DOI: 10.13009/EUCASS2017-183

Mechanism of ideal gas separation for bodies in a supersonic flow

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Abstract
A mechanism has been formulated that describes the formation of forces causing separation of gas fluxes in non-stationary and spatial flows. A condition has been analytically derived that describes the separation of the gas flow. This flow is torn off either from the body surface or from the mixing layer in the form of a vortex, which induces a velocity in the field of flow, opposite to the velocity of the main flow, while also decelerating it. In accordance with this analytically derived condition, the conditions of separation for cone and self-similar gas flows have also been found, which coincide with the results of experimental and numerical simulation.

1. Introduction

In problems of ideal gas separation, a gas deceleration occurs when under certain initial or boundary conditions a vortex appears in the stream. This vortex induces a velocity in the field of flow, opposite to the velocity of the main flow, while also decelerating the latter. The intensity of this vortex depends on the angle shape of the body [1-3], and angle of sideslip [4-5], i.e. forces that repel it from the surface. Depending on the intensity, the vortex rises to a particular height relative to the surface of the body. In the linear formulation this vortex is a Ferri point type. It should be noted that the class of separated flows due to inertial forces is much broader than that due to forces of viscosity. These separations occur most often when gas flows of different density collide, and then tangential or contact discontinuities are formed, which turn into a vortex under the influence of torques. This can occur either on the surface of the body [1-5] or in space. Another example is a complex pattern of vortex creation when a weak harmonic wave interacts with a mixing layer. Here, the resultant forces alternately "push" the created vortices up and down with respect to the central layer and lead to the flow turbulence [6]. In the case of an incompressible fluid, the forces causing an "inviscid" separation are a source of tangential shear instability described in the standard problem of the Kelvin-Helmholtz [7]. In nature, when viscous effects are taken into account, the gas separation can be observed both due to viscous and non-viscous effects at the same time. In this case, the separations due to inertial forces are global. Thus, [4] shows that when the conditions for the inviscid separations are met, it is implemented above the boundary layer, above the vortices caused by the boundary layer separation. This work shows the results of separation of two-dimensional nonstationary gas flux flowing around a convex corner. Analytical and numerical calculations and their comparison with experiment are provided. It is shown that the separation condition proposed in this work gives an analytical solution [2]. The second problem that has been researched refers to the stationary spatial flow around a V-shaped wing, which was solved in [4-5], where the existence of separated flow for inviscid gas at high angles of attack and sliding was shown both experimentally and numerically. For this task, the proposed separation condition allows to find analytically the conditions of separation, obtained experimentally and numerically in the mentioned works.

2. Problem formulation

A mechanism is proposed for formation of forces giving rise to vortices in an inviscid gas flow. On its basis the known solutions are analyzed (Fig. 1). It is shown that in the vicinity of a point E located on the surface of the body, with change of the initial or boundary conditions, streams are formed with different parameters in areas 1, 2 (Fig. 1, *a*). In the next moment, in accordance with the laws of conservation [8], the collision of flows forms areas 3 and 4,

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separated by a tangential discontinuity (Fig. 1, *b*). It does not matter in what area the point E is located; what is important is how the derivative of Vy normal to the surface behaves in areas 3 and 4. In most cases, it does not change sign, and then there is no separation. When the derivative does change its sign, the flow departs from the surface in region 4, and it adheres to the surface in region 3.

Consequently, due to a pressure difference at the point E, a force occurs starting a tangential gap. This leads to a vortex that floats up above the surface (Fig. 1, c).

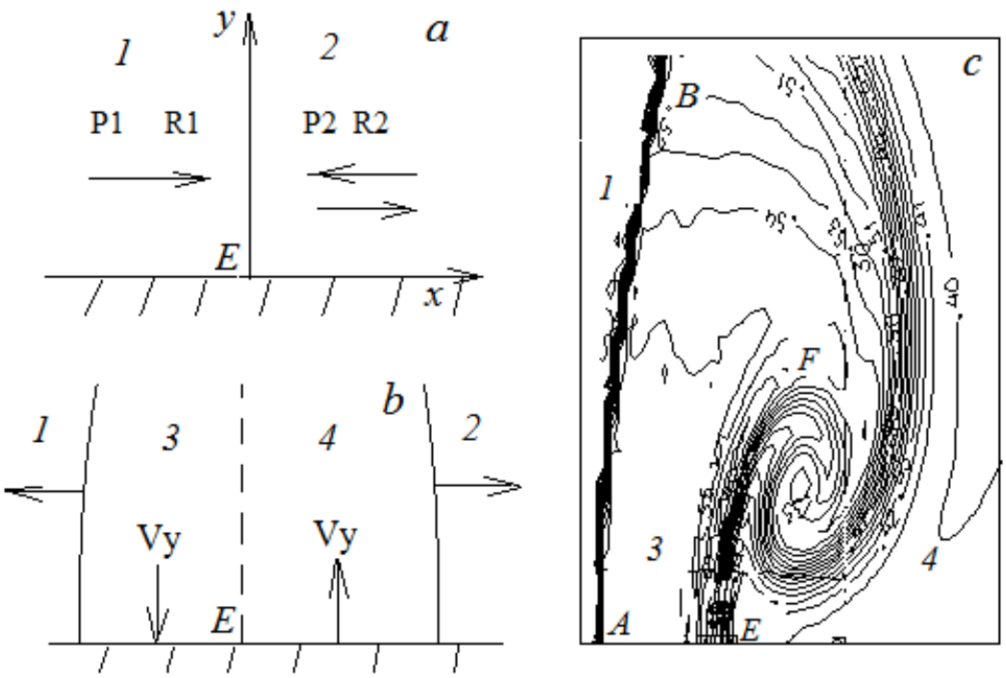

Fig. 1 Scheme of formation of the forces causing the separation of the inviscid gas flow: interaction of counter-flows (*a*) and the formation of a tangential discontinuity (*b*) on the surface of the body; the ascent of the vortex above the surface (*c*)

The results obtained in the framework of ideal gas model expand our theoretical knowledge about the mechanisms of development of separated flows. At the same time, they have a practical application in explaining the results of the flow of bodies obtained by numerical methods. Let's see how the proposed scheme of flow separation is implemented in the previously solved tasks.

3. Separation of inviscid gas in a non-stationary flow around a convex corner.

Let's consider the problem of a sudden movement of the gas flow around a convex corner (Fig. 2) when the gas suddenly begins to move alongside O_1O at supersonic speed Q. The basic parameters of the problem are: P – pressure, ρ – density, S is entropy, Q is the gas flow rate, and the parameter $M = Q/\sqrt{P}\ \gamma/\rho$, the Mach number and γ the ratio of specific heats.

Parameters P and ρ are related to their values in the unperturbed gas, the flow parameters in each region are indexed in accordance with its number. The task is self-similar and therefore the paper next discusses the flow patterns at time t = 1 for different values of the angle θ. At small θ, region of the disturbed gas ABE_1CD (Fig. 2, *a*) is restricted by the limit characteristics of the AB, CD and BC isobaric. Within the region there is a weak contact discontinuity EE_1. The parameters of the gas in regions 1 and 2, for given M and θ, are well-known formulas [8]. Thus the tangent velocity along the surface in region 2 (relative to the vertex (O) is equal to $Q_2 = Q\cos\theta$, and the entropy in regions 1 and 2 is equal to its value in the incoming gas. With increasing θ, the parameters of the flow in regions 1 and 2 differ significantly: the rate of Q_1 increases and Q_2 decreases. The area ABCD is limited by strong discontinuities AB, DC. Inside of the area, a tangential gap of the finite intensity EE_1 is implemented. The one-dimensionality of the flow in the near-wall region allows finding the parameters in fields 3 and 4 by one-dimensional formulas of decay of an

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arbitrary discontinuity [8]. The presence of a finite velocity discontinuity leads to a formation of vortex F in the vicinity of the point E. With increasing θ up to 55º vortex F floats above the surface (Fig. 1, *b*). Gas rotating around the vortex F in the boundary region AE inhibits the gas moving from point A to E. With further increasing of angle θ, the intensity and speed of the shock wave AB is growing so that its speed relative to the peaks becomes equal to the rate of gas flow in region 1: D_{13} (AB) = Q_1. This condition determines the value of the angle θ_k when the incoming flow of gas is separated. The angle θs is determined from the condition $M_3 = 1$, which uses self-similar variables to describe the floating of the vortex feature Ferri over the surface of the body and corresponds to the experimental data [1] and numerical simulations [2, 3].

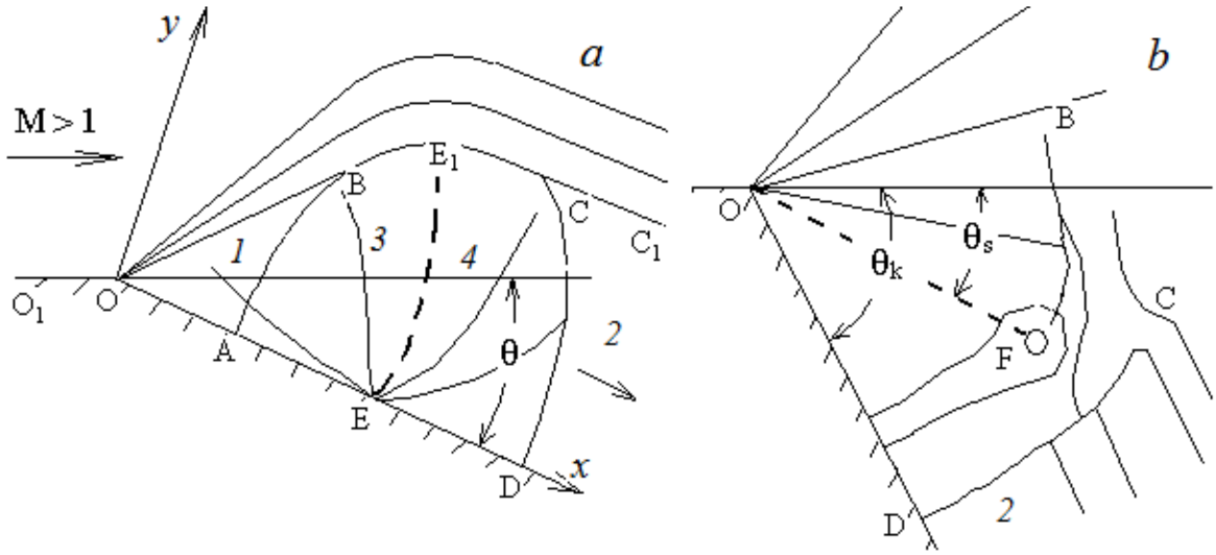

Fig.2 Separation of non-stationary gas flow around a convex corner: a non-separable wrap (*a*); a flow with separation (*b*).

Thus, for finding angles θ_k and θ_s there are two conditions: $D_{13} = Q_1$, $M_3 = 1$, necessary for the separation of gas flow from the apex: first, bottom disturbances should reach the apex, and secondly, the flow in the separation zone must be subsonic.

Let's consider how the proposed scheme of an inviscid separation is applicable to this problem. We will find v_y, normal derivative at the body surface in the vicinity of the point E and analyze its behavior on the basis of the non-stationary Euler equations. We write the equations of motion in self-similar variables ξ = x/t and η = y/t and based on the fact that the velocity on the surface is equal to 0, we obtain the normal derivative to the surface as: $v_\eta = -1 - P_\xi(M_3^2 - 1)/\rho U$ [2]. Here U, V – components of velocity in a moving coordinate system (ξ, η) associated with the components of the velocity in a stationary coordinate system (x, y) equations: U = u – ξ, V = v – η; the value of M_3 = U/a is the Mach number on the surface of the OD in region 3; a is the speed of sound. It can be seen that when M_3 = 1, the value $v_\eta = v_y = 0$. When $M_3 \neq 1$, we should take into account the behavior of the P_ξ, which is greater than 0 for θ close to θ_S [2].

In our case in region 4, $M_4 > 1$, and in region 3 $M_3 < 1$, i.e. in accordance with the scheme of separation, in region 4 there is a compression wave and in region 3 - rarefaction. It is this emerging force that "folds" the tangential gap; a vortex is formed that floats above the surface and induces a velocity field that rotates in a clockwise direction against the main flow. Thus, the sign change of the normal derivative of the velocity to the body surface in the non-stationary problem leads to a separation of the flow from the surface, and the condition $M_3 = 1$ ($v_\eta = 0$) in the problem determines the angle of flow separation θs, which is the exact solution.

4. The separation of the spatial gas flow; flow around a V-shaped wing

Let's consider a V-shaped wing with straight edges AB and BC and the central chord B' E (Fig. 3, where the cross section is at a distance from the front edge). Angle ABC is 90º. A supersonic gas flow with velocity U_0 is incident to the wing at angles of attack α and slip β is incident. This velocity can be broken into two components: B' F – speed, directed along the central chord, and DF is the cross velocity perpendicular to BE. Because of the cross component,

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the flow is asymmetrical. In this case, the intensity of the shock waves Ab and Cc is different. The gas flow along the console AB moves to a chord BE with greater speed than along the console BC. Thus, with the increase of the slip angle (and with the increasing intensity of separation on the contact surfaces bS' and cS') the point S' is increasingly moving away from point B, with which it coincides flow is symmetric. At certain angles of attack and slip point S' breaks away from the surface [4-5]. The characteristic regions, which are the result of the interaction of gas dynamic discontinuities, are indicated by the numbers 1-5 at Fig. 3. Discussed are flow regimes of a wing with supersonic edges.

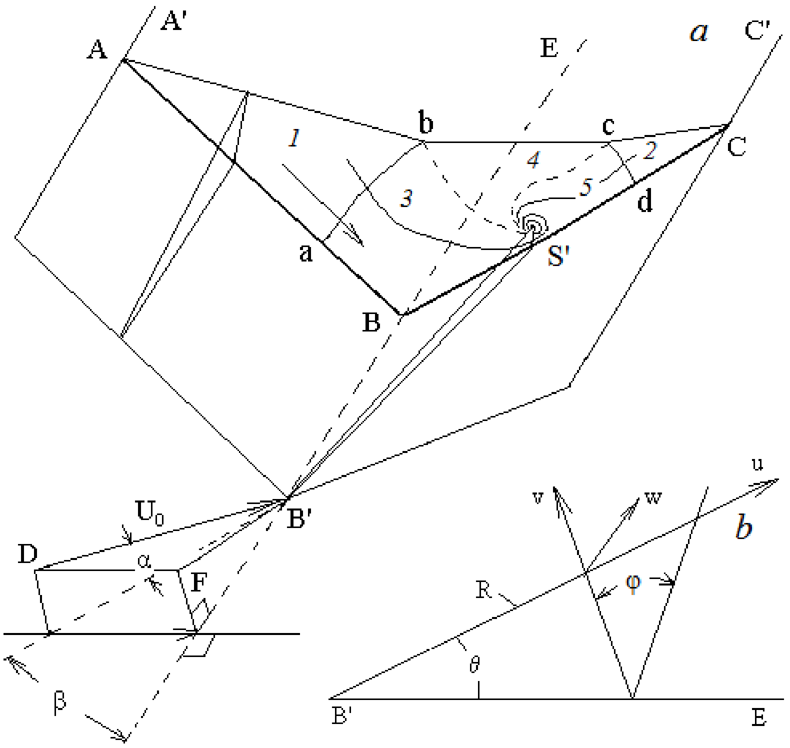

Fig 3. The flow diagram of the V-shaped wing: gas dynamic discontinuities (a): Ab, bc, cC, ab, cd of a shock wave; bS', cS' – tangential discontinuities; a characteristic region (1-5); a spherical coordinate system (b)

Figure 4 shows calculation results for a symmetric flow around a V-shaped wing when the angle of attack α increases. It can be seen that the conventional flow pattern with linear contact discontinuities ending at the corner point (see Fig. 4, a) is implemented for small α and moderate Mach numbers M in the vicinity of the corner point. The intersection point of contact discontinuities begins to move away from the corner point when the angle of attack α and the Mach number M increase (see Fig. 4b) and the contact discontinuities are curving.

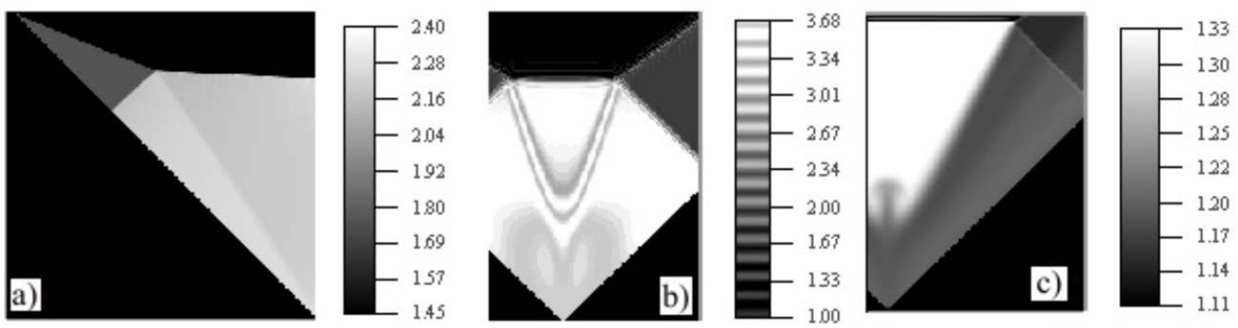

Fig 4. Density field fragments. Tangential discontinuity evolution depending on the angle of attack: a) M = 2, α = -22^o; b) M = 4, α = -20^o; c) M = 3, α = -30^o

The subsequent increase in the angle of attack leads to the appearance of a gas jet directed from the corner point (see Fig. 4c). In all cases, the shock wave starting from the triple point is slightly curved when it is incident on the wing, but it does not have a finite break.

On Fig. 5, for the initial parameters of the problem (M = 3, α = − 35°, β = 13°) the results of the flow around a V-wing are presented. Here, on the windward side of the console, when gas flows collide, a separation occurs in the form of surfacing Ferri point (point S').

Presented are the general flow pattern (Fig. 5, *a* − entropy field) and the fragments of isentropes in the flow region 4 (Fig. 5, *b*). The distribution of gas dynamic functions on the surfaces of consoles in the transverse direction is presented in (Fig. 5, *c*) where a vertical line in the center of the picture marks the separation boundary. It can be seen that in the region of separation a local maximum of the pressure P (curve 1) is reached, while density ρ(2) and entropy S(3) change abruptly. The transverse speed here u(4) = 0. Dotted lines (5) on the edges of the left and right consoles of the marked values of pressure and density can be found by the formulae for shock waves.

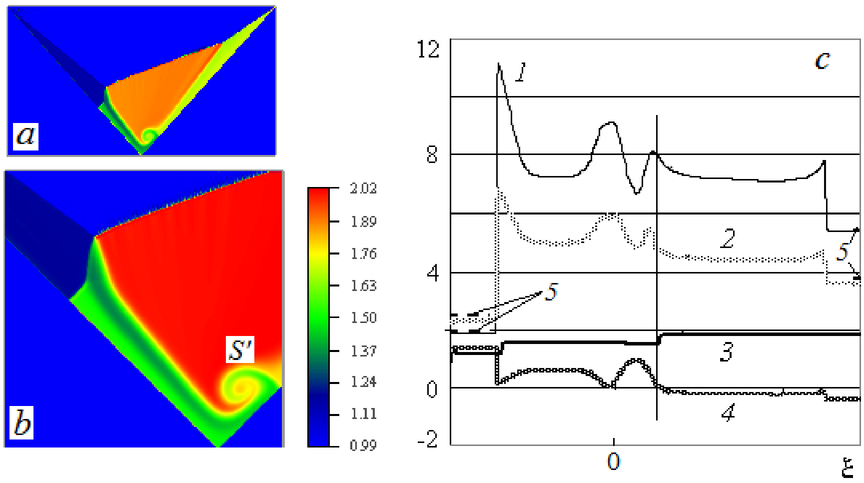

Fig 5. The results of calculation of separated flow wing at M = 3, α = –35°, b = 13°: field (a) and the fragments of entropy (b); distribution of flow parameters along the surface of the consoles in the transverse direction (c) – P (curve 1), ρ(2), S(3), u(4); dotted lines (5) – comparison of values of pressure and density with their exact values

We will determine, in accordance with the proposed scheme of separation, the conditions under which the flow tear-off is achieved.

We will write the equation of gas motion in a spherical coordinate system R, θ, φ with respective components of velocity u, v, w [9]. The scheme of associating a coordinate system to the surface of the wing is presented in Fig. 3, *b*. For conical flow, taking into account that along the beam d/dR = 0 and the normal to the surface component of velocity is equal to 0, it follows that the derivative of the normal component of the velocity is equal to the surface [5]: $w_\varphi = -\sin\theta(v_\theta + u)(1 - v^2/ a^2)$. From the analysis of this expression it follows that $w_\varphi = 0$, when $1 - v^2/ a^2 = 0$ or $v_\theta + u = 0$. The first expression, as shown by the calculations, does not become zero. With the second expression being zero, from the equations of motion it follows that on the surface of the console the value of $P_\theta = 0$. Therefore, in this case, a local extremum is realized on the beam on the surface of the wing. Indeed, data [3, 4] shows that the maximum pressure is realized here.

Analysis of the separation pattern in Fig. 5 and the results from [1-5] shows the single mechanism of separation of inviscid gas from the surface in the case of a conical spatial and two-dimensional unsteady (self-similar) gas flows. With two flows of gas colliding here, the resulting normal component of velocity tears the Ferri singularity from the surface.

In the case of the flow around the wing without slipping, the collision of different gas flows leads to a formation of a gas jet flowing up from the corner point. Flow patterns (Fig. 6) are very similar to those obtained in tasks where the Richtmyer-Meshkov instability is present. The explanation for this phenomenon is the following. Firstly, at higher angles of attack the pressure and density in the angle's vicinity (central chord) both increase to values greater than in the central upper region 5. As a result, heavier gas gets into a region with lower density, while the longitudinal and

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tangential components of velocity have a discontinuity at the interface. Such distribution of flow parameters in the mixing layer, with the tangential disturbances, gives rise to Richtmyer-Meshkov instability. The perturbations exist due to the movement of the gases along distorted discontinuities, the latter being a result of vibrations of weak waves between the weak tangential discontinuities emanating from the points g and h. Secondly, as a result of propagation of waves in a supersonic jet, a Kelvin-Helmholtz instability appears between tangential discontinuities, which leads to the vortices formation at the boundary. These vortices are visible in Fig. 6 as dark dots along the perturbed border of the tangential shear, where the pressure is at its minimum. That's why the boundary of the flow, emerging at the top of the wing, consists of vortices.

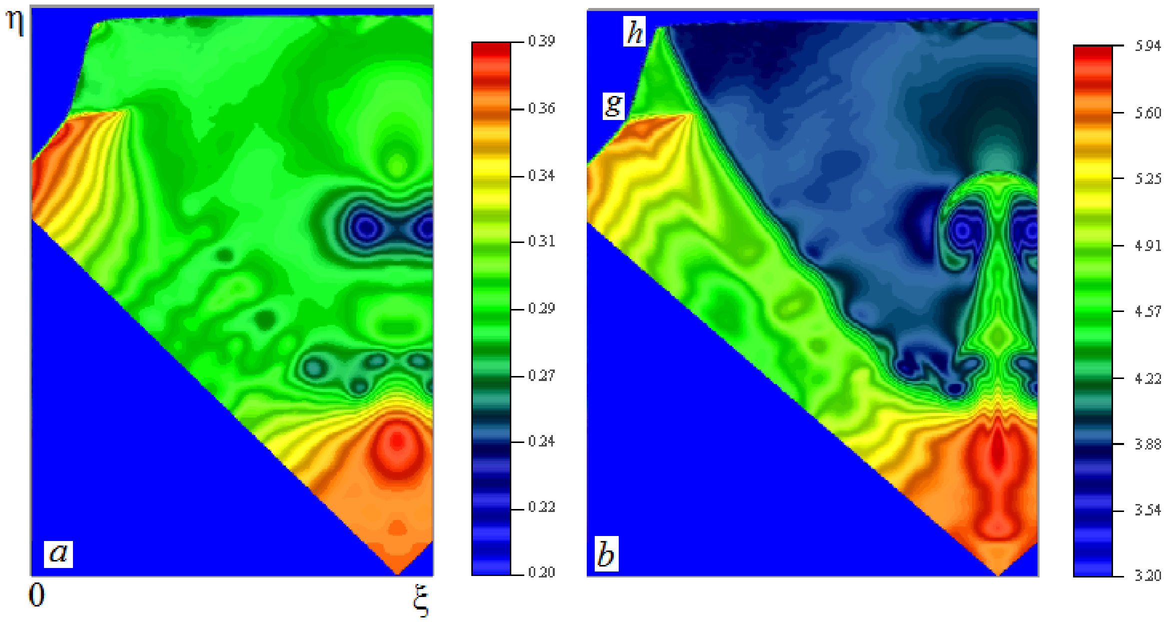

Fig 6. A special symmetric solution of the flow around the wing with M = 4, α = -35º: (*a*) a fragment of the pressure field in the middle of the wing; *b*) fragment of a density field

5. Conclusions

A mechanism has been formulated that describes the formation of forces causing separation of supersonic flows of ideal gas from surface in non-stationary and spatial flows. A condition has been analytically derived that defines the moment of vortex creation in the stream. The vortex induces a velocity in the field of flow, opposite to the velocity of the main flow. In the framework of the suggested scheme, the conditions of separation for cone and self-similar gas flows have been defined, which coincide with the results of experimental and numerical simulation.

Acknowledgements

Work is performed with a partial financial support of the Russian Foundation of Basic Research (Grant No. 17-01-00129).

DOI: 10.13009/EUCASS2017-183

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