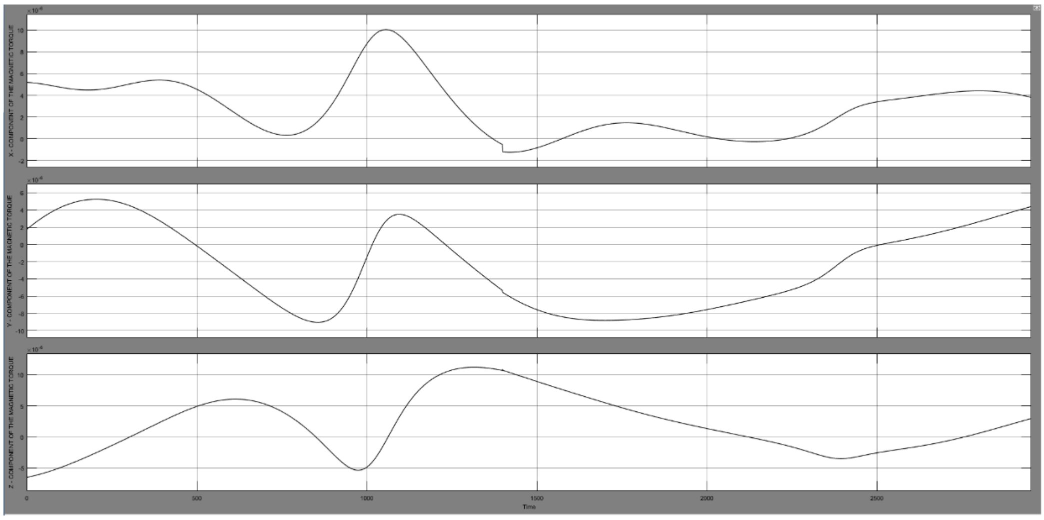


Figure 6: Block-scheme of calculating the geomagnetic torque acting on the spacecraft (Matlab-Simulink)

In figure 7 there are graphs of changing the gravitational torque vector components acting on the satellite in BCS in the calculating orbit for the half-period of the spacecraft movement along the orbit



(h = 400 km, $T \approx 1.5$ hr, $i = 90^0$, $\varepsilon \approx 0.0304$, a = 6990.6 km)

Figure 7: Gravitational torque vector components $\vec{M}_{mgn}$ acting on the spacecraft for the polar orbit

As it can be seen, the gravitational torque vector components in the calculating orbit fluctuate at the range from 10^{-5} N·m (maximal value) to 0. As well as in the case of the gravitational torque, numerical modeling has been made for the half-period of movement along the orbit.

6. Calculating scheme of block 6. Magnetic hysteresis torque. The phenomenon of the magnetic hysteresis is connected with magnetization and demagnetization of the ferromagnetic material when changing the intensity of the external magnetic field. During the process of the spacecraft movement and rotation in the geomagnetic field its shield material is magnetized and demagnetized along the hysteresis curve which depends on its features.

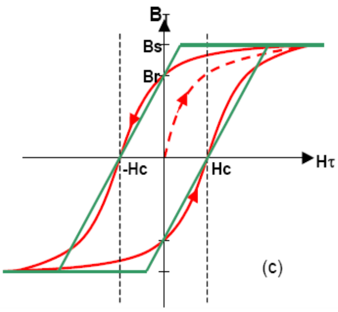


Figure 8: Modeling of hysteresis loop in Matlab–Simulink

Meanwhile the additional magnetic torque acting on the spacecraft rotation in the orbit appears. In figure 8 there is the curve of ferromagnetic material magnetization, where H_c is the coercitive force, B_z is the residual magnetization, B_s is saturation of the magnetic flow density[8-13].
At first, to calculate the hysteresis magnetic torque it is necessary to determine the magnetic induction vector B of the geomagnetic field in the spacecraft location using model WMM – 2015 which is described above. Then, with the help of the matrix of conversion from ICS to BCS, its components are determined in BCS.

$$\vec{B}^{BRF}_{Earth} = C_{I-B} \times \vec{B}^{IRF}_{Earth} \tag{12}$$

where C_{I-B} is the matrix of conversion from ICS to BCS.
The density of the magnetic field is calculated by the approximate model of the hysteresis loop mentioned above:

$$\vec{B}_{h.m} = F_{hyst}\left(\vec{B}_{Earth}\right) \tag{13}$$

where $\vec{H}_{Earth}$ is the vector of intensity of the geomagnetic field, F_{hyst} is the function which describes the curve of the hysteresis loop

$$\vec{H}_{Earth} = \vec{B}^{IRF}_{Earth} \Big/ \mu_0 \tag{14}$$

where μ_0 =1.256×10^{-6} H/m[6]. Magnetic torque of the hysteresis material is determined by the following formula:

$$\vec{m}_{hyst} = \frac{V_{hyst}}{\mu_0} \times \vec{B}_{h.m} \tag{15}$$

where V_{hyst} is the volume of the hysteresis material[6,11]. The hysteresis magnetic torque acting on the spacecraft is determined by the following formula:

$$\vec{M}_{hyst} = \left[\vec{m}_{hyst}, \vec{B}^{BRF}_{Earth}\right] \tag{16}$$

In figure 9 there is realization of the model of the hysteresis torque acting on a satellite as a result of magnetization of the ferromagnetic materials which are on the spacecraft board when moving through the Earth's magnetic field.

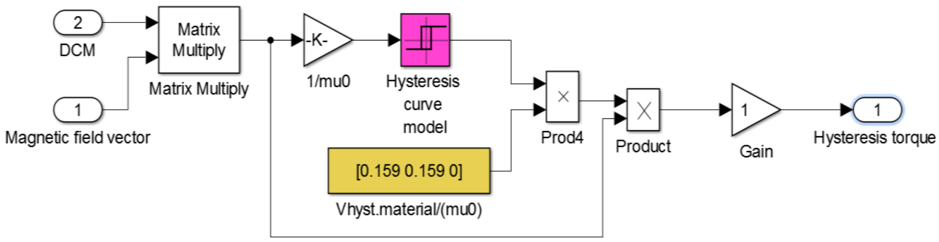


Figure 9: Block-scheme of calculation of the magnetic torque caused by the spacecraft ferromagnetic material

In figure 10 there are components of the vector of the hysteresis magnetic torque acting on a satellite in BCS for a single period of the spacecraft movement along the orbit. It is seen in the graph that at the height of 400 km from the Earth's surface the hysteresis torque vector components fluctuate at the range from - 3.5×10^{-6} to 3.5×10^{-6} N·m.

1.2 Modeling of the spacecraft movement in the polar elliptic orbit. Numerical experiment

Here there are the results that have been obtained on the basis of analysis and modeling of the spacecraft movement in the Matlab-Simulink software. When modeling we have taken into account the gravitational, magnetic and hysteresis torques acting on the spacecraft movement. Here the aerodynamic forces and the aerodynamic moment

DOI: 10.13009/EUCASS2017-205

have not been considered as they are insignificant at the heights of over 300 km and are almost zero at the taken height (the high-orbital spacecraft).

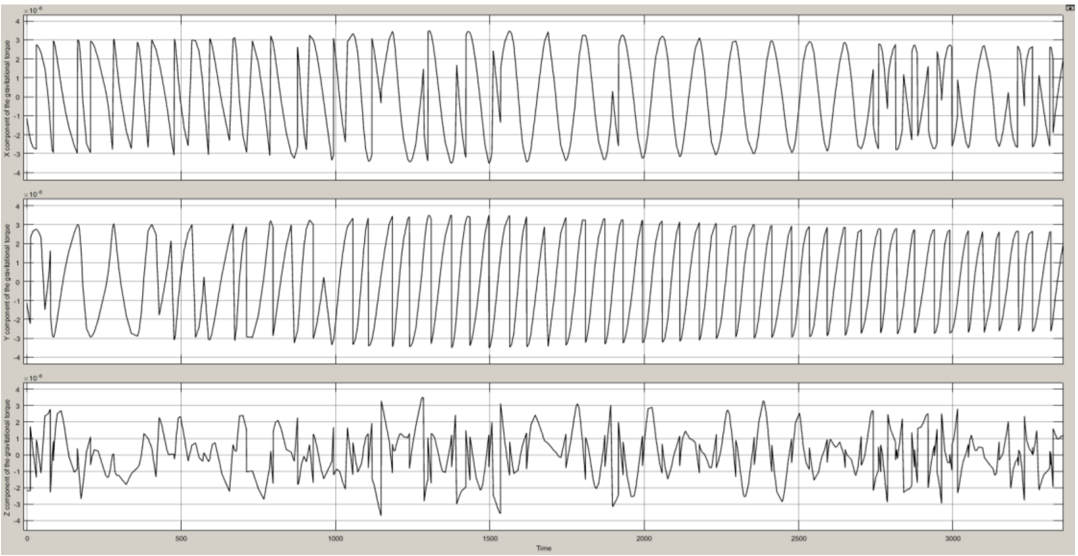


Figure 10: Components of the vector of the hysteresis torque in BCS acting on the spacecraft

In tables 1 and 2 there are data which have been used in modeling of spacecraft ‘Polytech-1’ dynamics in the Earth’s gravimagnetic field. The values of the main inertia moments about the spacecraft center of mass have been obtained in Solidworks software by mathematical calculation using the mass and dimensions data and the spacecraft units.

Table 1: Calculating characteristics of nano-satellite ‘Polytech-1’

Spacecraft parameters	**Values**
Mass	3 kg
Dimensions	10×10×30 sm^3
Inertia moments	(0.0815; 0.09;0.11) $kg{\cdot}m^2$
Magnets	**Values**
Full volume	0.59 sm^3
Magnetic dipole	0.34 $A{\cdot}m^2$
Ferromagnetic material	**Values**
Volume	0.60 sm^3
Saturation	0.73 Tesla
Coercitive force	1.59 A/m

In figures 9 and 10 there are the results of modeling the spacecraft movement in the elliptic orbit. In the first graph there are components of the vector of the satellite orbital position, in the second graph there are velocities the absolute value of which is approximately 7.7 km/s at the time moment of t = 0. As it should be, the components of satellite speed are changed periodically with the cycle of 1.5 hours. The movement occurs in XY plane as component V_z is equal to zero for the whole period of time.

In figure 11 there are graphs of the orbital position and the speed of the spacecraft center of mass in ICS. In the first graph there are curves of the components of the spacecraft center of mass vector radius for the polar orbit (with parameters in table 2) changing with the time. In the second graph there are curves of the components of the spacecraft center of the mass orbital velocity. As it has been expected, the curves have the periodic character with the cycle of ~ 1.5 hours.

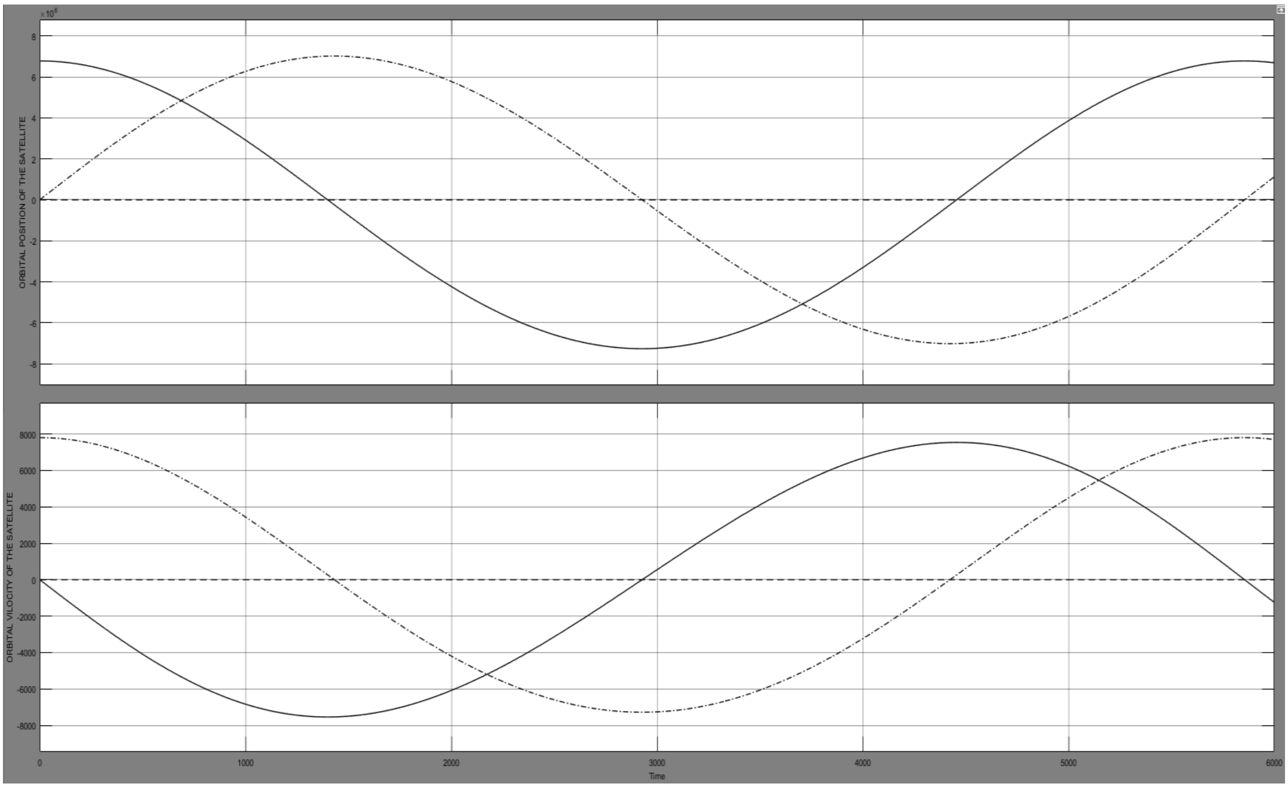
DOI: 10.13009/EUCASS2017-205

Table 2: Keplerian orbital parameters used in modeling

Satellite orbit parameters in ICS	**Values**
Altitude	~ 400 km
Orbital velocity	7,8 km/s
Period	~1.5 hr
Orbital inclination	90^0
Eccentricity	0.0304
Semi-major axis	6990.6 km
Semi-minor axis	6987.4 km

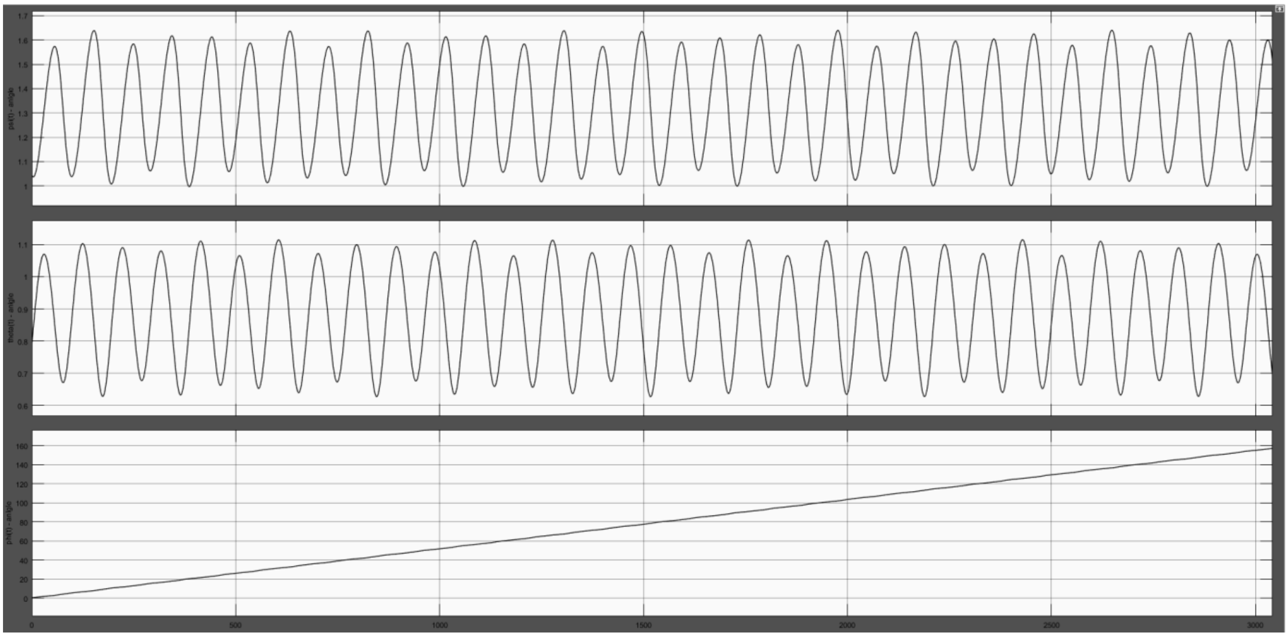
In figure 12 there are graphs of Euler angles (ψ, θ, ϕ), determining the spacecraft orientation in ICS. The initial angle velocities and positions (ψ, θ, ϕ) of the spacecraft have been chosen randomly in all cases.

For analysis and comparison of impact of different moments on the satellite behavior (orientation) the calculations have been done in two stages, at first without external moments impact, then with their summarized impact.



($\boldsymbol{V_0}$ = [0 0 7.8] km/s, $\boldsymbol{R_0}$ = [6778,137 0 0] km, T ≈ 1,5 hr; $i = 90^0$, $\varepsilon \approx 0.0304$, a = 6990.6km)
X – continuous curve, Y – dashed curve, Z – dot-and-dash curve

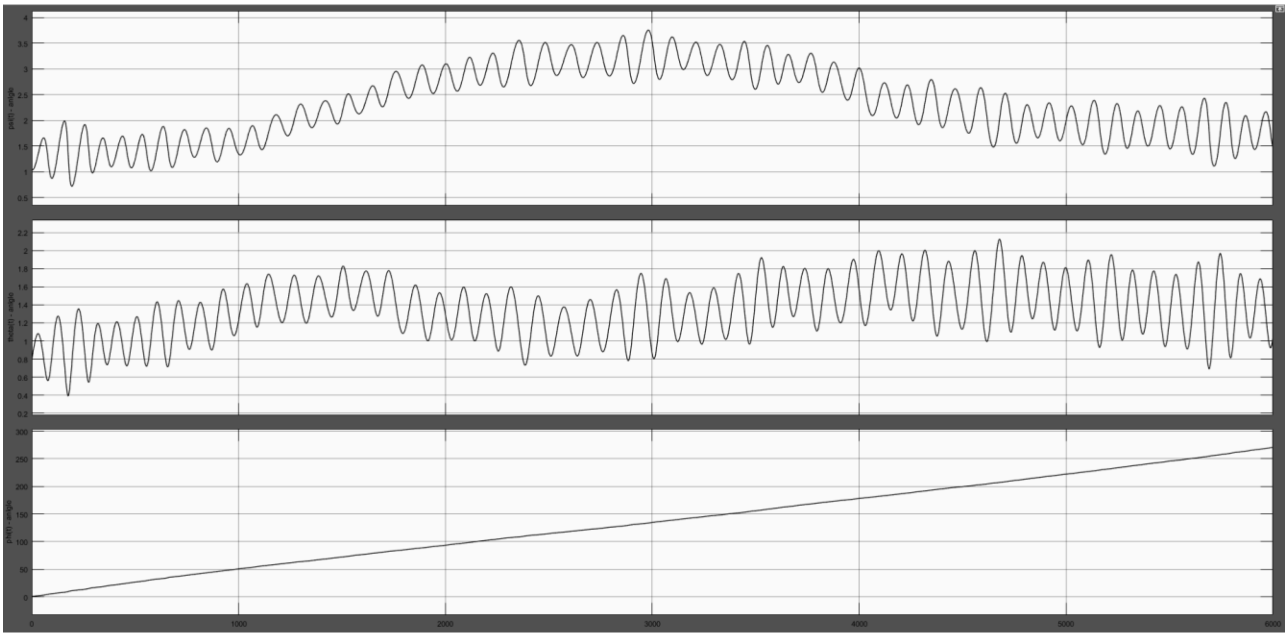
Figure 11: Spacecraft orbital position (upper graph) and orbital velocity (lower graph) for the polar orbit



$(p\ q\ r)$ = [0.01 -0.01 0.05] rad/s, $(\psi\ \theta\ \phi)$ = [π/3 π/4 π/6] rad, $T \approx 1.5$ hr, $i = 90^0$, $\varepsilon \approx 0.0304$, a = 6990.6 km

Figure 12: Changing of angles ψ, θ, ϕ in the polar orbit without the external moment

In figure 12 there is a case when the summarized external moment acting on the spacecraft is equal to 0. As it can be seen from the graph, the spacecraft rotates at approximately constant angle velocity (ω_3 = 0.05 rad/s) around axis X_3 (the third curve), which is tightly bound to the spacecraft, and at the same time it precesses (the first curve) around axis Z of the static coordinates system at first in one direction, then in the opposite direction with the period of approximately 1.5 minutes. Simultaneously with the spacecraft rotation and precession there is nutation observed (the second curve) which has the fluctuating character (with the period of 1.3 minutes) about the initial angle.



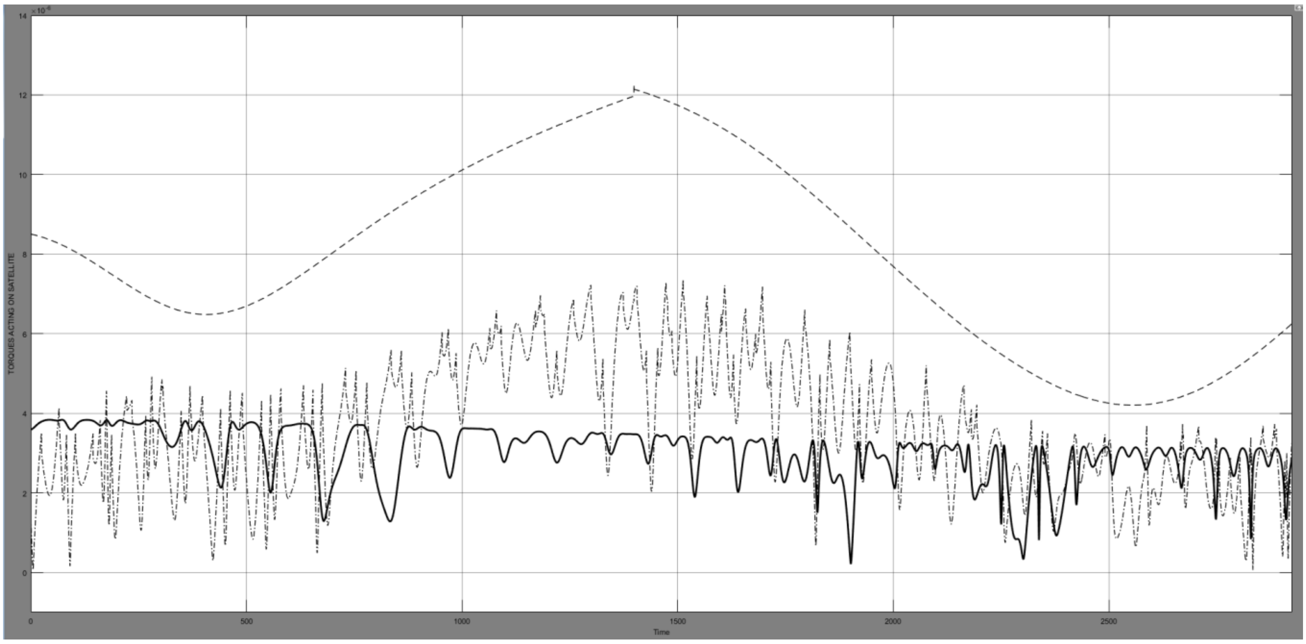
(p, q, r) = [0.01 -0.01 0.05] rad/s, (ψ, θ, ϕ) = [π/3 π/4 π/6] rad, $T \approx$ 1.5 hr, $i = 90^0$, $\varepsilon \approx 0.0304$, a = 6990.6 km

Figure 13: Changing of Euler angles ψ, θ, ϕ in the spacecraft movement in the polar orbit with consideration of the acting force moments

Considering all the acting force moments of the nano-satellite (figure 13) there is significant changing of angles ψ и θ. The spacecraft rotates at the constant angular velocity $\omega_3 \approx 0.05$ rad/s around axis Z_1 and precesses around axis

DOI: 10.13009/EUCASS2017-205

Z of the static coordinates system with minimal range $\psi_{min} \approx 0{,}5$ rad at 25th minute and with maximal $\psi_{max} \approx 1{,}5$ rad at the cycle completion. Also the nutation angle θ changes with fluctuation around $\theta \approx 1.4$ rad.



(M_{mgn}– dashed curve, M_{hyst}– dot-and-dash curve, M_g– continuous curve)

Figure 14: Changing of modules of the vectors of the magnetic, hysteresis and gravitational torques acting on the spacecraft in time

In figure 14 there are graphs of the external moments acting on the spacecraft in time. It is seen from the graph that the most dominant source of pertubance acting on the spacecraft at the height of 400 km from the Earth's surface is the magnetic torque the maximal value of which is equal to ~ 1.2×10^{-5} N·m. The second important source of pertubance is the hysteresis torque (caused by the spacecraft ferromagnetic material).

Table 3: Average values of the torques acting on the spacecraft at the altitude of 400 *km*

External types of pertubances	Average value of torques for the half-period in $N{\cdot}m$
Gravitational torque	3×10^{-6}
Magnetic torque	8×10^{-6}
Hysteresis torque	4×10^{-6}

For a nano-satellite having a mass of about 3 *kg* the gravitational torque impacts its rotation in the weakest way. The geomagnetic torque acting on the spacecraft is twice more than the hysteresis torque and it exceeds the gravitational torque by more than 2 times in this orbit. In table 3 there are average values of the moments of forces acting on the spacecraft at the height of 400 km for the half-period of its movement in the orbit.

Conclusion. The developed software complex allows modeling the orbital movement of satellites around the center of mass in different near-earth orbits. The inclination angle of an orbit to the equator as well as the orbit parameters is determined by the initial speed and the initial position of its center of mass. The rotation character depends on the initial angular velocities of a satellite and its position in the orbit. The variation of these parameters allows modeling a wide range of orbital movements of spacecrafts with consideration of their physical-mechanical features (mass, inertia tensor, its magnetization features and presence of magnetic systems).

References

[1] Beletsky V. V. 1977.The motion of the satellite around the center of mass in the gravitational field of the Earth .- Moscow: "Nauka" . P.300.
[2] Beletsky V.V., Khentov A.A. 1985.The rotational motion of a magnetized satellite M .: "Nauka." - The main edition of physical and mathematical literature.P.424.
[3] Suimenbaev B.T., Alekseeva L.A., Suimenbaeva Zh.B., Gusseinov S.R. 2016. Modeling the dynamics of the spacecraft in the gravimagnetic field of the Earth in the system"MatlabSimulink" . Izvestiya NAS RK, series physics and mathematics. №4 (308).P.192-207.
[4] Chulliat, A., S. Macmillan, P. Alken, C. Beggan, M. Nair, B. Hamilton, A. Woods, V. Ridley, S. Maus and A. Thomson, 2015, The US/UK World Magnetic Model for 2015-2020: Technical Report, National Geophysical Data Center, NOAA. doi: 10.7289/V5TB14V7
[5] Rawashdeh, S. A., and James E. L. 2013. Aerodynamic Stability for CubeSats at ISS Orbit. JOSS. JoSS, Vol. 2, No. 1, pp. 85-104.
[6] Rawashdeh, S. A. (2009): Passive attitude stabilization for small satellites, M.S. Thesis, Univ. Kentucky, Lexington.
[7] Rawashdeh, S. A. and Lumpp, J. E. (2010): Nano-satellite passive attitude stabilization systems design by orbital environment modeling and simulation, in Proc. AIAA Infotech@Aerospace Conf., Atlanta, GA.
[8] Levesque, J. (2003): Passive magnetic attitude stabilization using hysteresis materials, in Proc. 17th AIAA/ USU Conf. Small Satellites, Logan, UT.
[9] Psiaki, M. L. (2004): Nanosatellite attitude stabilization using passive aerodynamics and active magnetic torquing. J. Guidance, Control, and Dynamics, 27(3), pp. 347–355.
[10] Sidi, M. J. (1997): Spacecraft Dynamics and Control. Cambridge, UK, Cambridge Univ. Press.
[11] Flatley, T. W. and Henretty, D. A. (1995): Technical Report N95- 27801: A Magnetic Hysteresis Model, NASA Goddard Space Flight Center, Greenbelt, MD: NASA Technical Reports Server.
[12] Wertz, J. R. (1978): Spacecraft Attitude Determination and Control, Dordrecht, Holland, D. Reidel Publishing Co.
[13] Sarychev, V. A. and Ovchinnikov, M. Y. (1994): Dynamics of a satellite with a passive aerodynamic attitude control system. Cosmic Research, 32(6), pp. 561- 575.