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Aerodynamic shapes optimization on the base of method of local linearization

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Abstract
The optimization method is developed for aerodynamic shapes design at subsonic and supersonic flow conditions. The method is based on the local linearization of gas-dynamic functions dependence on the geometrical parameters, which results in a quadratic approximation of the objective function. The method provides a high convergence rate regardless of the number of the control variables and allows obtaining the optimization problem solution in an analytical form. The efficiency of the method is demonstrated on examples of critical Mach number increasing and aerodynamic drag minimization for two-dimensional and axisymmetric bodies.

1. Introduction

Method of local linearization is proposed to improve reliability of the optimization procedure and to accelerate the convergence through the elimination of numerical determination of the objective function derivatives. As opposed to standard linearization of motion equations, which is performed for perturbations of the uniform flow, in the case of local linearization data about the flow field over the body are used. The optimized surface is breaking up into elements, and in the vicinity of each element the flow parameters are averaged. The plane specified by the velocity vector and the normal to the surface element is allocated. Linearization relative to the averaged flow parameters is performed in this plane. As a result the objective function is approximated by a quadratic form, for which the gradient and the matrix of second derivatives are calculated, and the extremum location is determined. On the final stage of the optimization iteration the found shape variations are checked in the numerical computation. Processing aerodynamic constraints (for example, lift maintenance) and geometric constraints (for example, volume maintenance) is allowed.
For the first time the efficiency of the local linearization procedure was demonstrated on example of supersonic aircraft wing optimization [1, 2]. At supersonic flow conditions the simplest assessment of pressure variation on an element can be obtained from the wavy-wall theory for small disturbance. The spatial movement of the element requires a turn of the velocity vector on an angle so that it remains parallel to the element plane and the impermeability condition is satisfied. The turn of the flow results in the pressure change. Summation of the aerodynamic loads on all surface elements leads to the quadratic approximation of the objective function. Newton type method is used to determine shape variations that enable the aerodynamic performance to be improved. A fast convergence to the optimum in case of the large number (more then 100) of the variables is provided.
Unlike supersonic flows in the range of subsonic velocities there are no local models (such as the Ackeret's formula) connecting pressure on the body surface with local geometrical parameters and conditions in the free stream. However the local linearization based on a relation between the pressure and the geometry parameters is practicable in case of small variations of the aerodynamic shape [3, 4]. Application of local linearization in an unsteady computation is presented in [5]. A changing the spatial position of the element is considered as a motion of a flat piston into or out of gas and the pressure change on the element is related to the geometrical parameters through the Riemann invariants.

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2. Local linearization at supersonic flow conditions

The important advantages of the local linearization method consist in obtaining analytical solutions and establishing typical features of optimal aerodynamic shapes. The solution is found as a variation of the shape of the initial body, for which theoretical values of the flow parameters are known. For example consider constructing the airfoils with the minimum wave drag under constrain on the thickness c_{max}. The position of maximum thickness is not known in advance, therefore the additional geometric parameter is introduced, the distance from the leading edge to the appropriate section x_c.

The airfoil with the upper half contour composed of three line elements is taken as an initial one. The first two elements connected at the point of maximum thickness set the nose and tail parts. The third element having a height y_b, is the bottom face.

The nose and tail parts separately divided into N segments. The nodal points of the nose part segments have coordinates $x_{n1} = \frac{n}{N} x_c, \quad y_{n1}, \quad n = 0, N$. The boundary conditions are $y_{01}=0$ and $y_{N1}=y_c=0.5c_{max}$. The nodal points of the tail part have coordinates $x_{n2} = x_c + \frac{n}{N}(1 - x_c), \quad y_{n2}, \quad n = 0, N$ with boundary condition $y_{02}=y_c$.

For the nose part of the initial airfoil, the surface pressure p_s and Mach number M_s correspond to the conditions of the oblique shock wave. On the tail part the pressure p_v and the Mach number M_v are determined according to the theory of rarefaction flow. The wave drag coefficient is equal to:

$$c_{D0} = 4\left(p_s y_c - p_v\left(y_c - y_{N2}\right) - p_b y_{N2}\right) / \left(\gamma p_\infty M_\infty^2\right)$$

Here γ is the ratio of specific heats, p_∞ and M_∞ are the pressure and Mach number in the free stream, p_b is the pressure on the bottom end.

The wave drag coefficient of an arbitrary airfoil is represented by the sum:

$$c_D = \frac{4}{\gamma p_\infty M_\infty^2} \left\{ \sum_{n=1}^{N} \left[p_{n1}(y_{n1} - y_{n-1,1}) + p_{n2}(y_{n2} - y_{n-1,2}) \right] - p_b y_{N2} \right\}$$

The pressure and geometry parameters variations are connected by linear relations:

$$\Delta p_{n1} = k_1(\Delta y_{n1} - \Delta y_{n-1,1}), \quad k_1 = \frac{\gamma M_s^2 p_s}{\Delta x_1 \sqrt{M_s^2 - 1}}$$

$$\Delta p_{n2} = k_2(\Delta y_{n2} - \Delta y_{n-1,2}), \quad k_2 = \frac{\gamma M_v^2 p_v}{\Delta x_2 \sqrt{M_v^2 - 1}}$$

The local linearization leads to the quadratic approximation of the objective function, the wave drag change:

$$\begin{aligned} \Delta c_D = \frac{4}{\gamma p_\infty M_\infty^2} \Bigg\{ \sum_{n=1}^{N} \Big[&(p_s + k_1(y_{n1} - y_{n-1,1}))(\Delta y_{n1} - \Delta y_{n-1,1}) + \\ &+ k_1(\Delta y_{n1} - \Delta y_{n-1,1})^2 + k_2(\Delta y_{n2} - \Delta y_{n-1,2})^2 + \\ &+ (p_v + k_2(y_{n2} - y_{n-1,2}))(\Delta y_{n2} - \Delta y_{n-1,2}) \Big] - p_b \Delta y_{N2} \Bigg\} \end{aligned}$$

According to the boundary conditions, position of some points of the contour does not change: $\Delta y_{01}=0$, $\Delta y_{N1}=0$, $\Delta y_{02}=0$.

Extremum conditions for the nose and tail parts are divided. The optimal variations of the nose points are found from the equations system:

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$$
\begin{aligned}
&2\Delta y_{1,1} - \Delta y_{2,1} = 0 \\
&2\Delta y_{2,1} - \Delta y_{1,1} - \Delta y_{3,1} = 0 \\
&\ldots \\
&2\Delta y_{N-2,1} - \Delta y_{N-3,1} - \Delta y_{N-1,1} = 0 \\
&2\Delta y_{N-1,1} - \Delta y_{N-2,1} = 0
\end{aligned}
$$

It is easy to see that a trivial solution is realized – $\Delta y_{n1}=0$, n=1,N-1. Within the framework of the local linearization the optimal airfoil has a wedge-shaped nose.
Extremum equations for the tail part are:

$$
\begin{aligned}
&2\Delta y_{1,2} - \Delta y_{2,2} = 0 \\
&2\Delta y_{2,2} - \Delta y_{1,2} - \Delta y_{3,2} = 0 \\
&\ldots \\
&2\Delta y_{N-1,2} - \Delta y_{N-2,2} - \Delta y_{N,2} = 0 \\
&p_v + k_2\left(y_{N2} - y_{N-1,2}\right) + 2k_2\left(\Delta y_{N2} - \Delta y_{N-1,2}\right) - p_b = 0
\end{aligned}
$$

The first N-1 equations of the system allow to express variations of the second and subsequent points through the variation of the first point $\Delta y_{n2}=n\Delta y_{1,2}$, n=2,N. The last equation closes the solution. The optimal change of the nodal points ordinates are calculated according to the equation

$$
\Delta y_{n2} = n\left(\frac{p_b - p_v}{2k_2} - \frac{y_{N2} - y_{N-1,2}}{2}\right) = \frac{n}{2N}\left(\frac{\sqrt{M_v^2 - 1}}{\gamma M_v^2}\left(\frac{p_b}{p_v} - 1\right)(1 - x_c) + y_c - y_b\right)
$$

Increasing the number of segments to infinity it is established the correspondence between n/N and $(x-x_c)/(1-x_c)$, and the continuous dependence $\Delta y_2(x)$:

$$
\Delta y_2 = \frac{x - x_c}{2}\left(\frac{\sqrt{M_v^2 - 1}}{\gamma M_v^2}\left(\frac{p_b}{p_v} - 1\right) + \frac{y_c - y_b}{1 - x_c}\right)
$$

Function $\Delta y_2(x)$ is a linear one. So the optimal variation of the airfoil shape is reduced to changing the angle of the tail section. The variation does not change the windward part.
It is necessary to make clear that the purpose of the analytical solution is not to find a specific optimal shape. The accepted assumptions in the problem statement are sufficiently rough and the values of the pressure coefficient obtained analytically differ from the exact values. The theoretical analysis is directed at establishing the typical features of the optimal shapes and determining the search directions with a limited decrease of the number of geometrical parameters. Solving the problem is closed by direct optimization in the nonlinear statement. The geometric parameters are the coordinates x_c and y_b.
Consider the example at the following values of the governing parameters: $M_\infty=3$, $c_{max}=0.066$ and $\gamma=1.4$. The aerodynamic characteristics are compared relatively the diamond shaped airfoil ($x_c=0.5$ and $y_b=0$), which is optimal one in the framework of the linear theory. The drag coefficient of the rhombus is:

$$
c_{DR} = 2\left(p_s - p_v\right)tg\delta / \left(\gamma p_\infty M_\infty^2\right) = 0.00812
$$

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Assuming p_b=0, the decrease in wave drag compared to the rhombus is about 6%. The optimum airfoil has the sharp edges (Figure 1). In contrast to the rhombus the cross section with the maximum thickness shifts towards the rear – x_c=0.638. An increase in the bottom pressure is accompanied by a decrease of the wave drag. When $p_b=0.4p_\infty$ the advantage is characterized by ratio c_D/c_{DR}=0.911, and when $p_b=p_\infty$ the wave drag is reduced almost five times. The bottom pressure increase first leads to an increase in maximum thickness coordinate x_c, and then to the appearance of the bottom face, the height of which gradually increases. The ultimate solution is the wedge.

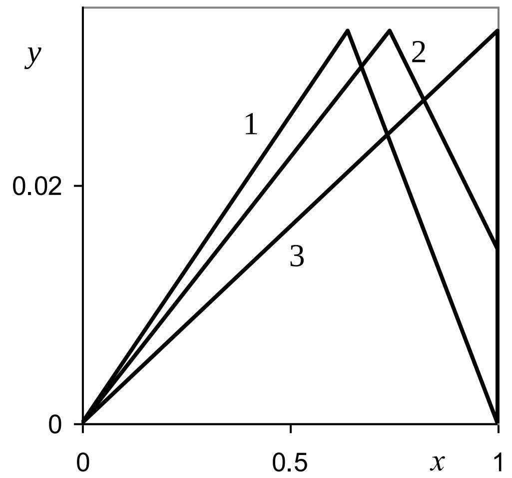

Figure 1: Optimal airfoils
1 – p_b=0, 2 – $p_b=0.4p_\infty$, 3 – $p_b=p_\infty$

3. Local linearization at subsonic flow conditions

In the subsonic range the analytical dependency of the pressure coefficient on the longitudinal coordinate can be determined within the framework of the linear theory for thin symmetrical flat bodies streamlined under zero angle of attack:

$$c_p(x)=\frac{2}{\pi\sqrt{1-M_\infty^2}}\int_0^b \frac{y'(\xi)}{\xi-x}d\xi$$

Here y' is the first derivative of the ordinate; c and b are the thickness and chord of the airfoil, respectively; and M_∞ is the free stream Mach number.

The local linearization at subsonic flow is based on the analysis of the surface curvature [6]. For small deformation of the shape the local pressure coefficient change could be estimated by the relation with the element length and the variation of the second derivative of the ordinate y":

$$\Delta c_p=\frac{2b\Delta y''}{\pi\sqrt{1-M^2}}$$

The contour of the airfoil is represented by a set of line segments joined at nodal points. The ordinates of the nodal points are used as the geometrical parameters that are varied in the optimization process. The pressure on the element (segment) changes when the spatial position of this element and two adjacent elements changes. So the successive local analysis is performed for two pairs of the elements. The Mach number is averaged on these elements based on the data of the numerical flow field modeling, and the second order derivative is determined by finite-difference approximation through the ordinate values of the three nodal points. The pressure coefficient corresponds to the pressure increment relative to the averaged pressure value divided by the averaged dynamic pressure. The proposed linear dependence of the surface pressure on the geometrical parameters leads to the quadratic approximation of the objective function and determines the shape variations, which are directed to its reduction.

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3.1 Airfoil with increased critical Mach number

The problem of constructing bodies with increased values of the critical Mach number is associated with the study of the flow scheme corresponding to the cavitation model proposed by Ryabushinsky. The characteristic feature is a plan distribution of the surface pressure in the longitudinal direction. The effectiveness of the local linearization is demonstrated on the example of constructing the body contour with specified nose and tail parts. The optimization aim is the target pressure loading on the surface. A desired pressure coefficient distribution is specified and the least squares difference between the actual and target distributions is used as the objective function. This is the basic idea behind inverse design methods.
The body with contour composed by nose and tail vertical segments with connecting convex curved line is constructed at incompressible flow conditions. Figure 2 shows the pressure coefficient level lines with increments of Δc_p=0.1 near the optimum body. On the surface the pressure coefficient equals c_p=-0.45.
The optimization method demonstrated a sufficiently high rate of convergence. The objective function is the mean-squared deviation of the pressure coefficient from the specified value. The pressure distribution on the initial body with flat top line qualitatively differs from the required distribution and the objective function F is equal to 0.316. The first variation brought the pressure in the vicinity of the mid-length section nearer to the sought value (F=0.115). In subsequent cycles, gradual pressure smoothing in the directions toward the front and rear faces occurred. For the constructed optimal body, the mean-square deviation of the pressure coefficient is F=0.006. The numerical optimization results are in agreement with the exact solution [7]. For the values of the relative thickness the difference is 0.2%.

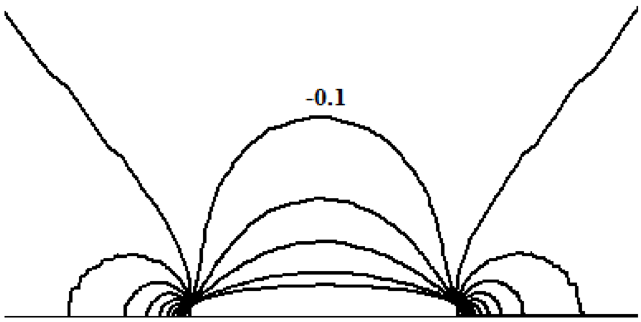

Figure 2: Pressure coefficient level lines, Δc_p=0.1

Optimization of the airfoil with wedge shaped nose and tail parts is performed. In this case, the pressure coefficient on the horizontal part of the initial airfoil depends on the longitudinal coordinate:

$$c_{p0}(x) = \frac{2tg\delta}{\pi\sqrt{1-M^2}} \ln\frac{(x-x_W)(1-x-x_W)}{x(1-x)}, \qquad x_W \le x \le 1-x_W .$$

Here x_W is the longitudinal coordinate of the plane joining the front wedge and the horizontal part; δ is the semi-angle of the wedge; and the airfoil chord is b=1.
To find an analytical solution of the problem it is made the additional simplifying assumption concerning the smallness of the Mach number that concludes that the second derivative of the ordinate of the airfoil contour is directly proportional to the difference of the sought and current pressure coefficients:

$$y'' = K(c_p - c_{p0}), \quad K = const.$$

After integration the equation for pressure coefficient we obtain the dependency of the first derivative of the ordinate:

$$y' = Kc_p(x-0.5) - \frac{2Ktg\delta}{\pi\sqrt{1-M^2}}\left[x\ln x + (1-x-x_W)\ln(1-x-x_W) - \right.$$
$$\left. -(x-x_W)\ln(x-x_W) - (1-x)\ln(1-x)\right].$$

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Here, the condition $y'(0.5)=0$ related to the vertical symmetry of the airfoil is used.
Integration with the boundary conditions $y(x_W)=y(1-x_W)=x_W tg\delta$ leads to the final result:

$$y = x_W tg\delta + A(x-x_W)(1-x-x_W) + B\big[(1-x-x_W)^2\ln(1-x-x_W) + \\ +(x-x_W)^2\ln(x-x_W) - (1-x)^2\ln(1-x) - x^2\ln x - \\ -(1-2x_W)^2\ln(1-2x_W) + (1-x_W)^2\ln(1-x_W) + x_W^2\ln x_W\big].$$

Here A and B are the shape parameters which equal $A=-0.5Kc_p$, $B=\dfrac{Ktg\delta}{\pi\sqrt{1-M_\infty^2}}$ within the framework of the accepted assumptions. In order to obtain a more exact solution of the problem it is necessary to perform a direct numerical optimization by varying parameters A and B.
The results of the optimization are presented in Figure 3 for the following conditions: M=0.672; c_p=-0.9; γ=1.4, x_W=0.2 and δ=22.15° [4]. Numerical simulation of the flow over airfoil is performed based on solving the full potential equation. For the initial body with the flat cut, the data obtained by numerical simulation and within the framework of the linear theory are compared. The theoretical curve correctly describes the qualitative feature of the aerodynamic load distribution.
Despite the low accuracy of the theoretical model and the significant differences between the initial and optimal bodies, the analytical representation of the airfoil contour with two shape parameters provides a near to optimum solution. The longitudinal distribution of the pressure coefficient on the body surface is characterized by insignificant perturbations relative to the required value c_p=-0.9. A more exact result with the flat pressure distribution is obtained on the base of multi-parametric optimization. The local linearization provides a fast convergence of the optimization process. A mean-square residual on the Mach number on the optimized surface is taken as the objective function.
The aerodynamic shapes obtained by two-parametric and multi-parametric optimization almost coincide. Differences are observed in the vicinity of the points joining the nose and tail wedges. Figure 4 shows the contours of the initial and optimal airfoils. The relative thickness of the optimum airfoil achieves 22.64%. The results are in good agreement with results of [4]. The difference on areas bounded by the airfoil contours is less than 0.5%. Figure 5 shows the lines of equal values of the Mach number with increments of 0.05 near the airfoil that has wedge-shaped nose and tail parts. The central part of the airfoil is characterized by achieving the local sound conditions at free stream Mach number M=0.672.

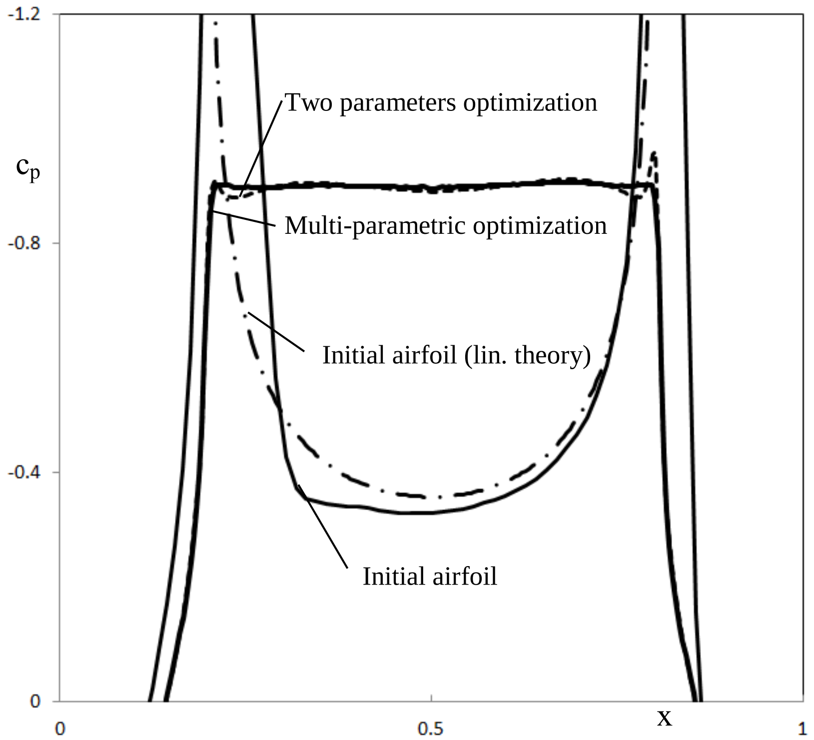

Figure 3: Pressure coefficient distribution on airfoils

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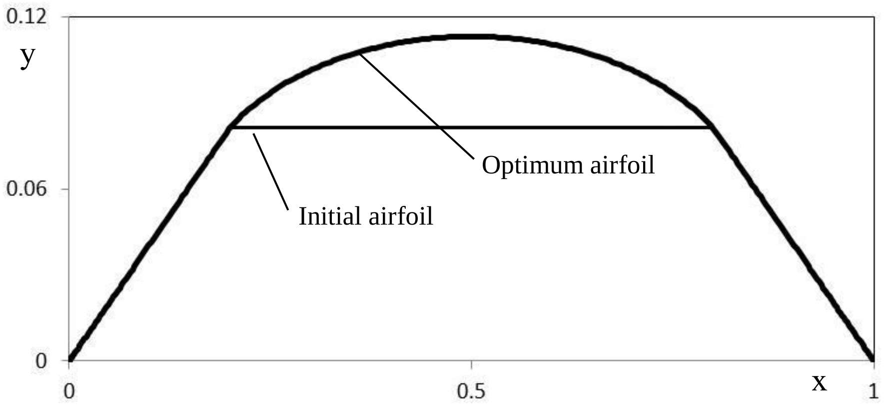

Figure 4: Initial and optimum airfoils

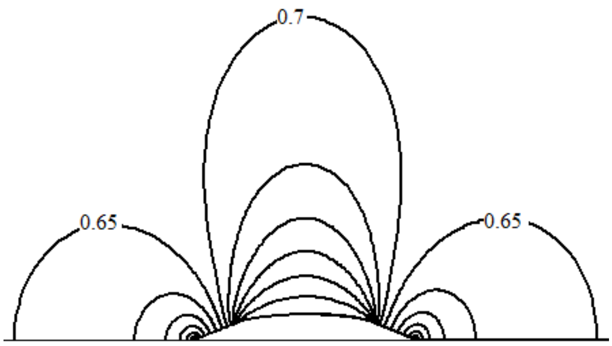

Figure 5: Mach number level lines

3.2 Airfoil with low aerodynamic drag

Symmetrical airfoils are designed to produce low drag. The direct design method involves the specification of the geometry and the calculation of pressures, skin friction and aerodynamic drag. The given shape is evaluated and then modified to improve the performance.
The transonic design problem is to create the airfoil section for desired thickness without causing strong shock waves and boundary layer separation at a given speed. The maximum local Mach number on a supercritical airfoil should not exceed 1.3. This leads to limit of the minimum pressure coefficient that can be tolerated and bounds contour curvature. On the other hand the pressure recovering behind the maximum thickness section with the steepest possible gradient causes separation. To prevent separation additional constraint is imposed on the trailing edge angle.
The optimization of airfoils with low aerodynamic drag is performed under dimensional constraint on the maximum thickness. The aerodynamic drag coefficient c_D is minimized at zero angle of attack. Airfoils with sharp trailing edges are considered.
Symmetrical supercritical airfoil SC(2)-0012 is taken as the base configuration [8]. The airfoil has the bottom face (5% of the thickness) and its tail part geometry is modified to provide a sharp edge. The trailing edge semi angle is 7.5° that corresponds to the first derivative of the ordinate y'=-0.131. The minimum of aerodynamic drag and the optimal airfoil shape are found at Mach number M=0.8 and Reynolds number $Re_b=9 \bullet 10^6$ based on the chord length. Optimization is carried out for y'=-0.131, -0.19 and -0.25.
Comparison of the airfoil contours is shown in Figure 6. The constructed airfoil with y'=-0.131 has greater area in comparison with modified SC(2)-0012 airfoil. It leads to redistribution of the pressure coefficient in the midsection of the airfoil (Figure 7). Increasing the trailing edge angle is accompanied by the area increase and redistribution. Thickness of the nose part decreases and thickness of the tail part increases. The maximum on absolute value pressure coefficient diminishes. The observed effect is due to reduction of the contour curvature near maximum thickness point. On the other hand the pressure recovering gradient enhances and gives rise to flow separation. The maximum local Mach number is 1.1 at y'=0.131 and 1.03 at y'=0.25.

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The relative contribution of the surface friction drag to the airfoil drag is about 50% at Mach number M=0.8. Advantage on pressure drag varies from 2% up to 20% in dependence on trailing edge angle. With Mach number increase the wave drag enhances and portion of friction drag is reduced. At M=0.9 aerodynamic drag coefficient decrease achieves 11% in comparison with supercritical airfoil.

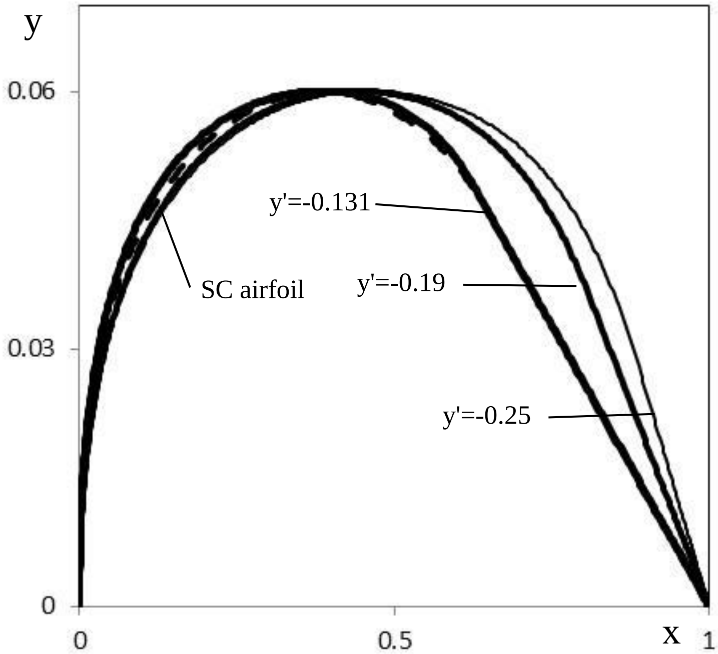

Figure 6: Airfoil contours

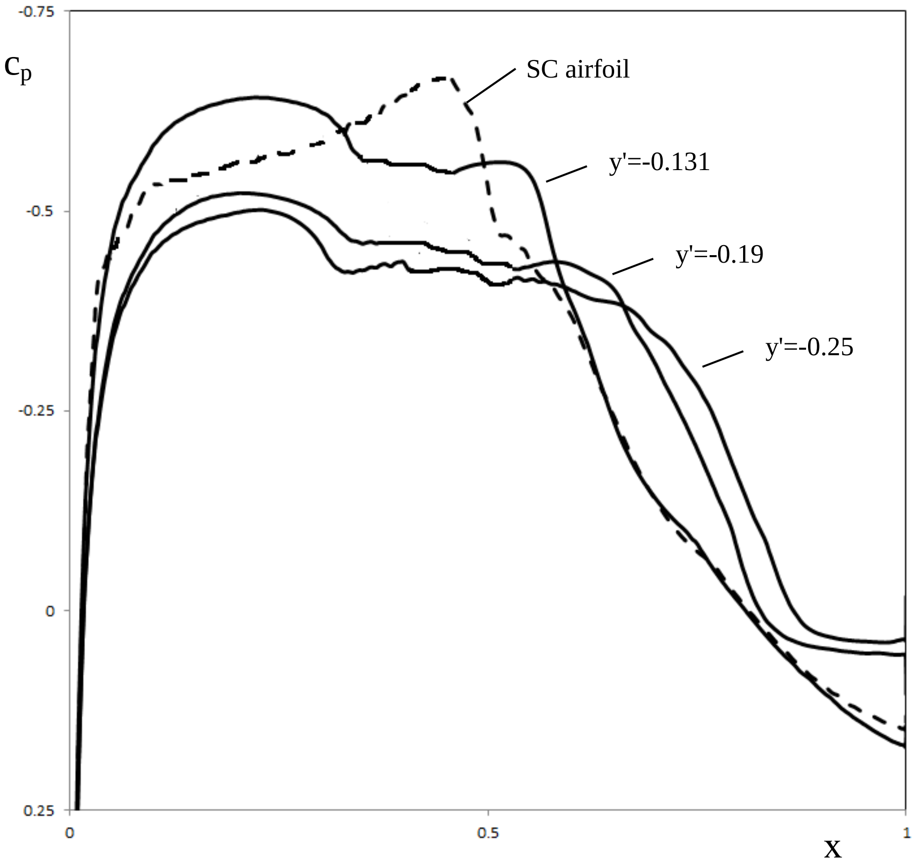

Figure 7: Pressure coefficient distribution on airfoils

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3.3 Axisymmetric Riabouchinsky problem

The aerodynamic characteristics of the fuselage depend on the distribution of cross-sectional area in the longitudinal direction. Under the assumptions of the theory of thin bodies the von Karman ogive nose cone has the minimal wave drag among the noses of the same lengthening. Wind tunnel tests revealed the superiority of noses with power law generatrixes in the supersonic speed range. Additional reduction of the wave drag is achieved by blunting the nose face. Computational and experimental studies of noses with Ryabushinsky generatrix confirmed a flat distribution of the gas-dynamic functions on the surface that meets the requirement of increasing the critical Mach number [9, 10].
The method of local linearization is applied to construct the fuselage nose in the form of Ryabushinsky cavity. The problem is solved under the incompressible flow conditions. The mean-squared deviation of the pressure coefficient from the specified value is taken as the objective function. The optimum nose has the lengthening λ=0.87. The front end radius is 54% compared to the base radius.
The flow parameters and aerodynamic characteristics of the nose are investigated in the framework of the system of Navier-Stokes equations. Numerical simulation is performed for the following defining parameters: the free stream Mach number is M_∞=0.3÷0.9, the Reynolds number calculated on the nose length is Re=$6{\cdot}10^6$. The equations are closed by the algebraic turbulence model of Baldwin-Lomax. Nose part is smoothly connected with the cylindrical part of the fuselage, the lengthening of which is ten times more than the nose lengthening.
The main integral characteristic is the aerodynamic drag of the nose consisting of the skin friction drag and wave drag due to volume. The drag coefficient c_D dependence on the Mach number shows a sharp increase of Ryabushinsky nose drag at M_∞=0.85.
The pressure distribution (relative to free stream pressure p_∞) on the nose surface is shown in Figure 8 for the three Mach numbers. With increasing Mach number the rarefaction on the surface is intensified. On considerable part of the nose the flow parameters variation in the longitudinal direction has a monotonic character until the local Mach number is less than 1. The pressure dependence on the longitudinal coordinate is close to a linear one.
Figure 9 shows the longitudinal section of flow field at M_∞=0.9 around axisymmetric fuselage with forebody in the form of half of the Riabouchinsky cavity. The lines of equal pressure coefficient values are plotted with increments of Δc_p=0.1. The maximum value of the local Mach number is about 1.15. The supersonic flow region is closed by the shock wave located in the vicinity of the junction of the nose and cylindrical parts of the fuselage.

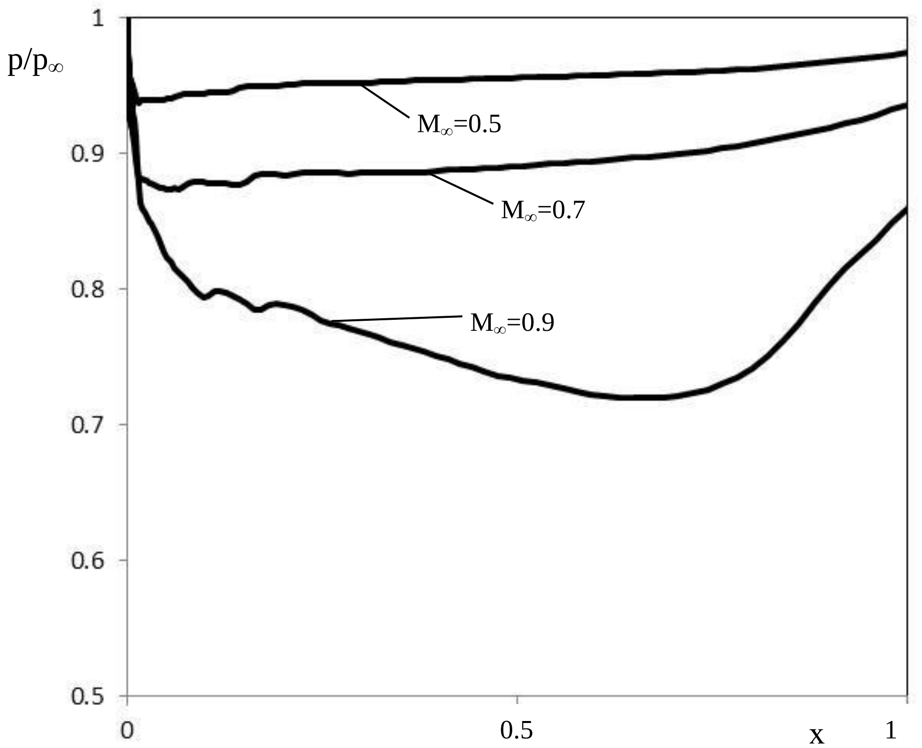

Figure 8: Pressure distribution on the nose surface

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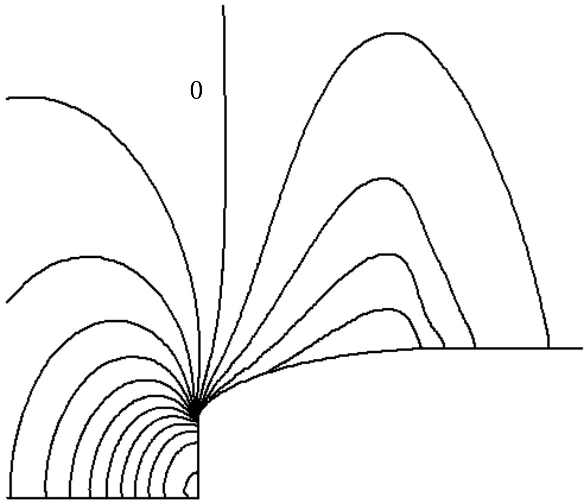

Figure 9: Pressure coefficient level lines, Δc_p=0.1

4. Conclusion

It is developed the method of aerodynamic shapes optimization on the base of local linearization of the relation between gas-dynamic functions and geometrical parameters. Efficiency of the method is demonstrated at subsonic and supersonic flow conditions. Examples of construction of symmetrical airfoils and axisymmetric forebodies are considered. Distributed and integral aerodynamic characteristics are used as the objective functions. Flow fields modeling and analysis are performed within the framework of local models, linear theory, and high-level models of flow physics.

Acknowledgments

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References

[1] Takovitsky, S.A. 2002. Numerical optimization of the wing of a supersonic airplane. In: *Proc. 23rd Congress of International Council of the Aeronautical Sciences. Paper ICAS 2002-2.3.2*. 1-8.

[2] Takovitskii, S.A. 2015. Optimization problems of supersonic aerodynamics. Moscow: Nauka (in Russian).

[3] Davis, W.H. 1979. Technique for developing design tools from the analysis methods of computational aerodynamics. *AIAA Paper 79-1529*.

[4] Brutyan, M.A., and S.V. Lyapunov. 1981. Optimization of shape of symmetrical flat bodies with the purpose of increase of the critical Mach number. *Uch. Zap. TsAGI*. 12, 5 : 10–22.

[5] Kraiko, A.N., A.V. Grachev, and S.A. Takovitskii. 2015 A new direct method of optimal profiling aerodynamic shapes, streamlined by sub-, trans- and supersonic flows and demonstrations of capabilities of this method. In : *Proc. of Russia-Wide Science-Technical Conference on Aviation Engines in the 21st Century*. Moscow-CIAM : 24–27 (in Russian).

[6] Takovitskii, S.A. 2016. About construction of flat bodies with increased critical Mach number. *TsAGI Science Journal*. 47, 6 : 1–17.

[7] Guzevsky, L.G. 2006. Boundary integral equation method for solution of plane and axisymmetric Riabouchinsky problems. *Vychisl. Tekhnol*. 11 : 68–81.

[8] Mineck R.E., and P.L. Lawing.1987. High Reynolds number tests of the NASA SC(2)-0012 airfoil in the Langley 0.3-meter transonic cryogenic tunnel. NASA TM-89102.

[9] Vyshinsky V.V., and E.N. Kuznetsov. 1995. Study of flow around nose parts of bodies of revolution with a Ryabushinsky generatrix. *Trudy TSAGI*. 2571.

[10] Barinov V.A., A.L. Bolsunovsky, N.P. Buzoverya, E.N. Kuznetsov, S.I. Skomorokhov, and I.L. Chernyshev. 2007. Study of transonic gas flow around a model of the aircraft with the fuselage nose part in the form of Ryabushinsky half cavity. *Reports of Academy of Sciences. Mechanics*. 416, 4.