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Modeling of radiative cooling of disperse flows of the low-grade heat rejection frameless systems in space.

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Abstract
The research on formed of liquid droplet radiators' dispersed flow temperature profile was done. It was shown that the process is accompanied by wave phenomena. An analysis of these waves' influence on radiative system charachteristics was carried out.

1. Introduction

For solving some urgent problems of the use of space it is necessary to essentially raise the power-to-weight ratio of space vehicles. The most problem constituent of their power systems is the low-grade heat rejection system. Micrometeorite vulnerability of conventional panel radiators and those out of heat pipes increases quickly with increasing rejecting power (and the surface area along with it). Armoring of radiators surfaces inadmissibly raises the space vehicle mass. The use of the frameless systems of low-grade heat rejection may be a way out. The idea of liquid droplet radiator (LDR) involves the heat rejection with the aid of the heated droplet flow. The droplet generator disperses the heated heat-transfer agent, the collector picks up the cooled droplet flow (Fig.1).
The problem of calculating the equilibrium profile of the temperature of the fine-dispersed sheet of high-temperature ($T>600$ K) LDR under the radiation cooling is considered in [1]. The solution was found for an absorbing-dissipative flat layer. Its emissivity depends on the optical thickness and the albedo of the dissipation of particles. This regularity becomes true after the initial transient period, in the course of which the agreement of the temperature distribution and the dissipation source function takes place. At the same time, for low-grade LDR ($T<500$K) the time to reach the steady-state process may exceed the droplet transit time in a sheet. The thermal calculation of such radiators should be carried out taking into account the transient processes.

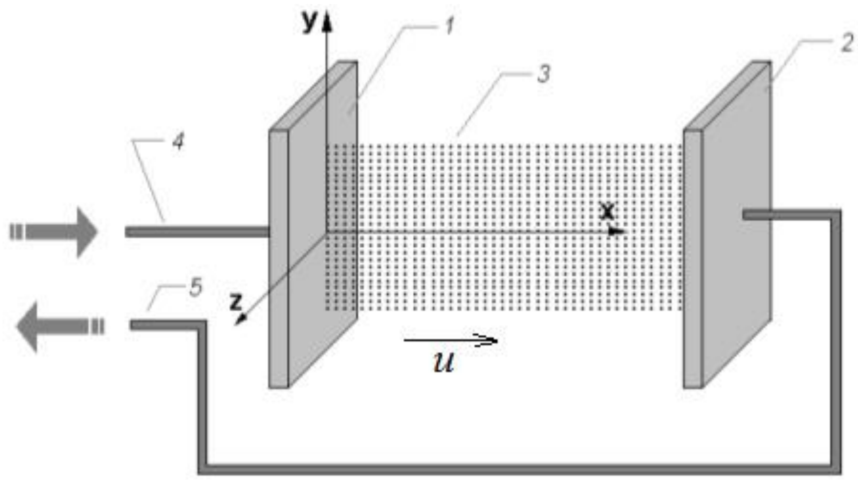

Figure 1: Liquid droplet radiator. 1 - The generator of drop flow; 2 – Hydro-collector; 3 - Droplet flow; 4, 5 - The pipeline. The velocity of droplets u is directed along the axis x.

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2. Mathematical formulation of the problem of cooling the disperse flow

We study the droplet sheet, consisting of several parallel thin droplet layers - the elements of its structure. The size of drops is around $2\cdot10^{-4}$ m, and optical thickness is much greater than unity. The index *y* is used to identify the particles. Taking into account the mutual overradiation of drops, the cooling equation of the test particle in the flow is written as:

$$mcu\frac{dT(x,y)}{dx} = -4\pi\varepsilon(r,T)\sigma T^4(x,y) + Q(x,y) \qquad \textbf{(1)}$$

where *T(x,y)* - temperature; *x*, *u*, *r* and *c* – are the coordinate of the center, velocity, radius of the drop and heat capacity, respectively; σ – is the Stefan-Boltzmann constant; ε(*r*,*T*) – total emittance of the droplet of radius *r* at given temperature. Measured in [2] infrared emission of working fluid DC-705 gives value ε ≈ 0.6 in a practically interesting range of radius and temperature of drops. Summand *Q(x,y)* describes the radiative interaction of drop with the sheet and the external radiation. In the case of the solitary drop, *Q* = 0. If the drop interacts with external (eg solar) radiation, then $Q = \varepsilon_{\odot}P_{\odot y}$, where $P_{\odot y}$ – the power of solar radiation, falling on the drop with the index *y*, $\varepsilon_{\odot}$ - coefficient absorption of solar radiation. Overradiation of a certain particle *y'* from the drop stream with the drop under study is described by the function $Q(x) = 4\pi\varepsilon(T)\sigma\varphi\, T^4(x-\Delta_x, y')$, where φ – is the coefficient of overradiation of drops *y* and *y*', Δ_x – is the distance between the droplets under consideration. If $\alpha = 4\pi\varepsilon\sigma/mcu$, the overradiation of drop with the sheet is described by the dependence:

$$Q(x,y) = \varepsilon\alpha\int_{\tilde{y}\neq y}\int_{x}^{l+x} f_{x,y}(\tilde{x},\tilde{y})T^4(\tilde{x}-x,\tilde{y})d\tilde{x}d\tilde{y} \qquad \textbf{(2)}$$

where *l* – is the length of drops fligh in the sheet, $f_{x,y}$ - is the distribution function of the radiation coefficient of the particle with the index y located at the point with the coordinate *x*. The physical meaning of the integral is the summation of the energy of the overradiation of all drops of the sheet with the drop under study.

The conditions for cooling of drops in each droplet layer are the same, so the index y determines the number of the structure element. It is assumed that the distance between the droplet layers is much greater than their thickness, which is why for layers with numbers *i* and *y*: $f_{x,y}(\tilde{x},\tilde{y}) = f_{|i-y|}(\tilde{x})$. In this case, the problem of cooling of drops (1) can be represented as the problem of interaction of structural elements

$$-\frac{1}{\alpha}\frac{dT_y(x)}{dx} = T_y^4(x) - \varepsilon\sum_{i=1}^{N}\int_{-x}^{l-x} T_i^4(\tilde{x}+x)f_{|i-y|}(\tilde{x})d\tilde{x} - \varepsilon_{\odot}P_{\odot y}/\alpha \qquad \textbf{(3)}$$

where T_y - is the temperature field in the drop layer with the number *y*, *N* – is the number of layers in the sheet [3].

3. Idealized problem of establishing the temperature profile

We neglect the influence of external radiation on the process of cooling of drops. In addition, we will consider that drops in LDR cool down slowly enough. Under these assumptions, the system of integral equations (3) reduces to the concentrated problem

$$-\frac{1}{\alpha}\frac{dT_y(x)}{dx} = T_y^4(x) - \varepsilon\sum_{i=1}^{N}\varphi_{|i-y|}T_i^4(x) \qquad \textbf{(4)}$$

where $\varphi_{|i-y|} = \int_{-l/2}^{l/2} f_{|i-y|}(\tilde{x})d\tilde{x}$.

It is seen from equation (4) that in the case of the freely cooling drop (values of all the coefficients $\varphi_{|i-y|} = 0$), the temperature depends on the coordinate according to the following law:

$$T_1(x) = T_0(1+3\alpha T_0^3 x)^{-1/3} \qquad \textbf{(5)}$$

where T_0 – is the initial temperature of the drops. We divide equation (4) by T_1^4. Further, the change of variables is realized.

$$\xi = (1+3\alpha T_0^3 x)^{-1/12}$$
$$T_y(x) = T_0\xi^4\tau_y(\xi) \qquad \textbf{(6)}$$

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The physical meaning of the value ξ – is the cooling rate of the solitary drop; dimensionless temperature, τ is the ratio of temperatures of the drop, which cools down in the drop stream of DCR to the temperature of the free cooling drop. In new variables, the system (4) is written in the form:

$$\frac{1}{4}\xi\frac{d\tau_y}{d\xi} = {\tau_y}^4 - \tau_y - \varepsilon\sum_{i=1}^{N}\varphi_{|i-y|}\tau_i^4 . \qquad \textbf{(7)}$$

We will simulate the cooling of a droplet flow consisting of two structure elements – the core and periphery (for example, an optically thick cylindrical sheet – see Fig. 2). The temperature field at its center $\tau_c$varies slowly, at the periphery τ_p – much faster. The cooling equations take the form:

$$\begin{aligned} \frac{1}{4}\xi\frac{d\tau_я}{d\xi} &= (1-\varphi_1){\tau_c}^4 - \tau_c - \varepsilon\psi_1{\tau_p}^4 \\ \frac{1}{4}\xi\frac{d\tau_p}{d\xi} &= (1-\varphi_2){\tau_p}^4 - \tau_p - \varepsilon\psi_2{\tau_c}^4 \end{aligned} \qquad \textbf{(8)}$$

where ψ_1, ψ_2 – are the angular emission coefficients of the core of the flow to the periphery and vice versa, and φ_1 and φ_2 - are the self-irradiation coefficients of the core and the periphery of the flow.

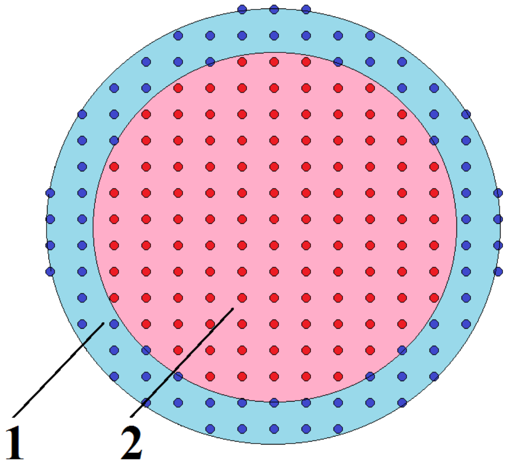

Figure 2: The model of droplet flow consisting of core and periphery. The numbers denote: 1 - the periphery of the flow, 2 - the core of the flow.

Let's consider the system of equations (8). At the moment of establishing the equilibrium temperature profile, its right side is zero. For values of the angular emission coefficients satisfying the energy conservation law, the system (8) has only one physically meaningful equilibrium position τ_c^* и τ_p^* (the remaining three correspond to negative temperature values). To analyze the behavior of the system (8) in a small neighborhood of the equilibrium values τ_c^* and τ_p^*, we introduce the local variables δx and δy such that: : $\tau_c = \tau_c^* + \delta x$ и $\tau_p = \tau_p^* + \delta y$. The linear approximation of the system (8) in the vicinity of the equilibrium position is written in the following form:

$$\begin{aligned} \frac{1}{4}\xi\frac{d}{d\xi}\delta x &= [4{\tau_c}^{*3}(1-\varphi_1)-1]\delta x - 4\varepsilon\psi_1{\tau_p}^{*3}\delta y \\ \frac{1}{4}\xi\frac{d}{d\xi}\delta y &= [4{\tau_p}^{*3}(1-\varphi_2)-1]\delta y - 4\varepsilon\psi_2{\tau_c}^{*3}\delta x \end{aligned} . \qquad \textbf{(9)}$$

For the compactness of the representation, we introduce the following notation:

$$\hat{A} = \begin{pmatrix} 4{\tau_c}^{*3}(1-\varphi_1)-1 & -4\varepsilon\psi_1{\tau_p}^{*3} \\ -4\varepsilon\psi_2{\tau_c}^{*3} & 4{\tau_p}^{*3}(1-\varphi_2)-1 \end{pmatrix},\ \delta\vec{\tau} = \begin{pmatrix}\delta x \\ \delta y\end{pmatrix}. \qquad \textbf{(10)}$$

In them, the system (9) is written in the form:

$$\frac{1}{4}\xi\frac{d}{d\xi}\delta\vec{\tau} = \hat{A}\delta\vec{\tau}. \qquad \textbf{(11)}$$

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Equation (11), written for the system of two equations, can easily be generalized to the case of higher dimensionality. Since ξ decreases with increasing of *x*, the trajectories of the solution of system (11), corresponding to the motion along the eigenvectors of the matrix $\hat{A}$ with positive values, turn out to be stable and vice versa. The analysis of the eigenvalues of the matrix $\hat{A}$ is carried out for the values of coefficients ψ_1, ψ_2, φ_1 and φ_2, which satisfy the law of conservation of energy. In this case, the equilibrium point of the system (8) corresponding to the equilibrium temperature profile of the droplet flow and represents the stable node. For the parameter values of the problem ψ_1=0,3, ψ_2=0,12, φ_1=0,85 and φ_2=0,3, ε=1, the equilibrium position has the following coordinates: $\tau_c^*\approx 2{,}36$ a $\tau_p^*\approx 1{,}67$. The eigenvalues of the matrix $\hat{A}$ are: $\lambda_1\approx 15{,}97$, $\lambda_2\approx 3{,}09$. Figure 3 shows the characteristic phase portrait of the system (8). When moving along the trajectory 1, the function τ_c, which characterizes the cooling of the core of the flow, varies monotonically, and the change in the peripheral temperature τ_p is non-monotonic. The non-monotonic behavior of the function τ_p has a noticeable effect on the emissivity of the disperse flow.

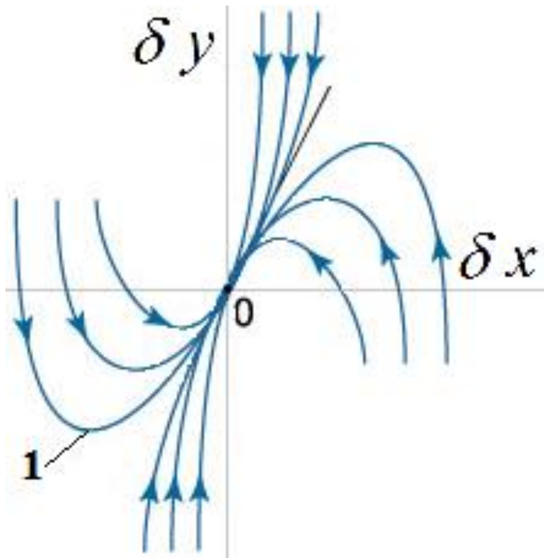

Figure 3: Phase portrait of the dynamic system (8).

Let’s estimate the velocity of establishment of the equilibrium temperature profile in the droplet flow. We change the variables, denoting by the symbol l the distance between the coordinate of some point in the space $\{\tau_c, \tau_p\}$ and the equilibrium position of the system (8). We assume that the trajectory of the system in the space $\{\tau_c, \tau_p\}$ is a straight line connecting the point with the coordinates {1, 1} and $\{\tau_c, \tau_p\}$. The motion along this straight line will be described by the equation:

$$\frac{1}{4}\xi\frac{dl}{d\xi} = \lambda_1 l, \tag{12}$$

where λ_1 – is the maximum eigenvalue of the matrix $\hat{A}$. It is not difficult to see that the solution of the differential equation (11) with the condition $l_0 = l(\xi = 1) = \left((\tau_c{}^* - 1)^2 + \left(\tau_p{}^* - 1\right)^2\right)^{1/2}$, can be written in the form:

$$l/l_0 = \xi^{4\lambda_1} = (1 + 3\alpha T_0^3 x)^{-\lambda_1/3} \approx \left(T_f/T_0\right)^{-\lambda_1}, \tag{13}$$

where T_f – is the final average temperature of the droplet flow. In the case of a drop in the temperature of the droplet flow in DCR 360-310 K and $\lambda_1 \approx 15$, the right side of the relation (12) is equal to ≈ 0,1. In this example, the droplet temperature field near the tank is ten times closer to the equilibrium position of the dissipative system (7) than at the beginning of the cooling process.

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4. Determination of the temperature profile in the presence of perturbations

Let's investigate how the presence of perturbations can effect on the process of establishing the profile of the temperature of the radiation-cooling droplet flow: external radiation, long-range radiative interactions, a decrease in the integral degree of blackness of the working medium during cooling, etc. The effect of these factors complicating the calculation will be modeled by the value of Ψ (x) in the cooling equation:

$$-\frac{1}{\alpha}\frac{dT_y(x)}{dx} = T_y^4(x) - \varepsilon\sum_{i=1}^{N}\varphi_{|i-y|}T_i^4(x) - \Psi_y(x)\,. \quad \textbf{(14)}$$

After the passing to variables ξ and τ, this cooling equation is written in the form:

$$\frac{1}{4}\xi\frac{d\tau_y}{d\xi} = {\tau_y}^4 - \tau_y - \varepsilon\sum_{i=1}^{N}\varphi_{|i-y|}\tau_i^4 - \widetilde{\Psi_y}(\xi). \quad \textbf{(15)}$$

We assume that perturbations have a weak effect on the solution of problem (13). We calculate the influence of the value $\widetilde{\Psi_y}(\xi)$ in the linear approximation. In a neighborhood of the equilibrium position of the unperturbed problem (7), the trajectory of the system is described by equation:

$$\frac{1}{4}\xi\frac{d}{d\xi}\delta\vec{\tau} = \hat{A}\delta\vec{\tau} - \vec{\Psi}(\xi). \quad \textbf{(16)}$$

where $\vec{\Psi}(\xi)$ - the vector whose y-component is equal to $\widetilde{\Psi_y}(\xi)$. The equilibrium position of systems (11) and (15) does not coincide. We introduce the vector $\delta\vec{r}$, which connects the equilibrium positions of systems (11) and (15). From equation (15) it follows that

$$\delta\vec{r}(\xi) = \hat{A}^{-1}\vec{\Psi}(\xi) \quad \textbf{(17)}$$

where $\hat{A}^{-1}$ - matrix inverse to the matrix $\hat{A}$. Taking this equation into account, the system (15) can be written in the following form:

$$\frac{1}{4}\xi\frac{d}{d\xi}\delta\vec{\tau} = \hat{A}[\delta\vec{\tau} - \delta\vec{r}(\xi)]\,. \quad \textbf{(18)}$$

From this relation, it is seen that the equilibrium position of system (15) moves with velocity of $\frac{d}{d\xi}\delta\vec{r}(\xi)$. Because of the presence of perturbations, the solution of the system (14) is at some distance Δ from the equilibrium position. We shall evaluate Δ on the basis of relation (11). The calculation shows the following result:

$$\Delta = \frac{1}{\lambda_1}\left\|\frac{d}{d\xi}\delta\vec{r}(\xi)\right\|, \quad \textbf{(19)}$$

where $\left\|\frac{d}{d\xi}\delta\vec{r}(\xi)\right\|$ – the norm of velocity vector $\frac{d}{d\xi}\delta\vec{r}(\xi)$.

Let's investigate the effect of external radiation on the solution of the problem (8) of establishing the temperature profile in droplet flow consisting of the core and periphery. We will consider that the Sun influences on the periphery of the flow, heating it with radiation with a thermal power of $\varepsilon_{\odot}P_{\odot}$. We neglect the direct effect of solar radiation on the cooling of the core of the flow. In this case, Ψ_c=0, a $\Psi_p=\varepsilon_{\odot}P_{\odot}$. Then $\widetilde{\Psi_я}(\xi) = 0$, $\widetilde{\Psi_п}(\xi) = \frac{\varepsilon_{\odot}P_{\odot}}{\alpha T_0^4\xi^{16}}$. For the abovementioned parameters of the problem ψ_1, ψ_2, φ_1, φ_2 and ε, the value

$$\Delta \approx \frac{\varepsilon P_{\odot}}{\alpha T_0^4}\frac{1}{\xi^{17}}.\Delta = \frac{1}{\lambda_1}\left\|\frac{d}{d\xi}\delta\vec{r}(\xi)\right\|\,. \quad \textbf{(20)}$$

For full-scale DCR with size of drops of 10^{-4} m and the initial temperature of drops of T_0=600 K, the value is Δ≈0,4. Similarly, an analytical calculation of the effect on the process of establishing the temperature profile of the droplet flow of long-range radiation interactions is carried out.

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5. Numerical solution of establishment problem

Fig. 4 shows results of numerical solution of the problem of droplet flow cooling:

$$-\frac{1}{\alpha}\frac{dT_y(x)}{dx} = T_y^4(x) - \varepsilon\sum_{i=1}^{N}\varphi_{|i-y|}T_i^4(x-\Delta_x) - \varepsilon P_{\odot y}, \quad \textbf{(21)}$$

where the value Δx describes the effect of long-range radiation interactions. The problem was solved for the droplet flow consisting of the core and the periphery (analog of (8)). In the calculation, the following parameter values were used ψ_1=0,3, ψ_2=0,12, φ_1=0,85, φ_2=0,3, ε=1. Initial condition is: $\tau_c = \tau_p = 1$. The value of $\varepsilon_\Theta P_\Theta/(\alpha T_0^4)$ was assumed to be equal to 0.15, and the ratio of the length of the radiation interactions to the length of the transit of drops in LDR - Δ_x/L = 0,05. The initial temperature of drops was assumed to be equal to 800 K, and the value α of such that the final value ξ was 0,4. The numbers indicate solutions with the following parameters. Curve 1 is a calculation with allowance for solar radiation but without distant overradiations. Curve 2 is an idealized problem of establishing the temperature profile (without solar radiation and without distant overradiations). Curve 3 - both solar radiation and distant overradiations were taken into account. Curve 4 - only distant overradiations are taken into account.

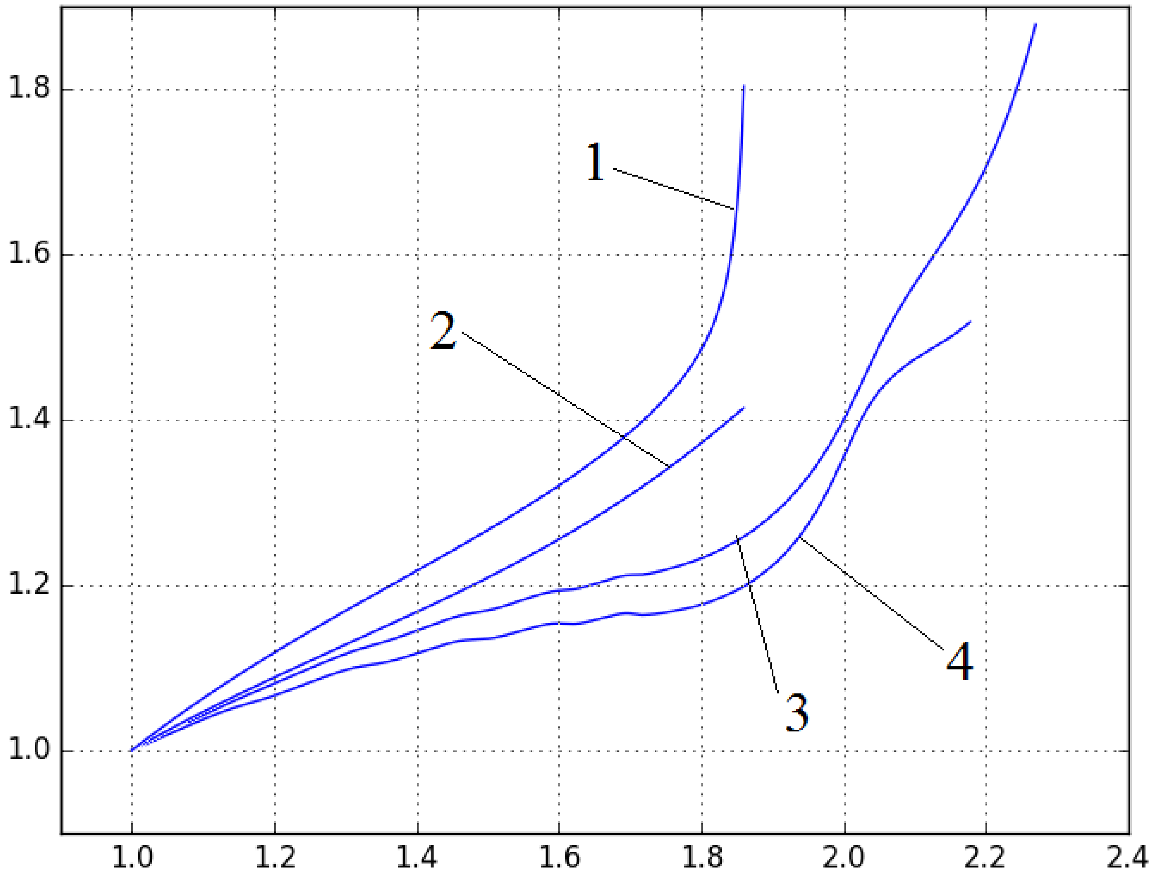

Figure 4: Numerical solution of the cooling problem (20). On the abscissa axis τ_c is plotted, and τ_p is plotted along the ordinate axis. 1 - Only solar radiation is considered; 2 - Both solar radiation and distant radiation interactions weren't taking into account; 3 - both solar radiation and distant radiation interactions are considered; 4 - only distant radiation interactions are taken into account. The initial condition is: $\tau_c = \tau_p = 1$.

Figure 4 shows that the vector connecting the ends of trajectories 2 and 3 (vector 2-3) with the high accuracy is equal to the sum of vectors 2-1 and 2-4. From this it can be concluded that the linear expansions on δx and δy with the sufficient accuracy describe the process for estimating of temperature profile.

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6. The discussion of results

The establishment of temperature in a freely cooling dispersed flow can occur non-monotonically and is accompanied by wave phenomena those characteristic for the flow model consisting of the core and the periphery. The non-linear process of establishing the temperature profile affects the energy characteristics of frameless systems for the removal of low-potential heat in space.

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