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A multi-objective $\mathcal{H}_\infty$ design framewok for helicopter PID control tuning with handling qualities requirements

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Abstract

Helicopters are strongly coupled and naturally unstable plants. This is why stability augmentation systems such as "attitude command attitude hold" (ACAH) are generally implemented to alleviate the pilot workload and reduce the risk of a spatial disorientation accident. These systems must be tuned carefully according to specific **Handling Qualities** requirements, as those specified in the ADS-33 standards.[6] Useful guidelines based on both frequency and time-domain criteria are thus provided to facilitate the design of safe control systems. However, these are not directly compatible with the $\mathcal{H}_\infty$ robust control design framework which is quite relevant for robustness improvement. The core of this paper then consists of a rewriting of the ADS-33 constraints so that robust $\mathcal{H}_\infty$ controllers can be designed by solving a reference model following problem. The main motivation is to introduce robustness requirements in the design of helicopter flight control laws while preserving pilot-in-the-loop handling qualities requirements.

1. Introduction

Piloting a "natural" (without any stability augmentation device) helicopter is a difficult task, because of its natural instability. Fortunately, assistance systems such as autopilots are now commonly available on board, in order to improve the helicopter pilotability. Along each axis (pitch, roll and yaw), such systems implement control laws which must be tuned according to Handling Qualities criteria from the ADS-33 standards.[6] Such a tuning process is generally time-consuming and difficult to automate since the criteria are not in a standard format to be used by modern control techniques. However, based on a work by J.C. Antoniolli et al,[3] which makes connections between the requirements to be fulfilled and reference model matching, it can be observed that the H_∞ design framework combined with poles placement constraints can finally be helpful to systematize the tuning process. Until recently, this observation was yet not so useful since standard H_∞ design techniques only provided non structured high-order controllers whose complexity was not compatible with industrial constraints. One of the major issues when designing helicopter control laws is indeed to enforce a pre-specified PID structure. This is required for many reasons among which security and maintenance play a keyrole. Whatever the chosen design technique, as soon as the controller structure is fixed, a non convex optimization problem is generally obtained. The resolution is then quite difficult without any guarantee of convergence towards a global solution. This was especially true for the $\mathcal{H}_\infty$ control design approach. In addition, combining simultaneously H_∞-based and poles placement constraints further increases the numerical difficulties.

However, thanks to recent advances in nonsmooth optimization techniques (detailed by Apkarian et al[5]), the structured $\mathcal{H}_\infty$ control problem is now efficiently solved in many cases. Moreover, such optimization techniques offer a considerable flexibility which permits to consider not only structured controllers but also to introduce robust and multi-objective constraints. All these key features will be used in the sequel where a new framework for helicopter PID control with handling qualities requirements is proposed.

The remainder of the paper is organized as follows. The handling qualities requirements and their connections with a reference model matching control problem are described in section 2. Next, the multi-objective $\mathcal{H}_\infty$-based design framework is detailed in section 3. The main results are then presented in section 4. Finally, a concluding section ends the paper.

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2. Handling qualities requirements & reference model selection

2.1 Helicopter modeling and linearization

The helicopter flight dynamics based on standard aerodynamics models are now well established[9,11] and both simulation and control design oriented models are easily derived. A comprehensive simulation tool, named HOST[7] (Helicopter Overall Simulation Tool) jointly developed by Eurocopter, DLR and ONERA is available and has been used in this work. With such a tool, linearized models about various trim conditions are also quite easily obtained in the following state-space format:

$$G(s) : \left\{ \begin{array}{l} \dot{X} = AX + BU \\ Y = CX \end{array} \right. \tag{1}$$

where:

- the 9^{th} order state vector $X = [u, v, w, p, q, r, \phi, \theta, \psi]^T$ includes the helicopter velocity components, the rotational speed vector and the standard euler angles,
- $U = [\delta_{col} \;\; \delta_{lat} \;\; \delta_{lon} \;\; \delta_{ped}]^T$ captures the four control inputs (respectively: collective, lateral, longitudinal and pedals inputs).

2.2 Attitude control laws structure

In this paper, the collective control input (δ_{col}) is fixed and one focuses on the attitude control system, which is also referred to as the Attitude Command Attitude Hold (ACAH) system. Its quite standard structure consists of a PID controller on each axis so that only 9 gains have to be tuned. However, because of coupling effects, the tuning process is not trivial and must be performed globally.

$$\left\{ \begin{array}{lclclcl} \delta_{lat} & = & K_{p_\phi}(\phi - \phi_c) & + & K_{i_\phi} \int(\phi - \phi_c) & + & K_{d_\phi} p \\ \delta_{lon} & = & K_{p_\theta}(\theta - \theta_c) & + & K_{i_\theta} \int(\theta - \theta_c) & + & K_{d_\theta} q \\ \delta_{ped} & = & K_{p_\psi}(\psi - \psi_c) & + & K_{i_\psi} \int(\psi - \psi_c) & + & K_{d_\psi} r \end{array} \right. \tag{2}$$

2.3 Handling quality requirements

The above control system must be tuned according to specific handling qualities requirements defined in the ADS-33 norm.[6] In this study, the following three criteria are considered. For each, three levels are defined.

2.3.1 Stability criterion (HQ#1)

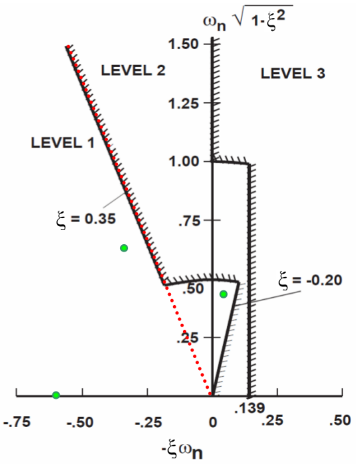

Figure 1: Handling quality requirement #1 - Stability criterion

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This criterion is evaluated on the **linearized** closed-loop plant. As illustrated by Figure 1, it consists of standard closed-loop poles constraints according to the required stability level. In the most demanding case (level 1), a minimum damping $\xi \geq 0.35$ is imposed beyond $0.5\,rad/s$. Note that the constraint is relaxed to $\xi \geq -0.2$ in the low frequency domain which is easily controlled by the human pilot's actions.

2.3.2 Attitude Quickness criterion (HQ#2)

This criterion quantifies the agility of the helicopter in response to **moderate amplitude** step inputs on each axis. It evaluates the capacity to realize safe and quick attitude changes. The computation is based on three parameters whose evaluation requires (possibly nonlinear) time-domain simulations.

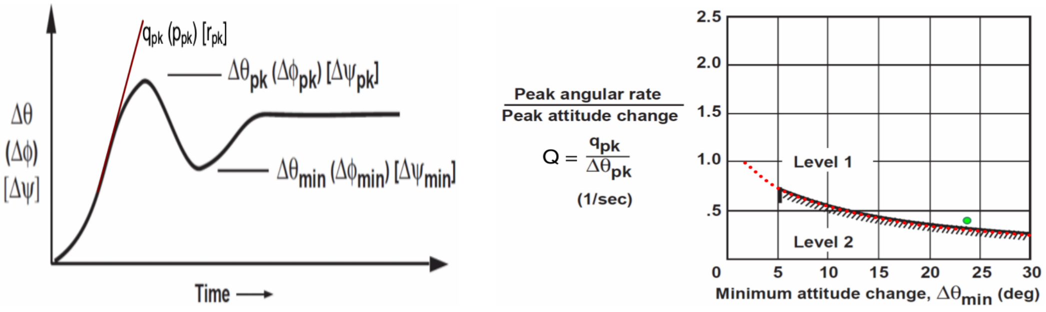

Figure 2: Handling quality requirement #2 - Attitude quickness (moderate amplitude change, agility)

2.3.3 Bandwidth / Phase delay criterion (HQ#3)

The third criterion quantifies the tracking capacity for small amplitude inputs. It is based on the notions of phase bandwidth and phase delay as shown in the left plot of Figure 3. The evaluation of this criterion is thus based on the frequency-domain responses of the closed-loop system, which in the nonlinear case can be evaluated by a frequency-sweeping technique.

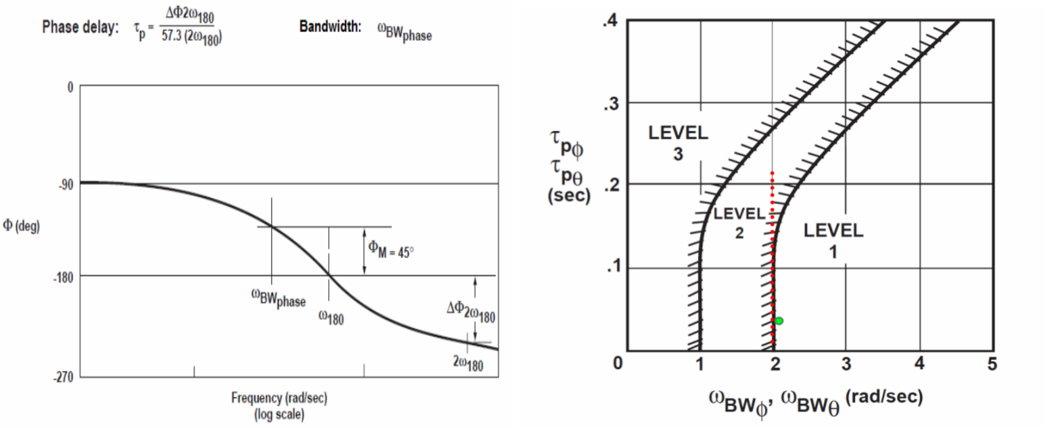

Figure 3: Handling quality requirement #3 - Bandwidth (small amplitude, short term response)

2.3.4 Numerical evaluation and exploitation of the handling qualities criteria

While the evaluation of the first presented criterion (the stability criterion) is straightforward (but restricted to the linear case), the last two ones require more attention and specific tools. Interfaced with HOST,[7] a user-friendly graphical and Matlab-based environment named EagHEL (**E**nvironment for designing and **A**djusting **H**elicoptEr flight control **L**aws) has been developed in Antonioli PhD thesis[1] and will be used in this work.

As is clear from above and was observed by Antoniolli et al,[3] the handling qualities criteria (except for the first one) are not in the appropriate format to be taken into account by "modern" control techniques such as the aforementioned $\mathcal{H}_\infty$ design approach. There is then a real need to adapt this norm in order to make it compatible with the most recent design tools and to take advantage of their versatility and robustness properties. But, thanks to recent contributions,[2–4, 13] close connections appear between the satisfaction of the criteria of the ADS-33 norm and the input/output behavior of the closed-loop model that must remain close to a diagonal reference model:

$$R(s) = \mathrm{diag}(R_\phi(s), R_\theta(s), R_\psi(s)) \tag{3}$$

For each axis, the structure of the model is the same. Thus, for the roll axis, one should have:

$$R_\phi(s) = \frac{1+\tau_{\phi_2}s}{1+\tau_{\phi_1}s}\,\frac{\omega_\phi^2}{s^2+2\xi_\phi\omega_\phi s+\omega_\phi^2} \tag{4}$$

with the additional constraint between τ_{ϕ_1} and τ_{ϕ_2}:

$$\tau_{\phi_2} = \tau_{\phi_1} + \frac{2\xi_\phi}{\omega_\phi} \tag{5}$$

The damping parameters ξ_ϕ, ξ_θ and ξ_ψ for each axis are fixed to values above 0.35 in order to enforce (if possible) the level 1 stability requirement (HQ#1). Next, the pulsations ω_ϕ, ω_θ and ω_ψ are tuned with the help of an HQ chart to enforce the attitude quickness (HQ#2) and bandwidth (HQ#3) requirements. Finally the last three parameters τ_{ϕ_1}, τ_{θ_1}, τ_{ψ_1} have no direct impact on the requirements but will yet slightly influence the response times.

3. Multi-objective $\mathcal{H}_\infty$ design framework

3.1 $\mathcal{H}_\infty$-based reference model matching

From the above observation, the control tuning problem can be re-written as an $\mathcal{H}_\infty$ optimization problem. With the notation introduced in the closed-loop model of Figure 4, the objective is to find the best structured gain K:

$$K = \begin{bmatrix} K_{p_\phi} & 0 & 0 & K_{i_\phi} & 0 & 0 & K_{d_\phi} & 0 & 0 \\ 0 & K_{p_\theta} & 0 & 0 & K_{i_\theta} & 0 & 0 & K_{d_\theta} & 0 \\ 0 & 0 & K_{p_\psi} & 0 & 0 & K_{i_\psi} & 0 & 0 & K_{d_\psi} \end{bmatrix} \tag{6}$$

such that the error z_p between the outputs $y = [\phi,\ \theta,\ \psi]'$ of the closed-loop plant and those (y_r) delivered by the bloc-diagonal reference model $R(s)$ is minimized. Simultaneously, the signal z_u will also be kept as "small" as possible in order to limit the control activity and thus to avoid any stability and performance degradations induced by actuators magnitude and rate constraints.

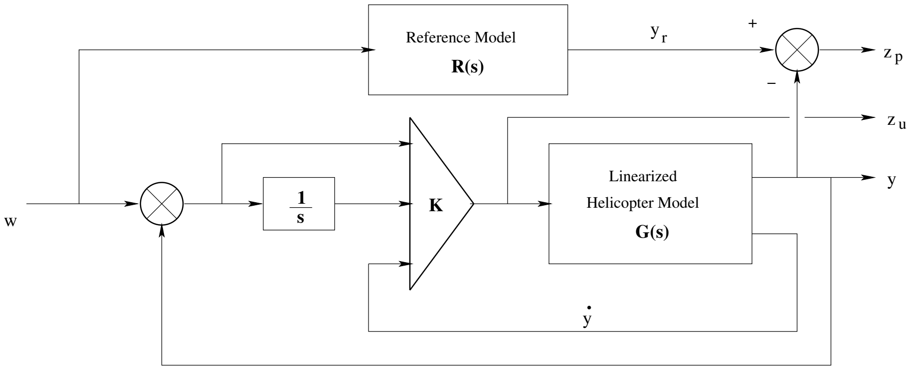

Figure 4: Reference model following design model with a PID-structured controller

In the $\mathcal{H}_\infty$ design framework, the "sizes" of the signals z_p and z_u are evaluated through weighted $\mathcal{H}_\infty$ norms of the transfer functions $\mathcal{T}_{w\to z_p}(s)$ and $\mathcal{T}_{w\to z_u}(s)$ from w to z_p and z_u respectively:

$$\begin{cases} \left\|W_p(s).\mathcal{T}_{w\to z_p}(s)\right\|_\infty = \sup_{\omega\geq 0}\overline{\sigma}(W_p(s).\mathcal{T}_{w\to z_p}(j\omega)) = \sup_{w\in\mathcal{L}_2}\frac{\|\tilde{z}_p\|_2}{\|w\|_2} \\ \left\|W_u(s).\mathcal{T}_{w\to z_u}(s)\right\|_\infty = \sup_{\omega\geq 0}\overline{\sigma}(W_u(s).\mathcal{T}_{w\to z_u}(j\omega)) = \sup_{w\in\mathcal{L}_2}\frac{\|\tilde{z}_u\|_2}{\|w\|_2} \end{cases} \tag{7}$$

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where $\tilde{z}_p$ and $\tilde{z}_u$ denote the "weighted" outputs. The weighting functions $W_p(s)$ and $W_u(s)$ are tuned to ensure a good tracking of the reference model $R(s)$ in the frequency domain of interest with a reasonable control activity. More precisely, $W_p(s)$ is typically a low-pass filter. The objective is indeed to penalize the low-frequency domain, to make sure that the closed-loop plant follows correctly the reference model in the low frequency region $[0\ ,\ \omega_c]$ where ω_c denotes the bandwidth pulsation. Conversely, $W_u(s)$ is a high-pass filter whose objective is to bound the fast variations of the control signal to avoid rate saturations.

As is highlighted above, the reference model $R(s)$ is parameterized to enforce the handling qualities constraints. But in this model, three parameters τ_{ϕ_1}, τ_{θ_1}, τ_{ψ_1} can be freely chosen inside pre-defined intervals. They can thus be considered as additional design parameters and integrated in an extended structured gain $\mathcal{K}$ to be optimized:

$$\mathcal{K} = \text{diag}\left(\begin{bmatrix} \tau_{\phi_1} & 0 & 0 \\ 0 & \tau_{\theta_1} & 0 \\ 0 & 0 & \tau_{\psi_1} \end{bmatrix}, \begin{bmatrix} K_{p_\phi} & 0 & 0 & K_{i_\phi} & 0 & 0 & K_{d_\phi} & 0 & 0 \\ 0 & K_{p_\theta} & 0 & 0 & K_{i_\theta} & 0 & 0 & K_{d_\theta} & 0 \\ 0 & 0 & K_{p_\psi} & 0 & 0 & K_{i_\psi} & 0 & 0 & K_{d_\psi} \end{bmatrix}\right) \tag{8}$$

Formally, the $\mathcal{H}_\infty$ design problem can thus be stated as:

$$\begin{aligned} \hat{\mathcal{K}} &= arg\ \min_{\mathcal{K}} \|W_p(s).\mathcal{T}_{w\to z_p}(s)\|_\infty \\ &\quad s.t.\ \ \|W_u(s).\mathcal{T}_{w\to z_u}(s)\|_\infty \leq c \end{aligned} \tag{9}$$

or also in the usual standard LFT-based format with reference to the notation of Figure 5.

$$\begin{aligned} \hat{\mathcal{K}} &= arg\ \min_{\mathcal{K}} \left\|W_p(s).\mathcal{F}_l\left(P(s),\mathcal{K}\right)_{w\to z_p}\right\|_\infty \\ &\quad s.t.\ \ \left\|W_u(s).\mathcal{F}_l\left(P(s),\mathcal{K}\right)_{w\to z_u}\right\|_\infty \leq c \end{aligned} \tag{10}$$

where the positive constant c is tuned according to the observed control activity.

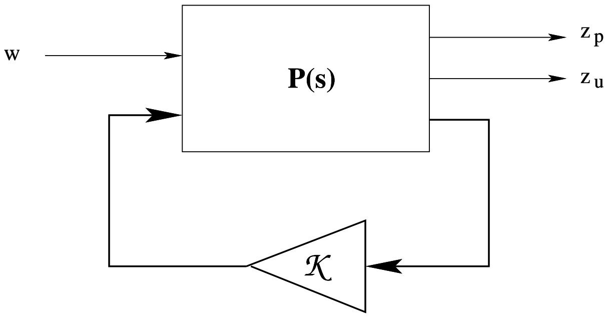

Figure 5: Standard form for H_∞ design

From a numerical viewpoint, the resolution of the above problem is rather difficult since either (9) or (10) describes a nonsmooth and nonconvex optimization problem. The nonsmooth character is easily understood from the definition of the $\mathcal{H}_\infty$ norm. Nonconvexity is implied by the research of a structured controller. In the specific context of systems stabilization and $\mathcal{H}_\infty$ norm minimization, these two difficulties are now efficiently addressed by specialized algorithms[5] which have been implemented in MATLAB^{TM} routines such as *hinstruct* or *systune*. Quite interestingly, these routines, beyond the $\mathcal{H}_\infty$ norm minimization also enable to:

- introduce regional constraints on the closed-loop poles (damping and time-response),
- to consider multi-objective and multi-models design problems

The first possibility will be of high interest to take into account the stability criterion of the ADS-33 norm. The second possibility will be used to improve robustness properties as is further discussed in this paper. However, because of the large variety of problems that can be solved with such routines, their use becomes quite tricky even for $\mathcal{H}_\infty$ control design experts. For this reason, a simple interface named **hinftune** has been developed. All details are provided in the appendix. The central objective of this routine is to provide the user with a standard $\mathcal{H}_\infty$ design tool with a formalism that remains close to the one used by **hinfsyn** which implements the famous "DGKF" initial algorithm.[10] With reference to Figure 5, the input arguments of the proposed interface are as follows:

- **P** : standard interconnection $P(s)$ in a state-space format such that the last inputs/outputs are connected to the controller, while the first are related to the transfer to be minimized,

- **Wi** : input weighting function $W_i(s)$ that may be optionally used to shape the exogenous input signal w
- **Wo** : output weighting function $W_o(s)$ to be used to shape the exogenous output signal z (here, $z = [z_p\ ,\ z_u]'$)
- **ns** : order of the controller to be optimized
- **CSTR** : additional constraints to be satisfied by the closed-loop system and the controller

Remark : *Note that standard algorithms as those implemented in* ***hinfsyn*** *do not allow to constrain the order of the controller or to consider addtional constraints such as poles placement.*

3.2 Robustness improvement by multi-model optimization

In the above approach, robustness issues have not been explicitely taken into account. In practice, in the context of this study, the robustness properties of the initial PID gain must be improved in order to cope with:

- a varying speed of the helicopter from 0 (hover flight) to $20\,ms^{-1}$,
- varying masses according to the payload,
- varying center-of-gravity locations.

To this purpose, a bank of linear models $G_i(s)$ of the helicopter is generated and next, a Linear-Fractional-Representation (LFR) illustrated in Figure 6 is obtained by polynomial interpolation using an advanced technique[12] implemented in the SMAC Toolbox.[8] By this approach, a unique model is to be handled instead of a possibly huge family. Morover, this model can be inserted into the design diagram as shown in Figure 7 and then directly treated by the *systune* routine as an **uncertain model**.

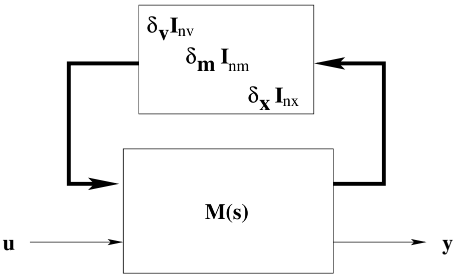

Figure 6: Linear Fractional Representation of the helicopter for varying speed, masses and center-of-gravity locations

The implemented algorithm is based on an iterative procedure based on multi-models $\mathcal{H}_\infty$ optimization and worst-cases search. Thus, at iteration $\#i$, the set of design models is enriched by worst cases for which the constraints are not satisfied and the new set is considered at iteration $\#(i+1)$ until all constraints are met or no further improvement is observed.

4. Application to an EC225 helicopter model

Let us now apply the above PID gain tuning approach to an EC225 helicopter model for which an AC controller (Attitude Command) is to be designed for a future evaluation of its benefits during ship landing. The control law is designed for hover and low speed flight conditions (below $40\,kts$).

4.1 Initial tuning in hovering flight conditions

The design process is first applied in nominal hovering flight conditions. The weight is fixed at $10\,t$ and neutral CoG (Center-of-Gravity) location is assumed. The trimming and linearization tools provided with HOST are used to generate a 9^{th} order linear model as detailed in (1). Next this model is truncated by elimination of u, v and w from the state-vector since one focuses on attitude control. Finally, the reference model to be followed is defined in order to enforce the attitude quickness criterion which appears to be the most demanding. The following values are selected:

Based on the design model of Figure 4, the structured $\mathcal{H}_\infty$ design interface ***hinftune*** (see Appendix A) is then invoked with an additional constraint on the damping of the closed-loop system $\xi_{min} \geq 0.35$ whose objective is to enforce the level 1 stability criterion. The next two criteria achieved with the designed controller are analyzed next.

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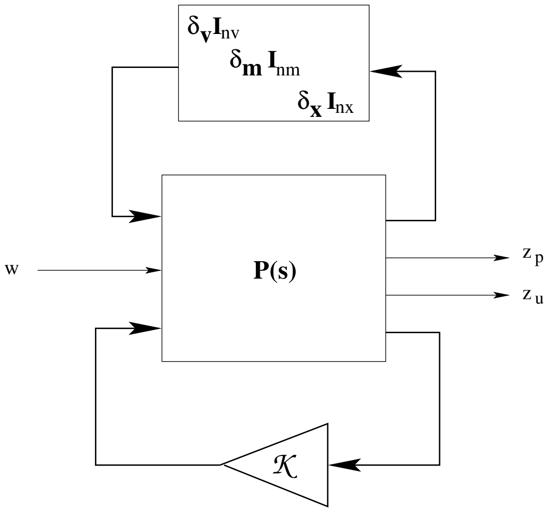
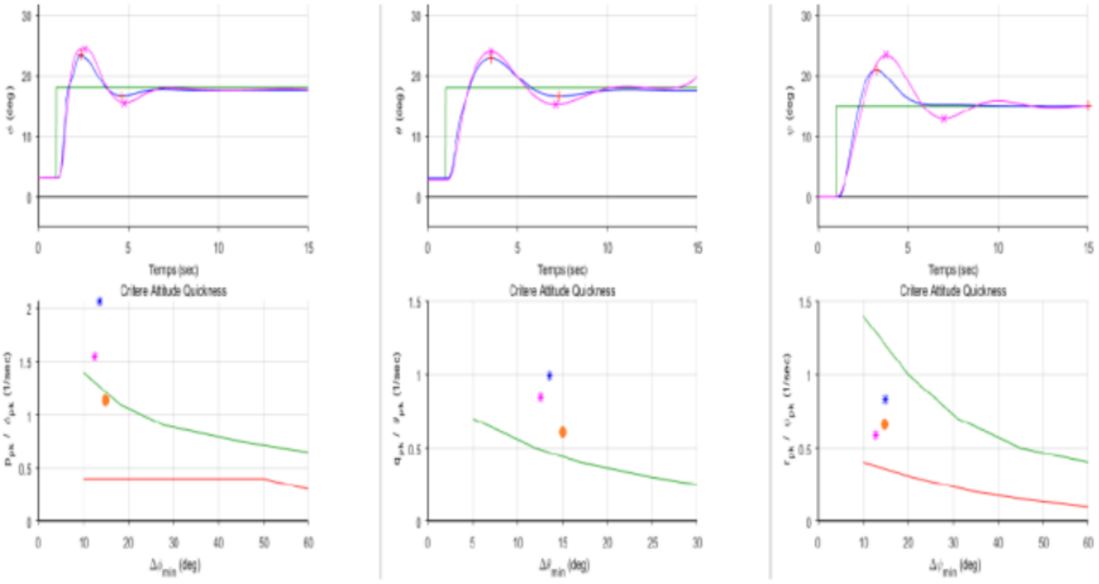


Figure 7: Standard form for parametrically robust H_∞ design

ξ_ϕ	ξ_θ	ξ_ψ	ω_ϕ	ω_θ	ω_ψ
0.6	0.6	0.6	1.9 rad/s	1.1 rad/s	1.1 rad/s

Table 1: Reference model parameters

4.1.1 Attitude quickness criterion evaluation



a) Roll
b) Pitch
c) Yaw

Figure 8: Desired (orange) and realized **Attitude Quickness Criterion** in the nominal case with linear (blue) and nonlinear (magenta) models

As observed in Figure 8 which presents the attitude quickness evaluation using first the linear (blue) and then the nonlinear (magenta) models, the level 1 is achieved for the three axes. The specified objectives (represented by orange circles) have been exceeded.

4.1.2 Bandwith criterion evaluation

For conciseness, the bandwith criterion is grapically displayed in Figure 9 along the pitch axis only. Here again, it is observed that the results (whatever the model used) are in compliance with the handling qualities standards (level 1).

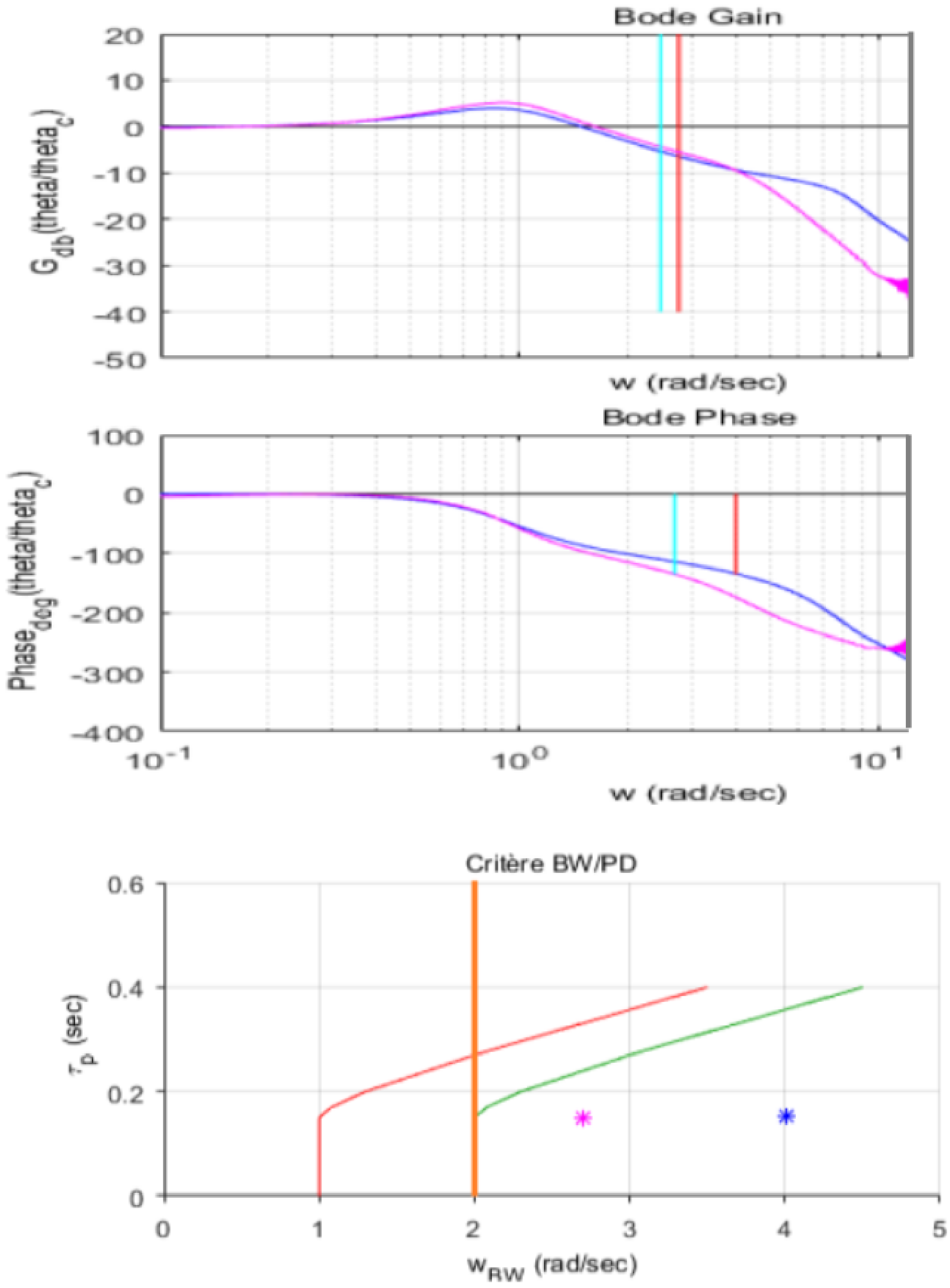


Figure 9: Desired (orange) and realized **Bandwidth Criterion** in the nominal case with linear (blue) and nonlinear (magenta) models

4.1.3 Summary of the results along the three axes

The results (attitude quickness and bandwidth criteria) along the three axes are summarized in Table 2. The graphical results are confirmed and the achieved values (obtained with nonlinear HOST models) are even better that expected for both roll and pitch axes. The only exception is the yaw axis for which both criteria remain 10% below the target but are

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still acceptable. Moreover, by "construction", this approach guarantees the first criterion since the damping constraint is an explicit parameter of the design algorithm.

	Roll target / achieved	**Pitch** target | achieved	**Yaw** target | achieved
Attitude quickness criterion	1.1 | 1.5	0.6 | 0.8	0.7 | 0.6
Bandwidth criterion	3 | 4.1	2.2 | 2.7	2 | 1.8

Table 2: Attitude quickness & Bandwidth criteria along the 3 axes

Therefore, at this stage, an initial controller is rapidly obtained which fulfills the requirements of the hover flight. The next step is to ensure robustness against changing flight conditions.

4.2 Robustness improvement to varying flight conditions

As is discussed in subsection 3.2, the robustness of the above controller is now improved to handle the following variations:

- varying speed of the helicopter from 0 (hover flight) to 40 knots,
- varying mass between 9000 kg and 11000 kg,
- varying center-of-gravity locations.

A set of linear models over 125 flight cases representing various weights, CG and speed configurations is first generated. Next the SMAC toolbox[8] is used to compute a Linear-Fractional- Representation (LFR) by polynomial interpolation, with a precision of less than 1%. As already clarified, this approach presents the advantage of manipulating a unique model as illustrated by Figure 6 instead of a large family.

With the same tuning of the reference model and weighting functions, the control design procedure is applied to this uncertain model. After 5 iterations, a new structured controller is obtained and a selection of worst case configurations is isolated.

Nonlinear HOST simulations are performed with this robust PID controller on the selection of worst case configurations. The maneuver consists of a commanded ramp input (with magnitude $-13\,deg$) on the pitch axis which induces significant speed variations. It is noted (see Figure 10) that in all cases, the reference input is correctly tracked whatever the flight conditions despite poorly damped oscillations on roll and yaw axes in extreme cases.

Next, the most demanding handling quality requirement, the attitude quickness, is evaluated on the same selection of worst case configurations with the same nonlinear model. The results are presented in Figure 11. As for the nominal case, it can be observed that the objectives are generally exceeded (for roll and yaw axes) or at least met (along the pitch axis).

5. Conclusion

In the framework of helicopter structured control laws development a novel approach has been developed in order to account for handling quality requirements from the early phase of the control law design. Thanks to the proposed methodology, an initial controller is rapidly obtained and its robustness properties are next easily improved with the help of a multi-model design approach. The obtained results are quite encouraging despite some deviations between the linear and nonlinear evaluations. This should be further improved in the future with the use of a more accurate linear models including the rotor states to model flapping.

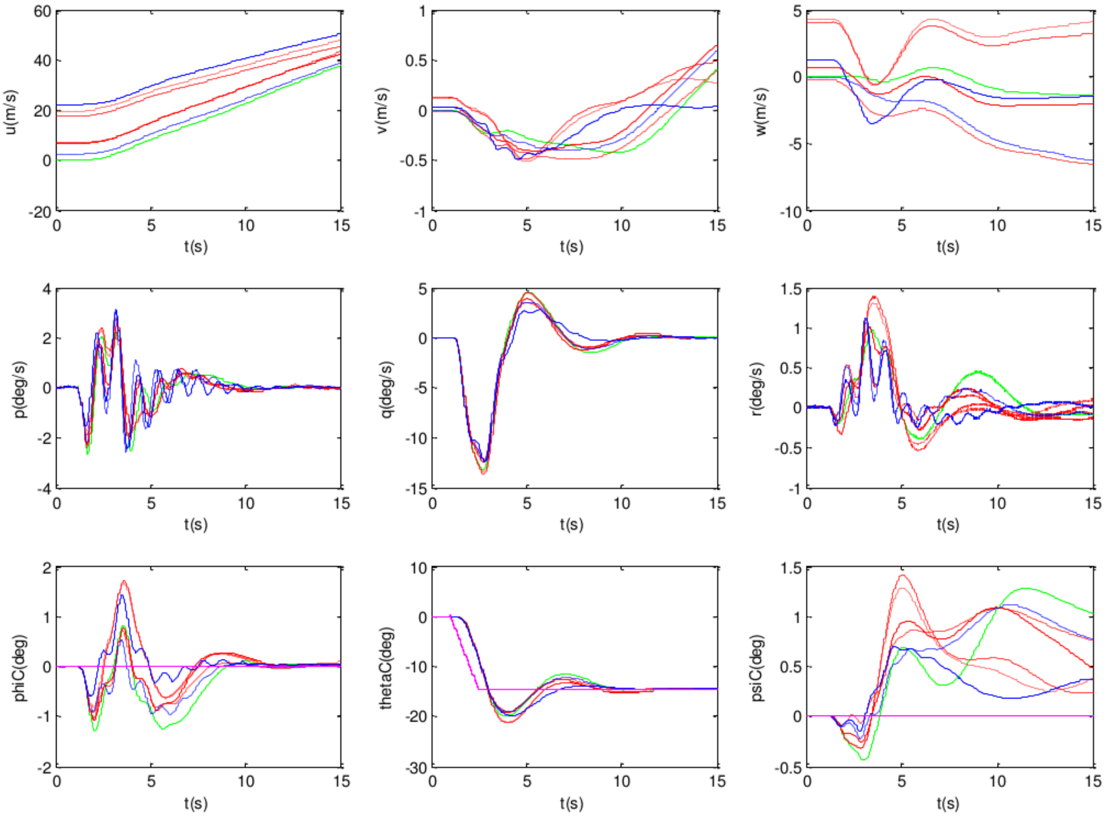

Figure 10: Nonlinear HOST simulations for a selection of worst-cases configurations

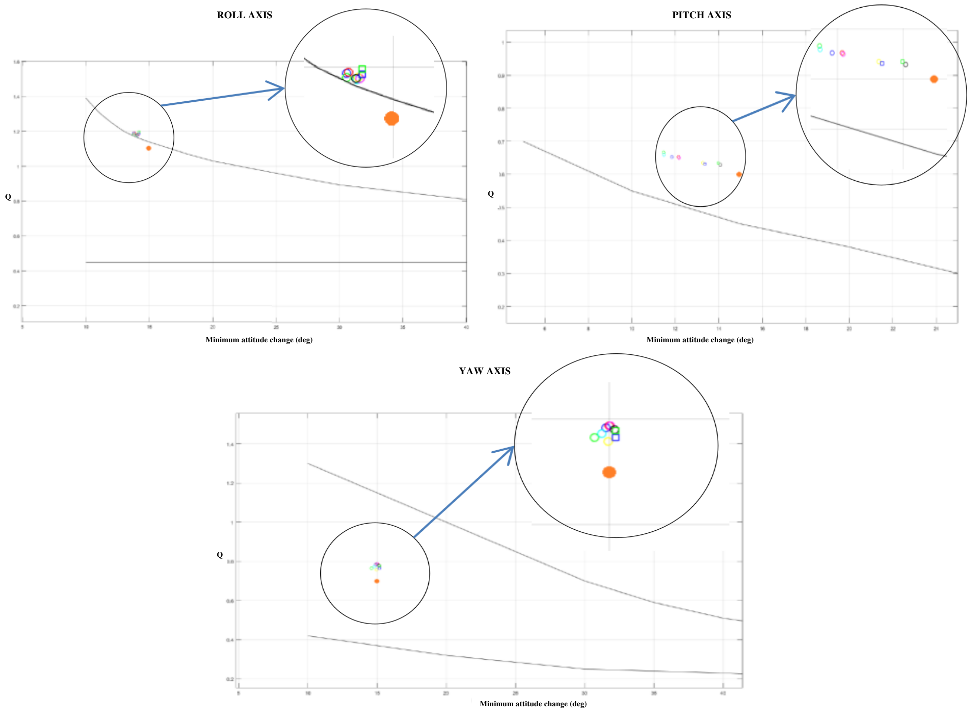

Figure 11: Achieved Attitude Quickness with Nonlinear HOST simulations for worst cases flight configurations

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A. HINFTUNE : A standard H_∞ design interface to "systune"

```
function [K,CL,CLW,gamma] = hinftune(P,Wi,Wo,ns,CSTR)
%
% [K,CL,CLW,gamma] = HINFTUNE(P,Wi,Wo,ns,CSTR)
%
%           -------   w    --------   z     -------
%  wf ---> | Wi(s) |----->|        |-----> | Wo(s) |--->  zf
%           -------       |  P(s)  |        -------
%  u  ------------------->|        |------------------->  y
%                          --------
%
%       u = K(s) * y
%       K(s) = argmin || Wo(s)* Fl(P,K) * Wi(s) ||_inf
%
% Fixed-order Hinfinity design with closed-loop poles constraints.
% This function is based on SYSTUNE and HINFSTRUCT routines.
%
% Given an LTI plant P in ss format with state-space equations:
%  dx/dt = A  * x + B1  * w + B2  * u
%      z = C1 * x + D11 * w + D12 * u
%            y = C2 * x + D21 * w + D22 * u
% HINFTUNE computes an output-feedback control u = K(s)*y that:
%  * minimizes the Hinfinity norm of the weighted transfer CLW:
%       CLW= Wo*CL*Wi with CL = Fl(P(s),K(s))
%  and, optionally:
%  * places the poles of the unweighted closed-loop CL in a defined sector
%  * keeps the poles of the controller in a prespecified region (dynamic controller)
%  * keeps the controller gains in specified intervals (static controller)
%
% Inputs:
%  P        LTI plant (ss format)
%  Wi       Input weighting function (size w)
%  Wo       Output weighting function (size z)
%  ns       Order of the optimized controller K (set to 0 for static gain)
%  CSTR     Structured variable with constraints in various fields:
%           CSTR.CLsec : 1x3 vector listing closed-loop poles constraints:
%             * CSTR.CLsec(1) : minimum decay rate (xi*wn)
%             * CSTR.CLsec(2) : minimum damping (xi)
%             * CSTR.CLsec(3) : maximum natural frequency (wn)
%           CSTR.Ksec : 1x3 vector listing controller poles constraints:
%             * CSTR.Ksec(1) : minimum decay rate
%             * CSTR.Ksec(2) : minimum damping
%             * CSTR.Ksec(3) : maximum natural frequency
%           CSTR.Kmin : no-by-ni matrix of lower-bounds for the gain K
%           CSTR.Kmax : no-by-ni matrix of upper-bounds for the gain K
%
%           Remark : The field Ksec is active for dynamic controllers only
%                    The fiekds Kmin, Kmax are active for static controllers only.
%
% Outputs:
%  K        Output-feedback controller
%  CL       Unweighted Closed-Loop plant
%  CLW      Weighted Closed-Loop plant
%  gamma    Best-achieved Hinfinity norm
%
% See also  HINFSYN HINFLMI HINFMIX HINFSTRUCT SYSTUNE
```

```
if nargin<5
    CSTR=[];
    if nargin<4
        ns=0;
    end;
end;
CLsec=[]; Ksec=[]; Kmin=[]; Kmax=[];
if isfield(CSTR,'CLsec') CLsec=CSTR.CLsec; end;
if isfield(CSTR,'Ksec')  Ksec=CSTR.Ksec;   end;
if isfield(CSTR,'Kmin')  Kmin=CSTR.Kmin;   end;
if isfield(CSTR,'Kmax')  Kmax=CSTR.Kmax;   end;

[p,m]=size(P); [pi,mi]=size(Wi); [po,mo]=size(Wo); ni=p-mo; no=m-pi;
HardReqCLsec=[];
if ~isempty(CLsec)
  HardReqCLsec=TuningGoal.Poles(CLsec(1),CLsec(2),CLsec(3));
end;
HardReqKsec=[];
if ns>0
  K0=tunableSS('K0',ns,no,ni);
  if ~isempty(Ksec)
    HardReqKsec=TuningGoal.ControllerPoles('K0',Ksec(1),Ksec(2),Ksec(3));
  end;
else
  K0=tunableGain('K0',no,ni);
  if ~isempty(Kmin)
     K0.Gain.Minimum=Kmin;
  end;
  if ~isempty(Kmax)
     K0.Gain.Maximum=Kmax;
  end;
end;

% closed-loop parameterized ss object (gss format) : CL = Fl(P,K)
CL0=lft(P,K0); % unweighted parametric closed-loop (genss)
CL0.InputName='w'; CL0.OutputName='z';

opt = systuneOptions('RandomStart',3);
SoftReq=TuningGoal.WeightedGain('w','z',Wo,Wi);
HardReq=[HardReqCLsec HardReqKsec];

if isempty(HardReq)
  [CL,fSoft] = systune(CL0,SoftReq,opt);
else
  [CL,fSoft,~] = systune(CL0,SoftReq,HardReq,opt);
end;

K=ss(CL.Blocks.K0); CL=ss(CL);  CLW=Wo*CL*Wi; gamma=fSoft;
```

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