DOI: 10.13009/EUCASS2017-137

Optimal Scheduling Algorithm in Point Merge System Including Holding Pattern Based on MILP

Somang Lee⋆, Youkyung Hong⋆, and Youdan Kim⋆†
⋆Department of Aerospace Engineering, Seoul National University
1 Gwanak-ro, Gwanak-gu, Seoul 08826, Republic of Korea
cjsomang@snu.ac.kr · youkyh1@snu.ac.kr · ydkim@snu.ac.kr
†Corresponding author

Abstract

An optimal scheduling algorithm of point merge system is proposed considering a holding pattern based on a Mixed Integer Linear Programming (MILP). Route structure of the point merge system is constructed by adopting virtual fixes, and appropriate constraints are included to integrate holding pattern in MILP formulation of the point merge system. The performance of the proposed scheduling algorithm is demonstrated by numerical simulation, which shows that the proposed algorithm can create aircraft schedule under severe traffic congestion through the holding pattern. The proposed optimal scheduling algorithm is expected to assist human traffic controllers and improve the capacity of terminal maneuvering areas.

1. Introduction

Air Traffic Management (ATM) has become one of the most important topics in aerospace engineering in recent years. The total amount of airborne delay is sharply increasing and workloads of human air traffic controllers become heavy due to the increasing flights. To relieve this situation, many studies have focused on ATM problems, and Point Merge System[1] (PMS) is one of those attempts. PMS was proposed by EUROCONTROL Experimental Centre in 2006 for efficient management of the arrival flows of aircraft, which has been widely adopted by many airports including Oslo, Dublin, and Jeju international airports. There are two main elements composing a PMS: the sequencing leg and the merge point. Once an aircraft enters the PMS, the aircraft flies along the prescribed sequencing leg until the air traffic controller provides an instruction for the aircraft to descend to the merge point. Due to equi-distance from the merge point along the sequencing leg, air traffic controller can efficiently manage the separation distance between the aircraft. It is also easy to estimate the trajectory of aircraft. On top of that, PMS allows Continuous Descent Approach (CDA) when the aircraft conducts descent operation between the sequencing leg and merge point. Several studies have proved that CDA can guarantee considerable amount of fuel saving, resulting in eco-friendly operation.[2]
There exist several research works on PMS. EUROCONTROL have studied PMS to implement PMS procedure to the airports.[3,4] Numerical and simple Human-In-The-Loop (HITL) simulations were performed to show that PMS can be beneficial in terms of staffing, predictability, and environment. HITL simulation with human air traffic controllers was also performed to analyze the potential of PMS.[1] HITL simulation proved that the instructions of air traffic controllers were reduced as much as 10% by adopting PMS. Unfortunately, few researchers have addressed modeling of PMS. Liang at al.[5] and Hong at al.[6] suggested the framework of scheduling algorithm based on PMS, but the holding pattern was not considered. The holding pattern may provide an additional choice for air traffic controllers to manage flights.
In this study, an optimal scheduling algorithm for PMS using Mixed Integer Linear Programming (MILP) formulation is proposed considering the holding pattern. In Section 2, route structures and some constraints are considered to apply MILP to the problem, and the formulation for PMS is provided. The performance of the proposed MILP formulation is demonstrated by numerical simulations in Section 3, which makes scheduling possible even under severe traffic congestion through the holding pattern. Section 4 describes the conclusion and future work.

2. Problem Formulation

To formulate PMS, the PMS operated in Jeju International Airport shown in Fig. 1 is considered in this study. There are two routes that merge into DANBI fix, the initial fix of PMS. From DANBI fix to WOODO fix, aircraft fly along

DOI: 10.13009/EUCASS2017-137

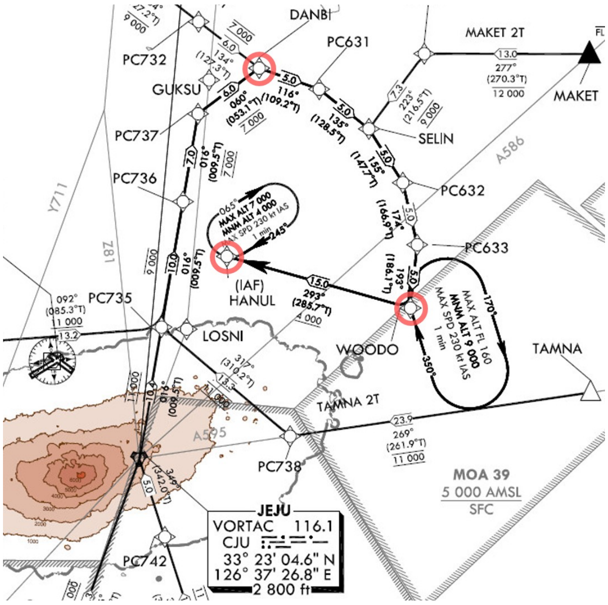


Figure 1: Standard terminal arrival route of Jeju international airport

the predefined leg to absorb the demanded airborne delay. If the separation requirement at the HANUL fix is satisfied, then the aircraft start to descent to HANUL fix, also known as the Initial Approach Fix (IAF) or the merge point in general. Note that the route between Jeju and Gimpo airports is one of the busiest air routes in the world by origin-and-destination passenger volume, resulting in saturation of the capacity of the sequencing leg.[7] To deal with thhe saturation, air traffic controllers often use holding pattern at the end of the sequencing leg (WOODO fix) and at IAF (HANUL fix). In this study, only the holding pattern in WOODO fix is considered to simplify the problem. Similarly, the incoming aircraft through SELIN fix are neglected, because the percentage of the traffic through this route is lower than 10%.

2.1 Route Transformation

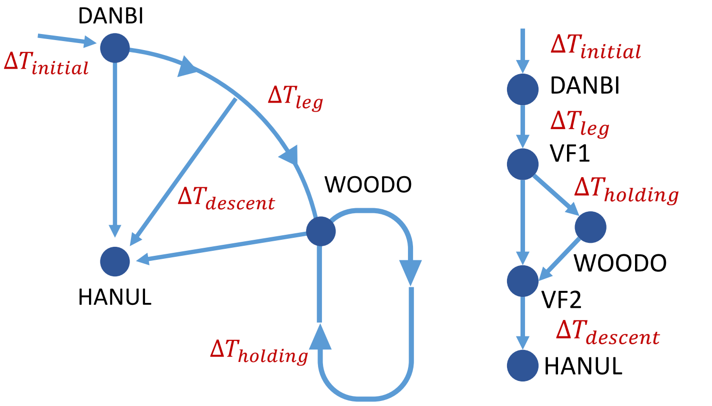
Figure 2(a) shows the simplified version of PMS with the holding pattern. The PMS with holding pattern has several complicated conditions compared to the typical PMS, and therefore it is difficult to apply MILP formulation directly to solve this problem. To deal with this problem, the original PMS structure is converted into the structure containing two virtual fixes as shown in Fig. 2(b). VF1 is the first virtual fix introduced in this study, which denotes the state when an aircraft finishes conducting delay absorption. VF2 is also one of introduced fixes, which is required in the situation when an aircraft in the sequencing leg or the holding pattern is ready to descent to HANUL fix. The aircraft drops by DANBI, VF1, WOODO, VF2, and HANUL fixes if the aircraft is scheduled to use the holding pattern. Otherwise, the aircraft only goes through DANBI, VF1, VF2, and HANUL fixes.

2.2 MILP Formulation

Several constraints including the holding entry condition and the separation constraint are introduced in this study. Note that the introduction of the holding pattern in modeling PMS is the main contribution of this study. Capozzi et al. represented the MILP framework of traditional route structure,[8] which is modified in this study to impose other important constraints.

There are three main variables deciding the result of scheduling. First, $A_{f,r}$ is a binary variable that denotes whether the flight f uses route r or not. It returns 1 if f utilizes r, otherwise 0. Second, $S_{f,f',r,r',p}$ decides the priority at a point p, a common point of routes r and r', when a flight f chooses route r and a flight f' chooses route r'. If f is prior to f' at a point p, then $S_{f,f',r,r',p}$ takes value 1, otherwise 0. Third, $T_{f,r,p}$ determines the arrival time of flight f at a point p on route r. These three variables are used to formulate MILP optimization problem. The detailed information about the routes and points are summarized in Table 1.

DOI: 10.13009/EUCASS2017-137



(a) Original structure (b) Transformed structure

Figure 2: The original and transformed structure of PMS

Let me define performance index to be optimized as follows

$$J = \sum_{f \in \mathbf{F}} \sum_{r \in \mathbf{R}} A_{f,r} T_{f,r,p_F} \tag{1}$$

where p_F is the last point on route r (the merge point), and $\mathbf{F}$ and $\mathbf{R}$ mean the sets of flights and routes, respectively. The performance index is set as the sum of transit time of all flights because the purpose of the proposed algorithm is to reduce airborne delay.
Following constraints are considered for the optimal scheduling in PMS.

- Constraint 1 - Single Route

$$\sum_{r \in \mathbf{R}} A_{f,r} = 1, \qquad \forall f \in \mathbf{F} \tag{2}$$

This constraint is required because a flight can only select one single route among $\mathbf{R}$.

- Constraint 2 - Ordering

$$S_{f,f',r,r',p} + S_{f',f,r',r,p} = A_{f,r} A_{f',r'} \tag{3}$$

Ordering constraint is imposed to relate variables $S_{f,f',r,r',p}$ and $A_{f,r}$. If there is a common point between routes r and r', the priority should be decided by setting $(S_{f,f',r,r',p}, S_{f',f,r',r,p}) = (1, 0)$ or $(S_{f,f',r,r',p}, S_{f',f,r',r,p}) = (0, 1)$.

- Constraint 3 - Safe Separation

$$S_{f,f',r,r',p}(A_{f',r'} T_{f',r',p} - A_{f,r} T_{f,r,p} - SEP_{f,f',p}) \geq 0 \tag{4}$$

If a flight f' on route r' is prior to a flight f on route r at a point p, then the separation distance between two flights should be maintained. In this study, safe separation in terms of distance is converted into separation time considering the flight speed.

Table 1: Notation of fix points and routes

Notation	Description
p_1	DANBI
p_2	VF1
p_3	WOODO
p_4	VF2
p_5	HANUL
r_1	$p_1 \rightarrow p_2 \rightarrow p_4 \rightarrow p_5$
r_2	$p_1 \rightarrow p_2 \rightarrow p_3 \rightarrow p_4 \rightarrow p_5$

DOI: 10.13009/EUCASS2017-137

- Constraint 4 - Transit Time

$$A_{f,r}[T_{f,r,p'} - T_{f,r,p} - (\Delta T_{f,r,p,p'})_{min}] \geq 0 \tag{5a}$$
$$A_{f,r}[T_{f,r,p'} - T_{f,r,p} - (\Delta T_{f,r,p,p'})_{max}] \leq 0 \tag{5b}$$
$$T_{f,r,p'} - T_{f,r,p} = 0 \qquad \text{if } (p, p') = (p_2, p_4) \text{ or } (p_3, p_4) \tag{5c}$$

This constraint is needed to define the transit time between two points. When a flight f on route r visits a point p first and a point p' second, the bounds of the transit time can be set as $(\Delta T_{f,r,p,p'})_{min} \leq \Delta T_{f,r,p,p'} \leq (\Delta T_{f,r,p,p'})_{max}$. Since the transit time between VF1 and VF2 is 0 in reality, Eq. (5c) is added in Eq.(5).

- Constraint 5 - Holding

$$A_{f,r}(T_{f,r,p'} - T_{f,r,p} - m_f P) = 0 \qquad \text{if } (p, p') = (p_2, p_3) \text{ and } r = r_2 \tag{6a}$$
$$T_{f,r,p'} - T_{f,r,p} - \Delta T^{max}_{leg} = 0 \qquad \text{if } A_{f,r} = 1 \tag{6b}$$

where m_f is an integer variable between 0 and m_{max}, which denotes the numbers of holding, and P is a parameter indicating the time delay of a single lap. In holding procedure, flights conduct discrete delay absorption leading to the different approach for modeling holding pattern. Because an integer variable is discrete, the discrete characteristics of the holding pattern is modeled in the proposed algorithm. Even though P is different according to the category of aircrafts, it is assumed as a constant in this study. The situation of a flight entering the holding pattern implies that the flight fully used the delay absorption in leg, which is applied in Eq. (6b).

- Constraint 6 - Initial Time

$$T_{f,r,p_I} - T^E_{f,r,p_I} \geq 0 \tag{7a}$$
$$T_{f,r,p_I} - T^L_{f,r,p_I} \leq 0 \tag{7b}$$

Similar to Eq. (5), the constraints of Eq. (7) are imposed on the entry time at the initial point. T^E_{f,r,p_I} is the lower bound of the entry time of flight f on route r, and T^L_{f,r,p_I} is the upper bound of the entry time of flight f on route r. In this study, T^E_{f,r,p_I} is considered as the earliest time, assuming that there is no delay at the initial point. T^L_{f,r,p_I} is defined as the sum of T^E_{f,r,p_I} and $\Delta T_{initial}$.

- Constraint 7 - Final Time

$$T_{f,r,p_F} \leq T_1 \quad or \quad T_{f,r,p_F} \geq T_2 \tag{8}$$

There may exist some situations that prohibit the landing procedure due to departing flights or bad weather condition. To deal with this situation, final arrival time at the merge point should be determined to avoid the time between T_1 and T_2.

In fact, the above constraints are not written in MILP form but written in nonlinear form. To convert those constraints into MILP constraints, conventional MILP techniques are utilized including auxiliary variables and large M. Table 2 summarizes the introduced variables. Finally, the MILP formulation of the proposed algorithm can be summarized as follows

$$J = \sum_{f \in \mathbf{F}} \delta^T_{f,r,p_F} \tag{9}$$

$$\delta^T_{f,r,p_F} \leq M A_{f,r} \tag{10a}$$
$$\delta^T_{f,r,p_F} \leq T_{f,r,p_F} + M(1 - A_{f,r}) \tag{10b}$$
$$\delta^T_{f,r,p_F} \geq T_{f,r,p_F} - M(1 - A_{f,r}) \tag{10c}$$

DOI: 10.13009/EUCASS2017-137

$$\sum_{r\in\mathbf{R}} A_{f,r} = 1, \qquad \forall f \in \mathbf{F} \tag{11}$$

$$-A_{f,r} + \delta^A_{f,f',r,r'} \le 0 \tag{12a}$$
$$-A_{f',r'} + \delta^A_{f,f',r,r'} \le 0 \tag{12b}$$
$$A_{f,r} + A_{f',r'} - \delta^A_{f,f',r,r'} \le 1 \tag{12c}$$
$$S_{f,f',r,r',p} + S_{f',f,r',r,p} = \delta^A_{f,f',r,r'} \tag{12d}$$

$$\delta^T_{f',r',p} - \delta^T_{f,r,p} - SEP_{f,f',p} + M(1 - S_{f,f',r,r',p}) \ge 0, \qquad \text{if } p \ne p_2 \tag{13}$$

$$\delta^T_{f,r,p'} - \delta^T_{f,r,p} - A_{f,r}(\Delta T_{f,r,p,p'})_{min} \ge 0 \qquad \text{if } (p,p') = (p_1,p_2) \tag{14a}$$
$$\delta^T_{f,r,p'} - \delta^T_{f,r,p} - A_{f,r}(\Delta T_{f,r,p,p'})_{max} \le 0 \qquad \text{if } (p,p') = (p_1,p_2) \tag{14b}$$
$$\delta^T_{f,r,p'} - \delta^T_{f,r,p} - A_{f,r}(\Delta T_{f,r,p,p'})_{max} = 0 \qquad \text{if } (p,p') = (p_4,p_5) \tag{14c}$$
$$\delta^T_{f,r,p'} - \delta^T_{f,r,p} = 0 \qquad \text{if } (p,p') = (p_2,p_4) \text{ or } (p_3,p_4) \tag{14d}$$

$$\delta^T_{f,r,p'} - \delta^T_{f,r,p} - \delta^m_f = 0 \qquad \text{if } (p,p') = (p_2,p_3) \text{ and } r = r_2 \tag{15a}$$
$$\delta^m_f \le MA_{f,r} \qquad \text{if } p = p_2 \text{ and } r = r_2 \tag{15b}$$
$$\delta^f_m \le m_f P + M(1 - A_{f,r}) \qquad \text{if } p = p_2 \text{ and } r = r_2 \tag{15c}$$
$$\delta^f_m \ge m_f P - M(1 - A_{f,r}) \qquad \text{if } p = p_2 \text{ and } r = r_2 \tag{15d}$$

$$\delta^T_{f,r,p'} - \delta^T_{f,r,p} - \Delta T^{max}_{leg} \le \quad M(1 - A_{f,r}) \tag{16a}$$
$$\delta^T_{f,r,p'} - \delta^T_{f,r,p} - \Delta T^{max}_{leg} \ge -M(1 - A_{f,r}) \tag{16b}$$

$$\delta^T_{f,r,p_I} - A_{f,r}T^E_{f,r,p_I} \ge 0 \tag{17a}$$
$$\delta^T_{f,r,p_I} - A_{f,r}T^L_{f,r,p_I} \le 0 \tag{17b}$$

$$\delta^T_{f,r,p_F} - A_{f,r}T_1 \le Mz_{f,r} \tag{18a}$$
$$\delta^T_{f,r,p_F} - A_{f,r}T_2 \ge M(1 - z_{f,r}) \tag{18b}$$

Table 2: Notation of auxiliary variables

Notation	Formulation	Type
$\delta^T_{f,r,p}$	$A_{f,r}T_{f,r,p}$	continuous
$\delta^A_{f,f',r,r'}$	$A_{f,r}A_{f',r'}$	binary
$z_{f,r}$	-	binary
m_f	-	integer
δ^m_f	-	continuous

DOI: 10.13009/EUCASS2017-137

3. Numerical Simulation

Numerical simulation is performed to demonstrate the performance of the proposed scheduling algorithm. Simulation parameters are decided as follows. The number of flights is set to eight. Separation time for safety between aircraft is determined as 60 seconds, and 30 seconds is added to the separation time as a safety buffer to deal with uncertainty. All aircraft are assumed as same type of aircraft, and therefore the separation time is same for every flight.

Note that there are three types of delay as shown in Fig. 2: i) $\Delta T_{initial}$, delay before entering PMS via speed control, ii) ΔT_{leg}, the time spent on the leg, and iii) $\Delta T_{holding}$, discrete time delay in holding pattern. These delays are used as control variables to schedule the flights. In this study, $\Delta T_{initial}$ is set between 0 and 20 seconds. In standard terminal arrival route procedure (Fig. 1), the speed of flight in leg is 220 kts and the length of leg is 25 NM. Thus, the maximum of ΔT_{leg} is calculated as 25 NM/220 kts = 409 sec. For holding procedure, the maximum number of laps is taken to 5, because too large delay in holding requires huge fuel consumption. The time required to complete one lap is set to 240 seconds (4 minutes), which means that the maximum holding time for an aircraft is 5×240 sec. = 1 200 sec.

All simulations are conducted with desktop PC (I7-7700 Intel Core processor and 32 GB memories). MATLAB is chosen as a simulation software and CPLEX is used to solve MILP problem.[9,10] Note that CPLEX is a commercial optimization software, which is considered as a standard to solve integer programing problem.

Three scenarios are generated to verify the performance of the proposed algorithm. The first scenario is the case when the holding pattern is not used. The second one is the case when some of flights conduct holding procedure. Finally, the last one is the case when the holding pattern is extremely utilized.

Figure 3 shows the scheduling result of scenario 1 by the proposed algorithm. Estimated Times of Arrival (ETAs) at the merge point are illustrated, and Scheduled Times of Arrival (STAs) results are also shown. ETA is computed assuming that there are no delays between fixes and no separation constraints among aircraft, i.e. ETA is not feasible. Based on ETA, the proposed algorithm calculates STA considering all constraints. Therefore, the differences between ETA and STA are overall delays of each aircraft. The red interval denotes the prohibited time window as described in Eq. (8), which makes Aircraft (AC) 6 and AC7 adjust their schedule. Except these two aircraft, other aircraft maintained their ETA, because the safety separation is already satisfied for those aircraft.

Figure 4 describes how the delay of each flight is composed. As shown in Fig. 3, only AC6 and AC7 conduct maneuver to absorb the delay. It can be found that $\Delta T_{initial}$ is used for both aircraft even though ΔT_{leg} can absorb as much as 409 seconds. The priority between those two delays is not considered in the current formulation, thus the proposed algorithm determines STA without considering the priority of delays when solving MILP optimization problem. In scenario 1, the total delay is 445 seconds. The detailed scheduling results are summarized in Table 3.

Figures 5 and 6 show the result of scenario 2, the case when holding is slightly used. The prohibited time duration of scenario 2 is longer than that of scenario 1, and therefore more time delay may be induced. The total delay of scenario 2 is 2 220 seconds. The detailed results of scenario 2 are summarized in Table 4. Note that the sequence between AC4 and AC2 is swapped to minimize the total delay. The characteristics of discrete delay of the holding enables this result. According to Fig. 6, AC2 flies 2 laps in holding procedure and AC4 does not enter the holding pattern. Even if it is desired for AC2 to reach the merge point early, it is impossible because reducing the holding delay of AC2 conflicts with the prohibited time constraint. The reduced STA of AC2 is 1 258 sec. − 240 sec. = 1 058 sec., and the upper bound of prohibited time is 1 145 seconds. Hence, rather than reducing the STA of AC2, putting AC4 before AC2 helps to minimize the total delay. However, there still exists surplus time delay between AC2 and AC4, whereas the separations between other aircraft are minimized (90 seconds). In addition, in Fig. 6, the maximum delay for aircraft is 889 seconds, which is significantly longer than the maximum of ΔT_{leg}. It means that it is impossible to make a feasible schedule for this scenario without the holding pattern, because it exceeds the limitation of leg delay. Thus, it can be stated that introducing the holding pattern in PMS can expand the region of scheduling-possible scenario. Therefore, the discrete characteristics of holding delay might make it possible to schedule under the situation that excessive delay is required, while exploiting some optimality.

The result of the case when the holding is extremely used (scenario 3) is shown in Figs. 7 and 8. The prohibited time is longest among all scenarios, so is the delay absorbed through the holding pattern. Similar to scenario 2, sequencing swap is generated, but the swapping result is much complicated. Table 5 describes the detailed scheduling results of scenario 3. It can be observed that scheduling is successfully performed without the intervention of traffic controllers, though the delays in leg are fully exploited for AC 2-4.

DOI: 10.13009/EUCASS2017-137

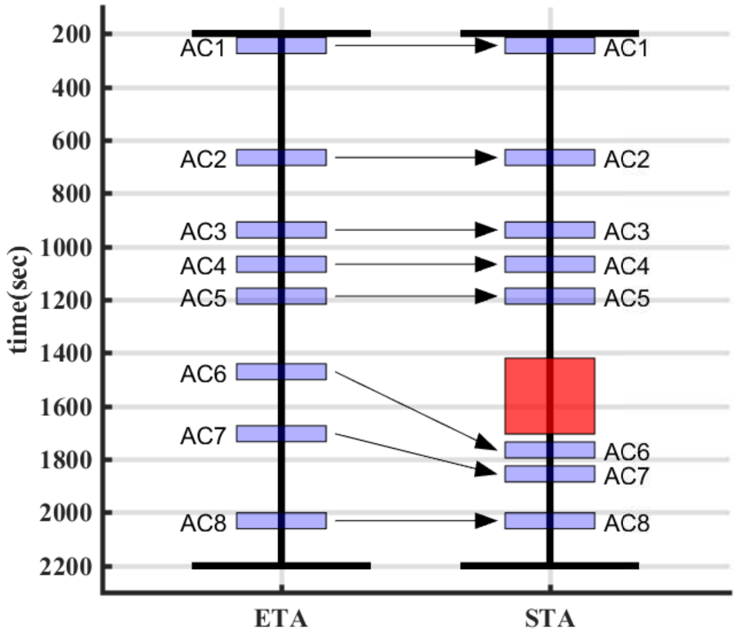


Figure 3: ETA and STA at the merge point for scenario 1

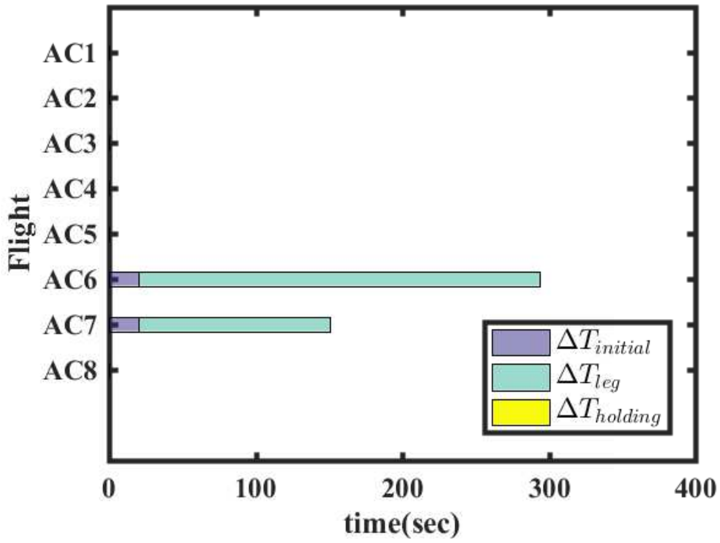


Figure 4: Delayed times in initial, leg, and holding at the merge point for scenario 1

Table 3: Scheduling result of scenario 1

Aircraft	Route	STA (sec.)		Delay (sec.)			
		T_{f,r,p_I}	T_{f,r,p_F}	$\Delta T_{initial}$	ΔT_{leg}	$\Delta T_{holding}$	$\sum \Delta T$
1	1	0	245	0	0	0	0
2	1	421	666	0	0	0	0
3	1	693	938	0	0	0	0
4	1	822	1 067	0	0	0	0
5	1	942	1 187	0	0	0	0
6	1	1 246	1 765	20	274	0	294
7	1	1 479	1 855	20	131	0	151
8	1	1 786	2 031	0	0	0	0

DOI: 10.13009/EUCASS2017-137

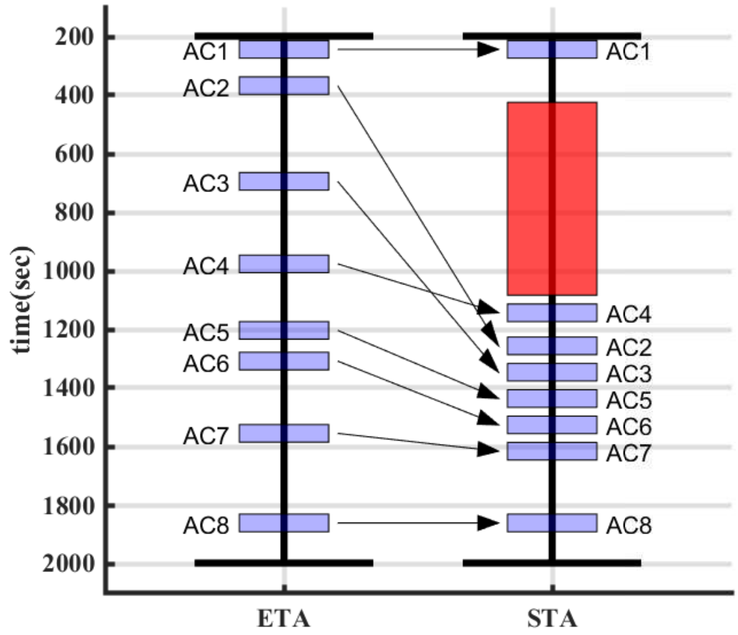


Figure 5: ETA and STA at the merge point for scenario 2

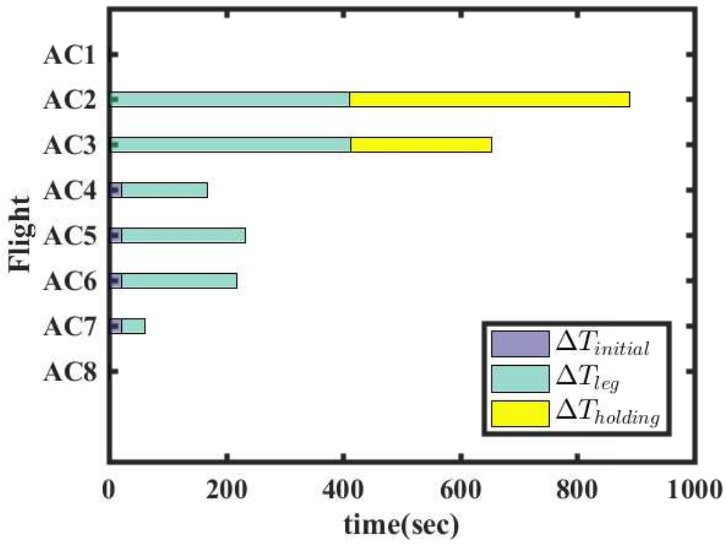


Figure 6: Delayed times in initial, leg, and holding at the merge point for scenario 2

Table 4: Scheduling result of scenario 2

Aircraft	Route	STA (sec.)		Delay (sec.)			
		T_{f,r,p_I}	T_{f,r,p_F}	$\Delta T_{initial}$	ΔT_{leg}	$\Delta T_{holding}$	$\sum \Delta T$
1	1	0	245	0	0	0	0
2	2	124	1 258	0	409	480	889
3	2	454	1 348	3	409	240	652
4	1	753	1 145	20	147	0	167
5	1	980	1 438	20	213	0	233
6	1	1 085	1 528	20	198	0	218
7	1	1 332	1 618	20	41	0	61
8	1	1 618	1 863	0	0	0	0

DOI: 10.13009/EUCASS2017-137

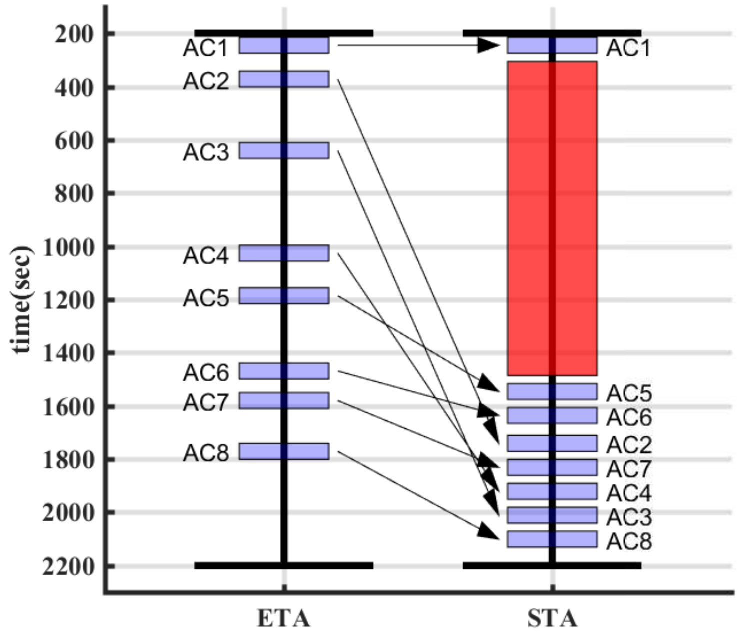


Figure 7: ETA and STA at the merge point for scenario 3

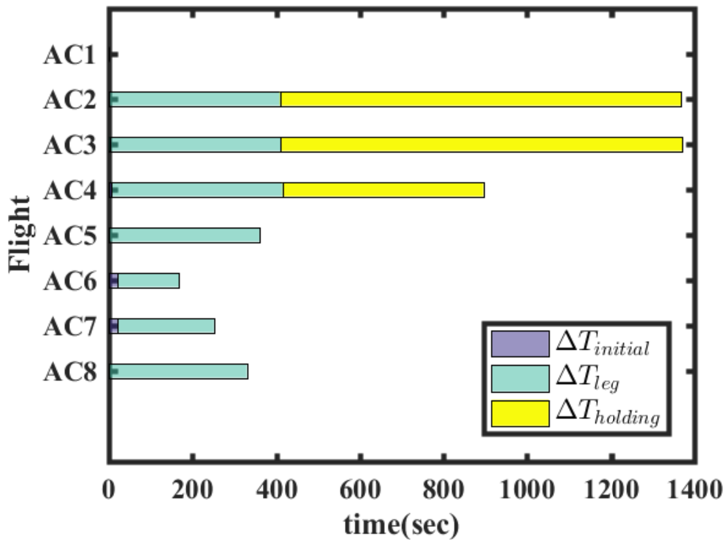


Figure 8: Delayed times in initial, leg, and holding at the merge point for scenario 3

Table 5: Scheduling result of scenario 3

Aircraft	Route	STA (sec.)		Delay (sec.)			
		T_{f,r,p_I}	T_{f,r,p_F}	$\Delta T_{initial}$	ΔT_{leg}	$\Delta T_{holding}$	$\sum \Delta T$
1	2	0	245	0	0	0	0
2	2	127	1 741	0	409	960	1 369
3	2	397	2 011	2	409	960	1 371
4	1	787	1 921	6	409	480	895
5	1	941	1 546	0	360	0	360
6	1	1 244	1 636	20	147	0	167
7	1	1 355	1 831	20	231	0	251
8	1	1 526	2 101	0	330	0	330

DOI: 10.13009/EUCASS2017-137

4. Conclusion

An optimal scheduling algorithm of PMS based on a Mixed Integer Linear Programming (MILP) was developed considering holding pattern. It was impossible to directly use the conventional MILP formulation for traditional route structure. The holding pattern has different characteristics which are difficult to be formulated. To apply the MILP formulation to the proposed algorithm, the route structure was changed and virtual fixes, which are still conceptual, were introduced. The existing MILP formulation was modified and suitable constraints were added to introduce the characteristics of holding procedure in the formulation. To demonstrate the performance of the proposed algorithm, numerical simulation was conducted. Jeju international airport was chosen as a test airport, and simulation parameters were selected considering aeronautical information. Three scenarios were used to check the proposed algorithm. The simulation results showed that the proposed scheduling algorithm might be useful to schedule scenarios which are impossible for PMS without holding pattern. In addition, the discrete nature of holding enables the swapping between aircraft to minimize the delay.
Several issues still remain for future work. The major portion of delay reduction actually comes from sequence changes between different types of aircraft, because the required safe separation distance depends on the aircraft type. Thus, the proposed algorithm Should be modified to deal with various aircraft types. Moreover, the holding pattern is estimated to improve the capacity of PMS by containing lots of aircraft. Hence, the analysis of capacity should be conducted when the holding procedure is introduced in PMS.

5. Acknowledgments

This work was supported by the Development of Integrated Departure/Arrival Management Technologies Project funded by the Ministry of Land, Infrastructure and Transport.

References

[1] B. Favennec, T. Symmans, D. Houlihan, K. Vergne, and K. Zeghal. Point merge in extended terminal area. Technical Report 2011-011, EUROCONTROL, Oct. 2010.

[2] J. P. B. Clarke, N. T. Ho, L. Ren, J. A. Brown, K. R. Elmer, K. O. Tong, and J. K. Wat. Continuous descent approach: Design and flight test for louisville international airport. *Journal of Aircraft*, 41(5):1054–1066, 2004.

[3] L. Boursier, B. Favennec, E. Hoffman, A. Trzmiel, F. Vergne, and K. Zeghal. Merging arrival flows without heading instructions. In *7th USA/Europe air traffic management R&D seminar*, Barcelona, Spain, Jul. 2007.

[4] B. Favennec, E. Hoffman, A. Trzmiel, F. Vergne, and K. Zeghal. The point merge arrival flow integration technique: Towards more complex environments and advanced continuous descent. In *9th AIAA Aviation Technology, Integration, and Operations Conference (ATIO)*, Hilton Head, SC, Sep. 2009.

[5] M. Liang, D. Delahaye, and P. Maréchal. A framework of point merge-based autonomous system for optimizing aircraft scheduling in busy TMA. In *5th SESAR Innovation Days*, Bolongna, Italy, Dec. 2015.

[6] Y. Hong, S. Lee, K. Lee, and Y. Kim. Optimal scheduling algorithm for air traffic point merge system using milp. In *EuroGNC*, Warsaw, Poland, Apr. 2017.

[7] J. Luiz, D. D. C. Fortes, D. A. Pamplona, and C. Müller. Analysis of flight trajectories in a terminal maneuvering area : São paulo-rio de janeiro route as a case study. *International Journal of Science and Engineering Investigations*, 4(42):80–85, 2015.

[8] B. Capozzi, S. Atkins, and S. Choi. Towards optimal routing and scheduling of metroplex operations. In *9th AIAA Aviation Technology, Integration, and Operations Conference (ATIO)*, Hilton Head, SC, Sep. 2009.

[9] MATLAB. *version 9.0 (R2016a)*. The MathWorks Inc., Natick, MA, 2016.

[10] IBM ILOG CPLEX. *V12. 7: User's Manual for CPLEX*. Int. Business Mach. Corp., Armonk, NY, 2016.