Fault Tolerant Integrated Barometric-Inertial-GPS Altimeter

Alberto MAÑERO CONTRERAS* and Chingiz HAJIYEV[†] * Politecnico di Milano School of Industrial and Information Engineering, Milano, ITALY [†] Istanbul Technical University Aeronautics and Astronautics Faculty, Istanbul, TURKEY amcamc92@gmail.com · cingiz@itu.edu.tr

Abstract

As a result of the development of modern vehicles, even higher accuracy standards are demanded. As known, Inertial Navigation Systems have an intrinsic increasing error and that is the main reason of using integrating navigation systems, where some other sources of measurements are utilized, such as barometric altimeter due to its high accuracy in short times of interval. Using a Robust Kalman Filter, error measurements are damped when a Fault Tolerant Altimeter is implemented.

1. Introduction

Since Earth's center is the most extended inertial coordinate frame taken by the scientist society, "where am I?" is probably the most asked question in all of our lifetime. Indeed, it does not matter where the origin is located. We all know how to orientate ourselves in relation to other places, or in other words, through relative positions. Navigation is a really ancient art. History talks by itself and there are several examples throughout human era. In the early times, mariners used to discover new coastlines pushed by their braveness sailing open oceans. They used the North Star as a compass, the same stars and heaven we use today for space navigation. With the advent of the magnetic compass, sextant and chronometers for timing, it was finally possible to determine them both precisely latitude and longitude at the sea. Air navigation has passed through similar evolutions and revolutions. The First World War was the catalyst for rapid advances in aviation. Since early twentieth century, a wide range of radio-based navigation systems were developed and they were determinants and widely used since World War II. After these milestones, new navigation systems appeared, such as Inertial Navigation Systems (INS), which were thought impossible to implement due to their unstable behaviour. Nevertheless, it was made practical by Charles Draper and is today the backbone of military navigation and weapon delivery systems. Today we are in the middle of a new technological revolution: the Global Positioning System (GPS) (United States). Accurate positioning on real time regardless of location, weather or time of day is equal to power on the worldwide equation. Nowadays, several countries are developing and improving similar navigation systems such as GALILEO (European), GLONASS(Russian), DORIS (French), BeiDou (Chinese) or IRNSS (Indian).

Integrated navigation systems with application of the Kalman filter (KF) were a milestone. Integrated navigation systems combine the best features of both autonomous and stand-alone systems and are not only capable of good short term performance in the autonomous or stand-alone mode of operation, but also provide exceptional performance over extended periods of time when in the aided mode. Hence, integration brings increased performance, improved reliability and system integrity, and of course increased complexity and cost.¹ Moreover, outputs of an integrated navigation system are digital, thus they are capable of being used by other resources of being transmitted without loss or distortion.

Inertial Navigation Systems (INS) are ubiquitously employed to provide position and velocity information. However, navigation information from INS gets degraded over time as INS errors are nonlinear and accumulate over time.⁷ The errors can be divided as horizontal errors and error in vertical channel. Error in the vertical channel directly contributes to error in altitude estimations. Measurements from other external sources are used in conjunction with the INS measurements, to keep the error growth in the vertical channel within bounds.

The inertial and barometric altimeters have different benefits and drawbacks. The reason for integrating these two navigators is mainly to combine the best features, and eliminate the shortcomings. In the case of inertial altimeters, the advantages are, mainly: the good characteristics of the INS with high bandwidth and frequencies are maintained, in certain operation periods acceptable accuracy is attained and easily dynamic system adaptation to the computer

Copyright © 2017 by Alberto Mañero Contreras and Chingiz Hajiyev. Published by the EUCASS association with permission.

environment. For the case of barometric altimeters, however, the benefits are: they give a high accuracy, usually they are cheap and ease of implementation, although they need recalibration after some periods of time.

Combining Global Positioning System and Inertial Navigation System has also another advantages. Despite GPS measurements are tipically less accurate than barometrical ones, it is also true that when GPS is used there is no need to recalibrate the sensor in function of the flight day, because the measurements have always the same mean error. In other words, measurements do not depend on pressure or temperature. The barometric altimeter augmented with integrated inertial GPS altimeter can provide more reliable and accurate navigation solutions under high maneuvering environments.

2. Integrated Altimeter Design

First of all, it is needed to illustrate the error models of the inertial, barometric and GPS altimeters. Every single estimation model has an unavoidable error. In this section these error models are presented for the case of barometric, inertial and GPS altimeters.

2.1 Error Models

The measurement model of the barometric altimeter used in this research includes the bias error of the first order Markov process and a random white Gaussian noise as stated in,¹³ and they are expressed in the next equations

$$h_B = h_T + B + \nu_B \tag{1}$$

$$\dot{B} = -\frac{1}{\tau}B + w \tag{2}$$

being h_B the altitude measured from the barometer, h_T is the true height, measurement noise is represented by v_B , τ is the correlation time of the bias error, w means the driving white Gaussian noise os the bias error and B is the bias error mentioned before.

Having numerous outputs from different channels, the INS has a complex error model. In this research, only the vertical channel of the INS is considered. Although the main parameter of study is the altitude, it is also interesting to consider other fluctuating values such as vertical speed W_z , vertical acceleration a_z and the gravitational acceleration g. The linear, discrete error model of the vertical INS channel is given by:¹⁴

$$\Delta H_I(k) = \Delta H_I(k-1) + \Delta t \Delta W_z(k-1) \tag{3}$$

$$\Delta W_z(k) = \Delta W_z(k-1) + \frac{2g\Delta t}{P_z} \Delta H_I(k-1) + \Delta t \Delta a_z(k-1) + \Delta t \Delta g(k-1)$$
⁽⁴⁾

$$\Delta a_{z}(k) = \Delta a_{z}(k-1) - \Delta t \alpha \Delta a_{z}(k-1) + \Delta t U_{\Delta a}(k-1)$$
⁽⁵⁾

$$\Delta g(k) = \Delta g(k-1) - \Delta t \beta_g \Delta g(k-1) + \Delta t U_{\Delta g}(k-1)$$
(6)

where $U_{\Delta a_z}$ and $U_{\Delta g}$ are white Gaussian noises with zero mean, α and β_g are terms of the correlation period, being Δt the discrete time and $R_I = R_0 + H$, where H is the flight altitude and R_0 is the radius of the Earth. For the case of GPS error model, a Gaussian random white noise error has been taken, with zero mean and without bias.

2.2 Scheme 1: Integrated Baro-Inertial Altimeter

Inertial Navigation Systems are ubiquitously employed to provide position and velocity information. However, navigation information from INS gets degraded over time as INS errors are nonlinear and accumulate over time. The errors can be divided as horizontal errors and error in vertical channel. Error in the vertical channel directly contributes to error in altitude estimations. Measurements from other external sources are used in conjunction with the INS measurements, to keep the error growth in the vertical channel within bounds. In this section, altitude measurements from the barometric altimeter and the INS measurements are fused with an Robust Kalman filter to obtain an accurate estimation of the flight altitude. While most of the references use a Kalman filter to directlyestimate the state variables of system and its derivatives, it is common in inertial navigation systems to instead use a Complementary Kalman Filter which operates only on the errors in those primary state variables.⁴ The scheme used for acquiring a precise altitude throughout IBI altimeters is depicted in Figure 1.⁸ The CPU block is necessary because the barometrical altimeter measures from sea level while the inertial system measures the relative altitude from an initial alignment location. In a system where these two navigators are combined, this difference could easily be eliminated via Central Processing Unit. In order to be consistent with the theory explained before it is important to note that the measurement matrix for this system is

$$H_{IBI}(k) = [1 \ 0 \ 0 \ 0 \ -1] \tag{7}$$

The scheme shown in Figure 1 symbolizes the acquiring process of vertical measurements when inertial barometrical altimeter is used.



Figure 1: Integrated baro-inertial altimeter scheme

2.3 Scheme 2: Integrated Inertial GPS Altimeter

There are a lot of cases where integrated inertial GPS altimeters are used. Indirect Kalman filtering technique is used for integrating two mentioned navigations system. Using IIG altimeter a smoother position and velocity estimations can be calculated that can be provided at a sampling rate faster than the GPS receiver. This also allows an accurate estimation of the aircraft attitude (roll, pitch, and yaw) angles. In general, GPS/INS sensor fusion is a nonlinear filtering problem, which is commonly approached using a Kalman Filter. The scheme used for implementing this integrated system is depicted in Figure 2. The CPU block is not longer necessary because both measurements are taken from the same reference. The measurement matrix of the system for this case is

$$H_{IIG}(k) = [1 \ 0 \ 0 \ 0] \tag{8}$$

In this case, the last value is zero because GPS bias error is not taken into consideration for this integrated system.



Figure 2: Integrated inertial GPS altimeter scheme.

2.4 Scheme 3: Integrated Baro-Inertial GPS Altimeter

The integrated inertial GPS system is designed using a loosely coupled integration architecture that incorporates the GPS navigation solution. The GPS typically has a poor accuracy in the vertical channel due to satellite geometry and

DOI: 10.13009/EUCASS2017-62

FAULT TOLERANT IBIG ALTIMETER

atmospheric effects. Even worse, the vertical channel is vulnerable to faults due to its inappropriate dynamic model coupled with the frequent loss of satellites when the aircraft would bank. From the bibliography, huge GPS faults can be observed in the vertical axis, causing the IIG navigation system to become unstable. By augmenting the barometric altimeter data, the vertical channel can be stabilized under GPS fault conditions. The results using the flight data show that the baro-altimeter augmented IIG navigation system can provide more reliable and accurate navigation solutions under high maneuvering environments. The scheme used for this solution is shown in Figure 3.⁸ As shown, the data acquired by the sensors pass through local filters and then arrive to the integration module, where they are fused in an Optimal Kalman Filter before providing to the user the estimation of the parameters of interest.



Figure 3: Integrated baro-inertial GPS altimeter scheme.

The algorithm used for fusing the data in the IBIG altimeter is

$$\Delta H_{IBIG} = \frac{P_1(1,1)\Delta H_{IIG}(k) + P_2(1,1)\Delta H_{IBI}(k)}{P_1(1,1) + P_2(1,1)} \tag{9}$$

where $P_1(1, 1)$ and $P_2(1, 1)$ represent a relation with the altitude variable in the covariance matrix of estimation error for IIG and IBI altimeters, repectively. Hence, the altitude estimation is given by

$$\hat{H}_{IBIG}(k) = \Delta H_I(k) - \Delta \hat{H}_{IBIG}(k) \tag{10}$$

The measurement matrix H(k), covariance matrix of the measurement noise R(k) and measurement vector Z(k) are the described above for each case.

3. Kalman Filtering for the Integration of Altimeters

A Kalman filter is most probably the most famous computer algorithm in order to bounding and updating navigation outputs such as position, velocity and attitude. Kalman filtering provides the manner and the method for combining updates that is practically useful as well as mathematically ingenious. Moreover, this technique can be extended to the integration of outputs from a wide variety of systems and continues to show a high degree of practical utility in flight applications. The Kalman filter is a linear filter. Recall that the actual differential equations for INS operation are nonlinear, but the error equations are valid for linearized versions of these differential equations. Hence the requirements for the errors themselves is to remain small, otherwise a linear analysis is not valid. Kalman filter applications presume that state-space dynamic modelling will be used to implement the algorithm, as stated in.⁴ The first widely known important Kalman filter application was for the Apollo moon flight. From that time, a lot of improvements have been developed in this field and these ones have been also implemented in different important missions and aircraft navigation systems. The model of Kalman filter in its most known universal form is shown along this section.

3.1 Optimal Kalman Filter for the Integration of Barometric and Inertial Altimeters

The state space model used throughout this study is expressed with the following equations

$$X(k) = \phi(k|k-1)X(k-1) + G(k|k-1)U(k-1)$$
(11)

$$Z(k) = H(k)X(k) + V(k)$$
(12)

where X(k) is the state space vector of the system, $\phi(k|k-1)$ is the system transition matrix (being k the k-iteration), G(k|k-1) is the noise transition matrix, U(k-1) represents the system noise, V(k) is the measurement noise and H(k) represents the measurement matrix. The state vector components are given by

$$X^{T} = [\Delta H_{I}(k) \ \Delta W_{z}(k) \ \Delta a_{z}(k) \ \Delta g(k) \ B(k)]$$
(13)

where $\Delta H_I(k)$ represents the altitude error, $\Delta W_z(k)$ expresses the vertical speed error, $\Delta a_z(k)$ is the vertical acceleration error, gravitational error is given by $\Delta g(k)$ and finally B(k), which represents the baroaltimeter bias error. The following statements are the final expressions of the optimal Kalman filter equations to be used in the calculations. The equation of the estimation value on the iteration k is

$$\hat{X}(k|k) = \hat{X}(k|k-1) + K(k)\Delta(k)$$
(14)

being K(k) the Kalman gain matrix and $\Delta(k)$ represents an innovation sequence defined in the following lines taken from.⁴ The variable $\hat{X}(k|k-1)$ estimates the extrapolation value of the space state vector specified above in equation 13. The expressions of these three variables are

$$\hat{X}(k|k-1) = \phi(k|k-1)\hat{X}(k-1|k-1)$$
(15)

$$\Delta(k) = Z(k) - H(k)\hat{X}(k|k-1) \tag{16}$$

$$K(k) = P(k|k-1)H^{T}(k)[H(k)P(k|k-1)H^{T}(k) + R(k)]^{-1}$$
(17)

where R(k) is the covariance matrix of measurement noise. On the other hand, matrixes *P* represent covariance matrixes of estimation and extrapolation errors, which respectively have the following form

$$P(k|k) = [I - K(k)H(k)]P(k|k - 1)$$
(18)

$$P(k|k-1) = \phi(k|k-1)P(k-1|k-1)\phi^{T}(k|k-1)$$

$$+G(k|k-1)Q(k-1)G'(k|k-1)$$
(19)

being Q(k-1) the covariance noise system matrix. The software used to implement these algorithms has been Matlab.

3.2 Robust Kalman Filter

Throughout this paper, a Robust Kalman Filter is proposed since it deals in a better way with abrupt faults cases in the estimation system such as computer malfunctions, abnormal measurements or noise increments, which are all described in the next section. The equation of the estimation value is

$$\hat{X}(k/k) = \hat{X}(k/k - 1) + p(1/k)K(k)\Delta(k)$$
(20)

Here, p(1/k), is the posterior probability of the normal operation of the estimation system, given for measurement results at the kth time step. The other parameters have been already defined along last section. When p(1/k) = 1, this filter will be exactly the same as the Optimal Kalman Filter, but when $p(1/k) \neq 1$ a new calculation in order to estimate the variables of interest is done. During normal operation of the measurement channel, $\overline{\Delta}(k)$ normalized innovation sequence of Kalman Filter is

$$\bar{\Delta}(k) = [H(k)P(k/k-1)H^{\top} + R(k)]^{-1/2}\Delta(k)$$
(21)

will satisfy the normal distribution N(0, 1). Consequently the adaptative filtration algorithm with the filter gain matrix correction can be presented in the form below

$$\hat{X}(k/k) = \hat{X}(k/k - 1) + K_A(k)\Delta(k)$$
(22)

$$K_A(k) = p(1/k)K(k) \tag{23}$$

$$\hat{X}(k/k-1) = \phi(k,k-1)\hat{X}(k-1/k-1)$$
(24)

$$p(1/k) = \frac{1}{\sqrt{2\pi\sigma_{\bar{\Delta}}(k)}} \int_{-3}^{3} \exp\left[-\frac{[\tilde{\Delta}(k) - \hat{\bar{\Delta}}(k)]^2}{2\sigma_{\bar{\Delta}}^2(k)}\right] d\bar{\Delta}$$
(25)

$$\Delta(k) = Z(k) - H(k)\hat{X}(k/k - 1)$$
(26)

$$\tilde{\Delta}(k) = [H(k)P(k/k - 1)H^{\mathsf{T}}(k) + R(k)]^{-1/2}\Delta(k)$$
(27)

$$K(k) = P(k/k - 1)H^{\mathsf{T}}(k)[H(k)P(k/k - 1)H^{\mathsf{T}}(k) + R(k)]^{-1}$$
(28)

$$P(k/k) = [I - K_A(k)H(k)]P(k/k - 1)$$
(29)

$$P(k/k-1) = \phi(k,k-1)P(k-1/k-1)\phi^{\top}(k,k-1) + G(k,k-1)G(k-1)G^{\top}(k,k-1)$$
(30)

where p(1/k) is the posterior probability of the normal operation of system, given for measurement result at the estimation step k. The rest of the parameters have been already defined throughout last section. If the fault probability changes, the the gain coefficient of the filter is automatically changed. Calculating the integral defined earlier, it gives to the Kalman Filter the ability to adapt to change in operating conditions. If there is a fault in the measurement channel, the value p(1/k) decrases, and the gain coefficient of the filter decreases too consequently. As a result, the correction effect of innovation sequence decreases and the estimated $\hat{X}(k/k)$ approaches to the extrapolation value $\hat{X}(k/k - 1)$.

4. Simulations

During simulations, for testing proposed Optimal KF and Robust KF algorithms, different kind of measurement malfunction scenarios are taken into consideration, such as instantaneous abnormal measurements, continuous bias and measurement noise increment. Throughout next subsections, a briefly explanation about every faulty single case is given.

Table 1: Simulations parameters				
Variable	Value	Units	Definition	
Δt	0.5	S	Discrete time	
R_0	6378150	т	Radius of the Earth	
H	1000	т	Flight altitude	
g	9.81	m/s^2	Gravitational acceleration	
β	0.01	s^{-1}	Term of the correlation period	
β_g	0.005	s^{-1}	Term of the correlation period	
a	0.001	s^{-1}	Term of the correlation period	
σ_w	0.0001	m/s	Vertical speed white noise mean	
$\sigma_{\Delta_{a_{7}}}$	0.00002	m/s^2	Vertical acceleration white noise mean	
σ_{Δ_g}	0.00002	m/s^2	Gravitational acceleration white noise mean	
σ_B	1	т	Bias white noise mean	
σ_{INS}	3	т	Inertial system white noise mean	
σ_{GPS}	2.23	т	GPS white noise mean	

The simulation section of this study is mainly based on the error models of the different navigation sources of the integrated altimeters. The simulations have been run on Matlab using the parameters shown in Table 1. The results shown in this section are basically all the outputs of the state space vector for each integrated system when an Robust Kalman Filter is used.

4.1 Definition of Faulty Cases

4.1.1 Instantaneous Abnormal Measurements

Instantaneous Abnormal Measurements algorithim is formed adding a constant term to the measurement in every 100 iterations in between of 200-400th iterations. These abrupt errors represent constant failure measurements condition at the channel, plotting some vertical and well defined lines on the measurement plots. The abbreviation for this faulty case is IAM.

4.1.2 Continuous Bias at Measurements

Continuous bias term is formed by adding a constant term to the measurement channel between the 200th and 400th iteration. The result of that measurement error is such as a step displacing the measurements upwards in that case, because the constant which has been introduced has positive sign. The abbreviation for this faulty case is CBM.

4.1.3 Measurement Noise Increment

Finally, the last measurement error is based on noise increments. In that third measurement malfunction scenario, measurement fault is characterized by multiplying the variance of the measurement noise by a constant term between 200th and 400th iteration. The effect of this error is an amplification on the noises. The abbreviation for this faulty case is MNI.

4.2 Scenario 1

As commented before, the abbreviations of the faulty cases are: IAM (Instantaneous Abnormal Measurements), CBM (Continuous Bias at Measurements) and MNI (Measurement Noise Increment). Three random experiments are simulated and the results are shown in Tables. In Figure 4 is shown the estimation of the altitude when the channel 1 (Integrated Baro Inertial) is operating in IAM and the channel 2 (Integrated Inertial GPS) in CBM. This is the experiment 1. The measures of the fused system are partially damped and the mean error of these distributions are presented in Table 2.



Figure 4: Results of Scenario 1

Kalman Filter	Mean Error [m]
IBI	5.0381
IIG	18.2586
IBIG	6.9580



4.3 Scenario 2

The next experiment has been done when both channels are performing in MNI faulty case. The results obtained are presented in Figure 5. As can be checked in the Table 3, the mean error values obtained for the IBIG system are much better than the ones calculated for each integrated system separately.



Figure 5: Results of Scenario 2

Table 3: Mean Errors of Scenario 2

Kalman Filter	Mean Error [m]
IBI	15.4731
IIG	13.3740
IBIG	8.2620

4.4 Scenario 3

The third experiment has been performed when the IBI altimeter is operating in CBM mode and the IIG altimeter is measuring in normal conditions. The result is depicted in Figure 6.



Figure 6: Results of Scenario 3

Table 4: Mean Errors of Scenario 3

Kalman Filter	Mean Error [m]
IBI	18.3939
IIG	1.3259
IBIG	13.0546

As shown in the Figure 6, the fused integrated system damps partially the errors introduced by the IBI altimeter, and the mean errors are presented in Table 4.

5. Conclusions

When some random faulty cases are introduced in the measuring system, only the Robust Kalman Filter case is considered. According to the shown Figures, the Fault Tolerant Integrated Baro-Inertial-GPS system damps the errors generated by the other two integrated systems, and in some cases, it gives less errors. This is the most relevant and an essential outcome of this research. As shown in the bibliography, there are similar studies which conclude with the same results. Hence, the present findings can be validated in agreement with those studies. The conclusion of this thesis holds significance, for it can be applied into new integrated systems in the future, providing safer flights.

References

- [1] Biezad, D. J., in *Integrated Navigation and Guidance Systems*. Reston VA: American Institute of Aeronautics and Astronautics, Inc. 1999.
- [2] Dinc, M. and C. Hajiyev, in Integration of Navigation Systems for Autonomous Underwater Vehicles. Journal of Marine Engineering and amp; Technology, 2015, Vol. 14, No. 1, pp. 32 - 43.
- [3] Hajiyev, C., in Adaptive Filtration Algorithm with the Filter Gain Correction Applied to Integrated INS/Radar Altimeter. Proceedings of the Institution of Mechanical Engineers (IMechE), Vol.221, Part G, Journal of Aerospace Engineering, 2007, pp. 847 - 855.
- [4] Hajiyev, C., in *Fault Tolerant Integrated Radar/Inertial Altimeter Based on Nonlinear Robust Adaptive Kalman Filter*. Aerospace Science and Technology, 2012, 17(1): 40-49.
- [5] Hajiyev, C. and R. Saltoglu, in *RKF-Based Fault Tolerant Integrated INS/Radar Altimeter*, Aircraft Engineering and Aerospace Technology: An International Journal, Vol.76, No.1, 2004, pp.38-46.
- [6] Jin, R., Sun, H., Sun, J., Chen, W., Chu, J. Integrated Navigation System for UAVs Based on the Sensor of Polarization.Proc. of the 2016 IEEE International Conference on Mechatronics and Automation, August 7 - 10, Harbin, China, pp.2466-2471.
- [7] Kayton, M. and W.R. Fried, Avionics navigation systems, 2nd edition, New York: John Willey & Sons, Inc, 1997.
- [8] Mañero Contreras, A. and Hajiyev, C., *Integrated Barometric Inertial GPS Altimeter*. Proc. of the International Symposium on Sustainable Aviation, 29 May - 1 June, 2016, Istanbul, TURKEY, ISSA-2016-0221, pp. 386-389.
- [9] Molnar, D.O, Baro-inertial altitude accuracy improvement for GPS monitoring and integration, Proc. of the 50th Annual Meeting-Institute of Navigation, Partnerships for Technology Conversion, Colorado Springs, CO, USA, 1994, pp.129-138.
- [10] R. A. Gray, P.S. Maybeck, in *Integrated GPS/INS/BARO and radar altimeter system for aircraft precision approach landings*, Proc. of the IEEE National Aerospace and Electronics Conference, Part 1, Dayton, KY, USA, 1995.
- [11] Rao, K.D., Integration of GPS and baro-inertial loop aided strapdown INS and radar altimeter, IETE Journal of Research, 43(5), 1997, pp. 383-390.
- [12] Rogers, R.M., in *Applied Mathematics in Integrated Navigation Systems*. Third Edition, Reston, VA, USA: American Institute of Aeronautics and Astronautics, Inc. (AIAA), 2007.
- [13] Sokolovic, V., Dikic, G. and Stancic, R., in Adaptive Error Damping in the Vertical Channel of the Ins/Gps/Baro-Altimeter Integrated Navigation System. Scientific Technical Review, Vol.64, No.2, 2014, pp.14-20.
- [14] Zhukovskiy, A. P. and V. V. Rastorguev, *Complex Radio Navigation and Control Systems of Aircraft*. Moscow: MAI (in Russian), 1998.