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Dimensional and scale analysis applied to the preliminary assessment of the environment criticality of large constellations in LEO

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Abstract

Several analytical expressions, based on reasonable simplifying assumptions, were developed for the assessment of the environment criticality of large constellations and huge numbers of small satellites in low Earth orbit. They can provide preliminary quantitative answers to difficult questions, with no need of complex models and computations. Moreover, a specific figure of merit was introduced for gauging the environment criticality of new large constellations: the collision rate percentage increase. Because several systems might be operated at the same time, an alert threshold of no more than 10% per single constellation seemed a wise suggestion. Finally, several quantitative examples and the associated results were presented and discussed.

1. Introduction

The dramatic surge of small satellite launches during the last few years and the current plans envisaging the deployment of very large constellations in low Earth orbit (LEO), some consisting of thousands of spacecraft, raised a growing concern in the orbital debris mitigation community regarding the long-term sustainability of the near-Earth space environment with the present-day guidelines recommended by the Inter-Agency Space Debris Coordination Committee (IADC) [1]. Assessing the impact of the proliferation of small satellites and large constellations has therefore been identified as a modeling priority, in order to evaluate if additional, and more stringent, mitigation measures might be needed to preserve the long-term access and utilization of the LEO protected region.

The task is quite complex, because traffic models and constellation deployment plans are, of course, subject to sudden changes, driven by economic and technical issues, and are anyway very uncertain beyond twenty years in the future. The same applies to technological developments and breakthroughs, which could change completely the nature of space systems, as in part is already happening with the impetuous flourishing of mini, micro and nano-satellites, and the widening scope and effectiveness of their applications. The purpose of this paper is to address the problem avoiding the detailed simulations of complex and highly speculative scenarios, concentrating instead on a simplified analysis, able to provide some preliminary clues and insights, perhaps useful for steering further refinements with more complex and time expensive tools and approaches.

1.1 Long-term simulations of the debris environment

Detailed simulations of the debris environment long-term evolution including five constellations in LEO, one of which consisting of 324 satellites, with a mass of 1400 kg each and a cross-sectional area of 12 m^2, placed at the altitude of 1375 km in orbits with an inclination of 85°, were completed in 2000 (Tables 1 and 2). In that case, it was shown that the implementation of the IADC mitigation guidelines, including the post-mission disposal of spacecraft in orbits with a residual lifetime < 25 years ("25-year rule"), would have been sufficient to guarantee the near stability of the LEO debris population ≥ 10 cm over one century, with an asymptotic collision rate in between 0.22 and 0.24 $year^{-1}$, and an expected cumulative number of collisions, among objects ≥ 10 cm, around 23 in 100 years [2][3][4]. Moreover, the results obtained did not change significantly either by enforcing the immediate spacecraft de-orbiting at the end-of-life, or relaxing the residual lifetime of the post-mission disposal orbits to < 50 years. The overall effect was a change by ± 3% in the slowly increasing number of objects after 100 years (the growth in LEO

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was approximately 8% in the 85 years following the implementation of the 25-year rule), by ± 8% in the asymptotic collision rate, and by ± 9% in the expected cumulative number of collisions among objects ≥ 10 cm [2][3].

Table 1: Orbital debris long-term evolution scenarios simulated in the study completed in 2000

Scenario	Description
Business as usual	The reference traffic scenario included 80 launches per year, 5 constellations and 5.5 explosions per year (4.25 low intensity, 1.25 high intensity)
Explosion prevention	Stop of explosions in orbit after 2010
Suppression of mission related objects	In addition to the explosion prevention after 2010, mission-related objects were eliminated after 2005
Full mitigation	In addition to explosion prevention and mission-related objects suppression, upper stages were immediately de-orbited after 2005 and satellites were immediately de-orbited at the end-of-life after 2015 (constellations included)
Full mitigation with re-orbiting (REO_DEO_0)	The same as full mitigation, but the satellites were managed as follows: a) End-of-life re-orbiting of geostationary spacecraft after 2015 b) End-of-life re-orbiting, in the super-LEO regime, of LEO spacecraft above 1400 km, after 2015 c) Immediate end-of-life de-orbiting of satellites below 1400 km after 2015
Full mitigation with re-orbiting (REO_DEO_25)	The same as REO_DEO_0, but the satellites below 1400 km were de-orbited, after 2015, in disposal orbits with residual lifetimes of 25 years
Full mitigation with re-orbiting (REO_DEO_50)	The same as REO_DEO_0, but the satellites below 1400 km were de-orbited, after 2015, in disposal orbits with residual lifetimes of 50 years

Table 2: Configuration of the satellite constellations simulated in 2000

System	Year of launch	Altitude [km]	Inclination [deg]	Satellites in orbit	Spacecraft average area [m^2]	Spacecraft mass [kg]
1	2002	1375	85	324	12	1400
2	1998	780	86	72	9	700
3	1999	1414	52	56	10	450
4	1998	775	45	28	9.6	42
5	2002	1457	55	80	12	800

More recent simulations, including a constellation of 1080 satellites, each with a mass of 200 kg and an average cross-section of 1 m^2, placed at 1100 km of altitude and at an inclination of 85°, were carried out by four European groups [5]. The new results obtained confirmed the main finding of the study completed at the beginning of 2000, i.e. that a full adherence to the IADC mitigation guidelines and the 25-year rule would be able to maintain under control and basically stabilize over the long-term the debris environment in LEO. This would still be possible with a limited post-mission disposal failure rate of 10%, but higher failure rates, as those currently recorded in LEO (50-60%), would lead to a substantial increase of the debris population [5].

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The couple of studies reviewed considered only one large LEO constellation each, placed in high near-polar orbits, where atmospheric drag is basically ineffective over many decades. The first one (2000 study) consisted of 324 satellites with a total average cross-section of 3888 m^2 and a total mass of 453 600 kg, while the second one (2016 study) consisted of 1080 satellites with a total average cross-section of 1080 m^2 and a total mass of 216 000 kg. Therefore, from an environmental impact point of view, they were roughly comparable. However, diverse large constellations are currently planned [6], some consisting of several thousands of satellites, and cubesats launched in bunches of various tens at a time are becoming increasingly popular in LEO.

This means rapidly changing traffic projections and reference scenarios, even in the short-term, to which suddenly adapt the analysis tools for evaluating if the adopted mitigation guidelines are still effective and sufficient, or something new is instead needed. But, unfortunately, the setup and running of detailed long-term evolution scenarios is complex, time consuming and affected anyway by considerable uncertainties [7], so more simple and flexible approaches would be desirable for a preliminary assessment of the impact of new large constellation proposals, or of the changing pattern of hundreds of cubesat launches.

1.2 Fermi estimation approach

Among many other things, like week nuclear interaction, slow neutrons, statistics of the particles obeying the Pauli Exclusion Principle, nuclear reactors and so on, the physicist Enrico Fermi was renowned for his legendary capability to attain good approximate results of complex problems carrying out back-of-the-envelope calculations with scant data. As recalled by the physicist Philip Morrison, «Fermi delighted to think up and at once to discuss and to answer questions which drew upon deep understanding of the world, upon everyday experience, and upon the ability to make rough approximations, inspired guesses, and statistical estimates from very little data» [8].

In many fields, like physics, engineering or economics, the accurate solution of problems often requests a tremendous amount of intricate modeling and demanding numerical calculations, which usually need long development times, a lot of qualified labor, a meticulous error checking and an exhausting debugging to be brought to fruition. It may be then frequently worthwhile to first deal with such complicated problems by introducing reasonable simplifications and approximations, scale and dimensional analysis, educated guesses about relevant variables and their variation range, and a subdivision of the problem in several pieces, which can be therefore estimated independently and factored in at the end [9].

Such "Fermi estimate" is generally able to provide at least an order of magnitude assessment, but often may result off only by a factor 2 or less, depending on the number of independent factors involved and on their accuracy. If no significant bias is put in the process, the unbiased errors affecting the individual factors tend to compensate each other, and if the various terms are estimated reasonably well, the final result may be quite more accurate than originally expected.

Even providing a quantitative answer to an intricate question with just the right order of magnitude makes this approach extremely valuable for various reasons. The preliminary answer obtained with relatively simple and easy to check calculations represents, in fact, a good guide to set up more refined and detailed estimates, setting yardsticks compared to which significant errors present in the more complicated methods can be early identified and eventually removed. Moreover, the simpler problem formulation helps a lot in gaining a deeper insight on the relevant aspects involved, driving the choice of the most appropriate models and numerical methods to attack the question in a more rigorous and accurate way. And finally, yet importantly, in many situations a rough estimate within a factor of 10 or 20 is more than sufficient to make a meaningful and useful decision.

For these reasons, the Fermi estimation approach was adopted in this paper for a preliminary assessment and sensitivity analysis of the environmental impact of small satellites and large constellations in LEO.

1.3 Simplifying assumptions

A set of simplifying assumptions was defined, with the goal of maintaining anyway the most important aspects of the problem with no loss of generality. It should also be remarked that some of them are often implicitly adopted as well by much more complicated and detailed long-term evolution models. The simplifying assumptions applied in this analysis were the following:

1. All the objects were modeled as spheres, so the mathematical relations linking the cross-sectional area A, the diameter d and the radius r were those valid for a three-dimensional sphere and a bi-dimensional circle;
2. The orbital debris background in LEO was approximated by two populations of identical objects, assuming "average" values applicable to the actual population of debris larger than 10 cm: intact objects, with radius $r_I = 1.9$ m and mass $M_I = 1000$ kg [10][11][12][13], and breakup fragments plus mission related objects, with radius $r_D = 10$ cm and mass $M_D = 1.2$ kg [2][10][14];

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3. In a given volume of space V, the average collision rate CR was computed with a “particle-in-a-box” approach, even though the main characteristics of the orbital motion were taken into account for the definition of the interaction volume V;
4. A collision occurred when the distance between the centers of the spherical objects involved in the close approach was smaller than the sum of the respective radii;
5. The time needed to deploy the new large constellations, and the operational lifetimes of their satellites, were considered short compared with the time required for changing the orbital debris background in a substantial way;
6. The cross-sectional area A_0 and mass M_0 of the satellites belonging to the new large constellations, and the corresponding values A_{CF} and M_{CF} for the fragments generated by new on-orbit collisions, were expressed according to the classical relationships [15]:

$$M_0[\mathrm{kg}] = \begin{cases} 62.013 A_0^{\,1.13} & A_0 \geq 8.04 \times 10^{-5}\,\mathrm{m}^2 \\ 2030.33 A_0^{\,1.5} & A_0 < 8.04 \times 10^{-5}\,\mathrm{m}^2 \end{cases} \qquad \textbf{(1)}$$

7. The current NASA breakup model was adopted for estimating the expected number of fragments N_{CF}, with characteristic length L_{c} or greater, generated by a catastrophic on-orbit collision involving a couple of objects with total mass M [16][17]:

$$N_{\mathrm{CF}}(L_{\mathrm{c}}) = 0.1 M[\mathrm{kg}]^{0.75} L_{\mathrm{c}}[\mathrm{m}]^{-1.71} \qquad \textbf{(2)}$$

8. L_{c} was set equivalent to the diameter of a corresponding spherical particle;
9. A specific energy of 40 000 J/kg in the center of mass of the colliding objects was needed in order to cause the complete (catastrophic) breakup of the orbiting bodies involved [18];
10. An average relative velocity $V_{\mathrm{Rel}} = 10$ km/s was considered for the objects resident in LEO [19][20].

2. Small vs. big satellites

Let us start with an ideal region of empty space around the Earth, putting new (constellation) satellites in it with nearly circular orbits at the same average altitude and with the same inclination. The volume of space V crossed by the satellites over a sufficiently long amount of time will have the shape of a toroidal shell, with a latitudinal excursion determined by the orbital inclination and a thickness around the mean altitude determined by the (small) orbital eccentricities and the radius vector variations induced by the perturbations, in particular the odd zonal terms of the geopotential. Anyway, the exact shape of V is not relevant for the following discussion: the only thing that really matters is that V represents, by averaging over a sufficiently long amount of time, the collisional interaction volume for the satellites placed in it.

Having set the stage, the first question was: from a debris mitigation point of view, small satellites should be preferred to big satellites, or not? In order to provide a scale and dimensional answer to this question, let us consider a set of N_0 uncontrolled satellites in V. They may be either operational spacecraft without maneuvering and collisional avoidance capabilities, articles failed during the operational phase or at the beginning of the end-of-life disposal phase, or just dead objects abandoned there. From the orbital debris perspective, the distinction is not important. With the goal of carrying out a parametric, small vs. big, analysis, a reference (“big”) satellite with mass $M_{\mathrm{R}} = 1000$ kg was introduced and a “satellite substitution ratio” N_0/N_{R} was defined as follows:

$$\frac{N_0}{N_{\mathrm{R}}} = \left(\frac{M_{\mathrm{R}}}{M_0}\right)^k \qquad \text{with } k \geq 0 \qquad \textbf{(3)}$$

where N_{R} represented the number of reference (“big”) satellites to be replaced by N_0 smaller satellites. The substitution ratio was driven by the positive constant k, with the following meanings:

- $k = 0 \Rightarrow N_0 = N_{\mathrm{R}}$, i.e. the replacement one by one of the big satellites with smaller ones;
- $0 < k < 1 \Rightarrow$ the replacement of the big satellites with a larger number of smaller ones, having however a total mass lesser than that of the big satellites;
- $k = 1 \Rightarrow$ the conservation of the total mass in space $M_{\mathrm{T}} = M_{\mathrm{R}} N_{\mathrm{R}} = M_0 N_0$;

- $k > 1 \Rightarrow$ the replacement of the big satellites with an even larger number of smaller ones, having a total mass greater than that of the big satellites.

As shown elsewhere [2][10], the average collision rate CR_{0-0} among a set of identical uncontrolled satellites, with radius r_0 and cross-sectional area A_0, interacting in a volume V, can be expressed as:

$$CR_{0-0} = 2\pi r_0^2 \rho_0 V_{\mathrm{Rel}} \left(\rho_0 V - 1\right) \tag{4}$$

where ρ_0 is the object spatial density N_0/V. Taking into account Eq. (3) and other obvious substitutions, Eq. (4) becomes:

$$CR_{0-0} = \frac{2A_0 V_{\mathrm{Rel}}}{V} N_{\mathrm{R}} \left(\frac{M_R}{M_0}\right)^k \left[N_{\mathrm{R}} \left(\frac{M_R}{M_0}\right)^k - 1\right] \tag{5}$$

A further simplification can be obtained by taking into account that if $N_0 >> 1$ then $N_0 (N_0 - 1) \approx N_0^2$, with an overestimation of the final result by 10% for $N_0 = 10$, by 5% for $N_0 = 20$, and by just 1% for $N_0 = 100$. Moreover, the cross-sectional area A_0 can be expressed as a function of the mass M_0 using Eq. (1). Eq. (5) can be therefore rewritten, in SI units, as follows:

$$CR_{0-0} \approx 5.185 \times 10^{-2} \frac{V_{\mathrm{Rel}}}{V} N_{\mathrm{R}}^2 M_{\mathrm{R}}^{2k} M_0^{1/1.13-2k} \approx \frac{5.185 \times 10^2}{V} N_{\mathrm{R}}^2 M_{\mathrm{R}}^{2k} M_0^{0.885-2k} \tag{6}$$

It should be remarked that for

$$k \approx \frac{0.885}{2} = 0.4425 \tag{7}$$

i.e. for a satellite substitution ratio

$$\frac{N_0}{N_{\mathrm{R}}} = \left(\frac{M_{\mathrm{R}}}{M_0}\right)^{0.4425} \tag{8}$$

corresponding to an increase by a factor 2.77 in the number N_0 of satellites for each decrease of the individual satellite mass M_0 by one order of magnitude, the collision rate CR_{0-0} would remain basically the same, irrespective of M_0. More generally:

- $0 \leq k < 0.4425 \Rightarrow CR_{0-0}$ would decrease with lower M_0;
- $k \approx 0.4425 \Rightarrow CR_{0-0}$ would be independent of M_0;
- $k > 0.4425 \Rightarrow CR_{0-0}$ would increase with lower M_0.

Consequently, for example, if 100 satellites of 1000 kg ($M_{\mathrm{T}} = 100\,000$ kg) were replaced by 277 satellites of 100 kg ($M_{\mathrm{T}} = 27\,700$ kg), the mutual collision rate would remain basically the same. A collision rate reduction would instead occur with 200 satellites ($M_{\mathrm{T}} = 20\,000$ kg), while 400 satellites ($M_{\mathrm{T}} = 40\,000$ kg) would cause a collision rate increase.

Looking at the expected average rate of production of new collisional fragments $\dot{N}_{\mathrm{CF}}(L_{\mathrm{c}})$ with a certain characteristic length L_{c} (e.g. 10 cm), or greater, it may be obtained, in SI units, by multiplying Eq. (6) with Eq. (2):

$$\dot{N}_{\mathrm{CF}}(L_{\mathrm{c}}) = CR_{0-0} N_{\mathrm{CF}}(L_{\mathrm{c}}) \approx \frac{5.185 \times 10^2}{V} N_{\mathrm{R}}^2 M_{\mathrm{R}}^{2k} M_0^{0.885-2k} 0.1(2M_0)^{0.75} L_{\mathrm{c}}^{-1.71} \approx \frac{87.20}{V} N_{\mathrm{R}}^2 M_{\mathrm{R}}^{2k} L_{\mathrm{c}}^{-1.71} M_0^{1.635-2k} \tag{9}$$

Again, it should be remarked that for

$$k \approx \frac{1.635}{2} = 0.8175 \tag{10}$$

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i.e. for a satellite substitution ratio

$$\frac{N_0}{N_\mathrm{R}} = \left(\frac{M_\mathrm{R}}{M_0}\right)^{0.8175} \tag{11}$$

corresponding to an increase by a factor 6.57 in the number N_0 of satellites for each decrease of the individual satellite mass M_0 by one order of magnitude, the average production rate of new collisional fragments $\dot{N}_\mathrm{CF}(L_\mathrm{c})$ with a certain characteristic length L_c, or greater, would remain basically the same, irrespective of M_0. More generally:

- $0 \le k < 0.8175 \Rightarrow \dot{N}_\mathrm{CF}(L_\mathrm{c})$ would decrease with lower M_0;
- $k \approx 0.8175 \Rightarrow \dot{N}_\mathrm{CF}(L_\mathrm{c})$ would be independent of M_0;
- $k > 0.8175 \Rightarrow \dot{N}_\mathrm{CF}(L_\mathrm{c})$ would increase with lower M_0.

Consequently, for example, if 100 satellites of 1000 kg ($M_\mathrm{T} = 100\,000$ kg) were replaced by 657 satellites of 100 kg ($M_\mathrm{T} = 65\,700$ kg), the average production rate of new collision fragments $\ge L_\mathrm{c}$ would remain nearly the same. A production rate reduction would instead occur with 500 satellites ($M_\mathrm{T} = 50\,000$ kg), while 800 satellites ($M_\mathrm{T} = 80\,000$ kg) would cause a production rate increase. Therefore, looking at the production of new collisional debris $\ge L_\mathrm{c}$, more small satellites would be needed to compensate large satellites, compared with the collision rate, due to the important role played by the fragmenting mass in Eq. (2). However, less massive satellites are vulnerable to less massive impacting debris, so if the attention is focused on the average rate of production of new collisional fragments able to induce the catastrophic breakup of the parent satellites, Eq. (9) is not applicable anymore.
Assuming a specific energy of 40 000 J/kg in the center of mass of the colliding objects, in order to cause the complete (catastrophic) breakup of the orbiting bodies involved, the number of catastrophic "projectiles" $N_\mathrm{CF}(d_\mathrm{cat})$ generated by a collision between two satellites of mass M_0 is listed in Table 3. As it can be seen, the number distribution is basically flat over the four orders of magnitude of interest, not much depending on the mass M_0, so the following average value can be adopted in the range considered:

$$N_\mathrm{CF}(d_\mathrm{cat}) \approx 650 \tag{12}$$

Table 3: Number of catastrophic "projectiles" generated by
a collision between two satellites of mass M_0

M_0 [kg]	**Mass of the catastrophic projectile M_cat [kg]**	**Area of the catastrophic projectile A_cat [m²]**	**Diameter of the catastrophic projectile d_cat [m]**	**Number of catastrophic projectiles generated by the collision of two satellites with mass M_0**
1	0.0008	5.375×10^{-5}	8.272×10^{-3}	612
10	0.008	3.615×10^{-4}	2.145×10^{-2}	674
100	0.08	2.773×10^{-3}	5.942×10^{-2}	664
1000	0.8	2.128×10^{-2}	0.165	654

The substitution of Eq. (12) in Eq. (9) then led to the expression, in SI units, for the expected average rate of production of new collisional fragments $\dot{N}_\mathrm{CF}(d_\mathrm{cat})$ able to induce the catastrophic breakup of the parent satellites of mass M_0:

$$\dot{N}_\mathrm{CF}(d_\mathrm{cat}) = CR_{0-0} N_\mathrm{CF}(d_\mathrm{cat}) \approx \frac{3.370 \times 10^5}{V} N_\mathrm{R}^2 M_\mathrm{R}^{2k} M_0^{0.885-2k} \tag{13}$$

where the same dependency on M_0 of the collision rate CR_{0-0}, given in Eq. (6), was recovered.

2.1 Conservation of the mass in orbit

An interesting boundary case is represented by a small vs. big satellite replacement in which the total mass in orbit

M_T is conserved. This corresponds to $k = 1$ in Eq. (3), leading to the following expressions:

$$CR_{0-0} \approx \frac{5.185\times 10^2}{V} M_T^2 M_0^{-1.115} \tag{14}$$

$$\dot{N}_{CF}(L_c) \approx \frac{87.20}{V} M_T^2 L_c^{-1.71} M_0^{-0.365} \tag{15}$$

$$\dot{N}_{CF}(d_{cat}) \approx \frac{3.370\times 10^5}{V} M_T^2 M_0^{-1.115} \tag{16}$$

Therefore, for a given total mass M_T in orbit, a reduction of the individual satellite mass M_0 by one order of magnitude, and a corresponding increase of the satellite number N_0, would result in a growth of the average collision rate CR_{0-0}, and of the average production rate of catastrophic "projectiles" $\dot{N}_{CF}(d_{cat})$, by a factor 13, while the increment of the average production rate of collisional fragments $\dot{N}_{CF}(L_c)$, with a certain characteristic length L_c (e.g. 10 cm), or greater, would be limited to about a factor 2.3.

Let us conclude this section with a numerical example, considering 1000 uncontrolled satellites with $M_0 = 100$ kg, placed in nearly circular polar orbits at the mean altitude of 1000 km and with radius vector excursions of ± 25 km, due to orbital perturbations and small eccentricity. It was found $CR_{0-0} = 0.0282$ collisions per year, $\dot{N}_{CF}(L_c = 10\text{ cm})$ = 7.7 fragments per year, and $\dot{N}_{CF}(d_{cat} \approx 6\text{ cm}) = 18.7$ fragments per year.

2.2 Conservation of the satellite number

Another interesting boundary case is represented by a small vs. big satellite replacement in which the total number of satellites in orbit remains the same. This corresponds to $k = 0$ in Eq. (3), i.e. to $N_0 = N_R$, leading to the following expressions:

$$CR_{0-0} \approx \frac{5.185\times 10^2}{V} N_0^2 M_0^{0.885} \tag{17}$$

$$\dot{N}_{CF}(L_c) \approx \frac{87.20}{V} N_0^2 L_c^{-1.71} M_0^{1.635} \tag{18}$$

$$\dot{N}_{CF}(d_{cat}) \approx \frac{3.370\times 10^5}{V} N_0^2 M_0^{0.885} \tag{19}$$

For a given total number of satellites in orbit N_0, a reduction of the satellite mass M_0 by one order of magnitude would result in a decrease of the average collision rate CR_{0-0}, and of the average production rate of catastrophic "projectiles" $\dot{N}_{CF}(d_{cat})$, by about a factor 7.7, while the average production rate of collisional fragments $\dot{N}_{CF}(L_c)$, with a certain characteristic length L_c (e.g. 10 cm), or greater, would decline by a factor 43.

These equations can be applied as well to the numerical example introduced at the end of the previous subsection, considering 1000 uncontrolled satellites with $M_0 = 100$ kg, placed in nearly circular polar orbits at the mean altitude of 1000 km and with radius vector excursions of ± 25 km, due to orbital perturbations and small eccentricity. Of course, they lead to the same results, as it should be expected, being consistent with the overall mathematical treatment, even though they look different. They just provide the right answers starting from a different perspective.

3. Criticality of large constellations

The envisaged deployment of very large constellations in LEO [6][21], some consisting of thousands of spacecraft, might radically change the debris environment for worse if too many satellites would fail or be abandoned close to their operational altitudes, generally high enough (above 700 km) to prevent a sufficiently fast orbital decay induced by atmospheric drag [21][22][23][24]. A possible simple criterion for evaluating the criticality of a new set of non-

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maneuverable satellites, deployed, lost or abandoned into a certain altitude band in LEO, could be a comparison of the additional collision rate due to their presence with the current overall collision rate among the background cataloged objects. Such figure of merit, the collision rate percentage increase *CRI*, can be defined as follows:

$$CRI\left[\%\right] = 100 \times \frac{CR_{0-0} + CR_{0-\mathrm{I}} + CR_{0-\mathrm{D}}}{CR_{\mathrm{D-I}} + CR_{\mathrm{D-D}} + CR_{\mathrm{I-I}}} \tag{20}$$

where the subscripts of the collision rate components *CR* refer to the impacts between new satellites (0–0), between new satellites and intact objects of the background (0–I), between new satellites and background debris (0–D), between debris and intact objects (D–I), between debris (D–D), and between intact objects (I–I). The sum of the first three terms $CR_{0-0} + CR_{0-\mathrm{I}} + CR_{0-\mathrm{D}}$ (numerator) represents the additional collision rate due to the new satellites, while the sum of the remaining three terms $CR_{\mathrm{D-I}} + CR_{\mathrm{D-D}} + CR_{\mathrm{I-I}}$ (denominator) represents the current collision rate among cataloged background objects.

Considering the current situation in orbit and the fact that a growing number of operational spacecraft routinely perform collision avoidance maneuvers, the present average collision rate between cataloged objects in LEO is [10]:

$$CR_{\mathrm{D-I}} + CR_{\mathrm{D-D}} + CR_{\mathrm{I-I}} \approx 0.20\ \mathrm{year}^{-1} \tag{21}$$

Concerning the additional contribution to the collision rate of the new satellites, CR_{0-0} can be computed using Eq. (4), while the two other terms can be estimated using the following relationships [10]:

$$CR_{0-\mathrm{I}} = \pi (r_0 + r_\mathrm{I})^2 V_\mathrm{Rel} \rho_0 \rho_\mathrm{I} V \tag{22}$$

$$CR_{0-\mathrm{D}} = \pi (r_0 + r_\mathrm{D})^2 V_\mathrm{Rel} \rho_0 \rho_\mathrm{D} V \tag{23}$$

where ρ_I and ρ_D are, respectively, the spatial densities in LEO of cataloged intact objects and debris in the volume *V* considered.

The potential criticality of a new satellite deployment in a volume of space *V* in LEO might be evaluated in the light of a violation of an agreed *CRI* alert threshold, e.g. 10% or 100%. In other words, if the launch of a new satellite system would imply, for example, an increase of the average collision rate in LEO by 0.02 collisions per year (10% increment), by 0.20 collisions per year (100% increment), or any other defined amount considered significant, specific constellation design changes should be implemented before deployment for restraining the predicted environmental impact to more acceptable levels. Of course, the applicable *CRI* alert threshold may be open to discussion, but due to the fact that several competing systems might be developed and launched at the same time, assuming a *CRI* alert threshold of no more than 10% per single constellation seems reasonable.

3.1 Quantitative examples

Based on the previous discussion, before planning a new large constellation, an answer to the following question should be provided: how many non maneuverable satellites, deployed, lost or abandoned at a certain altitude, might be tolerated without increasing the average collision rate in LEO by more than a fixed “criticality threshold”, e.g. 10%? In this subsection, the answer will be provided, as a function of the average altitude, for a representative example, taking into account the current distribution of cataloged debris (Figure 1).

In such a very preliminary assessment, the equivalent radius of the constellation satellites was assumed to be $r_0 = 1$ m, and their orbits were nearly polar. Concerning the collisional interaction volume, even assuming a small eccentricity of the order of 0.001, the long-period evolution of the orbital radius vector would display a significant overall excursion of about ± 25 km, dominated by the odd zonal harmonics of the geopotential (i.e. J_3, J_5, J_7, etc.), with further smaller contributions from the tiny eccentricity, radiation pressure, the remaining terms of the geopotential, and luni-solar third body attraction. Therefore, an overall radius vector excursion $\Delta R = 50$ km was considered for the definition of the interaction volume *V*.

The aim of this exercise was then to find, as a function of the average altitude, the number N_0 of non-maneuverable satellites able to increase by 10% the average collision rate among cataloged objects in LEO, i.e. leading to:

$$CR_{0-0} + CR_{0-\mathrm{I}} + CR_{0-\mathrm{D}} \approx 0.02\ \mathrm{year}^{-1} \tag{24}$$

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Using SI units and substituting Eq. (4), Eq. (22) and Eq. (23) in Eq. (24), it was possible to obtain the following quadratic equation in N_0:

$$\frac{2r_0^2}{V}N_0^2+\left[\left(r_0+r_{\mathrm{I}}\right)^2\rho_{\mathrm{I}}+\left(r_0+r_{\mathrm{D}}\right)^2\rho_{\mathrm{D}}-\frac{2r_0^2}{V}\right]N_0-\frac{6.338\times10^{-10}}{\pi V_{\mathrm{Rel}}}=0 \tag{25}$$

Solving it for N_0 with the assumptions made at the beginning of this subsection, and considering average altitudes in LEO in between 700 and 2000 km, where the atmospheric drag is not effective in removing the new satellites in a few decades, the results summarized in Figure 2, and detailed in Table 4, were obtained.

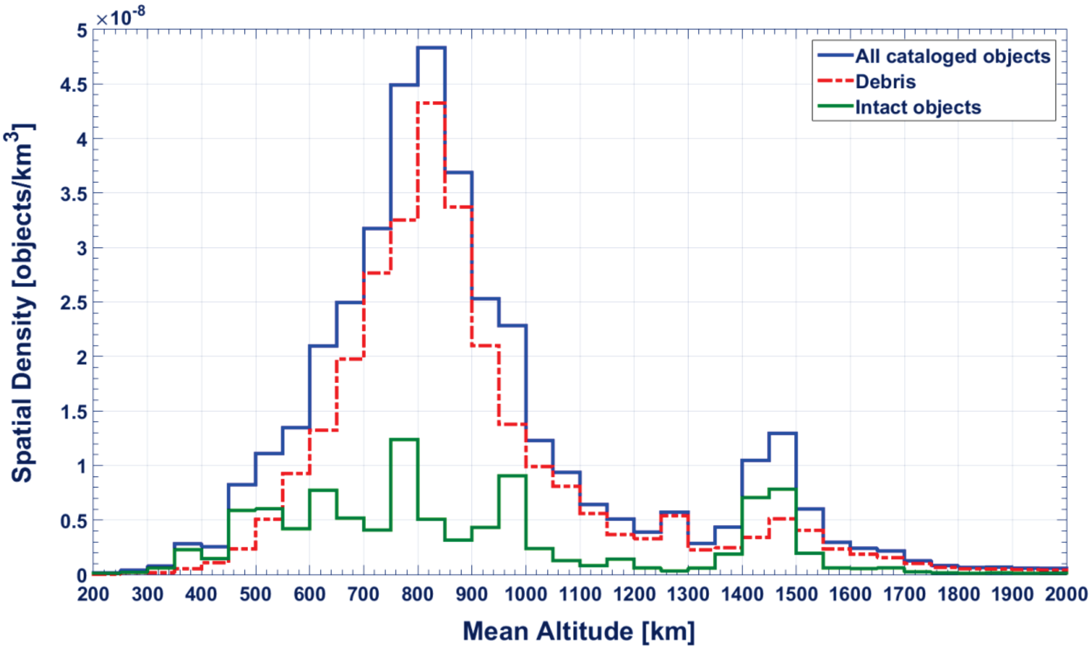

Figure 1: Spatial density of cataloged objects, debris and intact objects in LEO, as of May 3, 2017

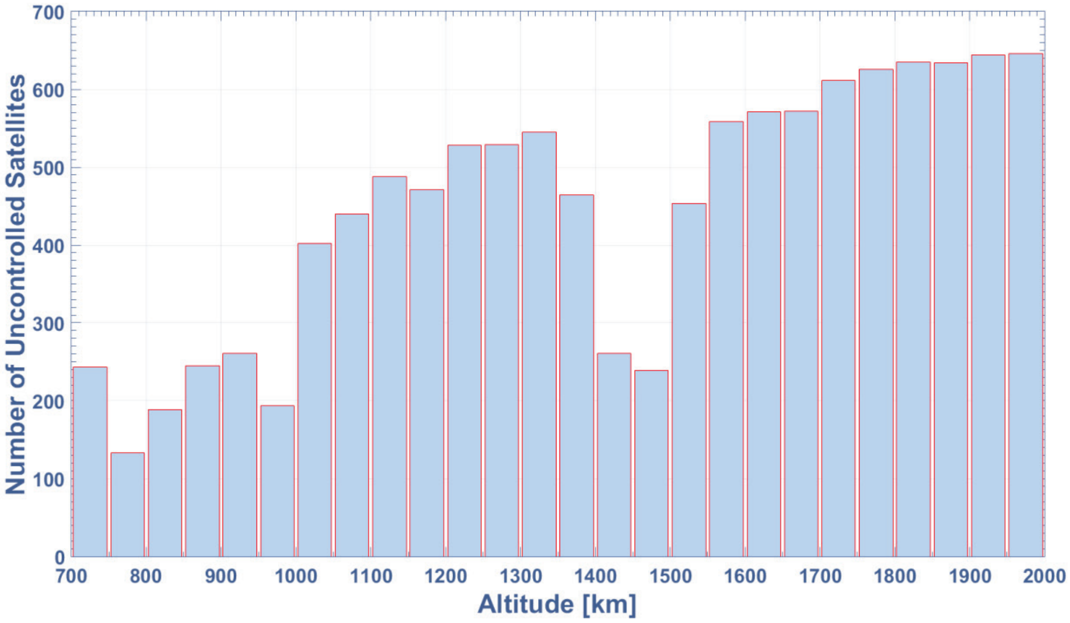

Figure 2: Number of new uncontrolled satellites, for each 50 km wide altitude shell, from 700 to 2000 km, able to increase the overall collision rate among cataloged objects in LEO by 10%

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Table 4: Number of new uncontrolled satellites, for each 50 km wide altitude shell, from 700 to 2000 km, able to increase the overall collision rate among cataloged objects in LEO by 10%

Central altitude of the 50 km wide shell [km]	**Number of uncontrolled satellites**
725	243
775	133
825	189
875	245
925	261
975	194
1025	402
1075	440
1125	488
1175	471
1225	528
1275	529
1325	545
1375	464
1425	261
1475	239
1525	453
1575	558
1625	571
1675	572
1725	611
1775	626
1825	635
1875	634
1925	644
1975	646

As a first superficial impression, the numbers shown in Figure 2 and Table 4 may seem generally quite high and not worrying, but, actually, the opposite is true. In fact, some of the new proposed constellations in LEO plan the deployment of several thousands of satellites and currently the level of worldwide compliance with appropriate post-mission disposal measures (e.g. the 25-year rule) is 60% at most [25], and very often, just in the most critical altitudes, it may be even significantly lower, i.e. 40% or less.

Let us consider, for example, a new constellation of 5000 satellites placed in high inclination and near circular orbits above 700 km. A post-mission disposal success rate of 60% would imply 2000 satellites failed or abandoned close to the original orbit, and even assuming the very ambitious and technologically challenging goal of a 90% success rate, as foreseen in various mitigation guidelines and standards, would lead to 500 uncontrolled satellites left to increase the average collision rate in LEO by a significant amount.

4. Highlights

The results presented would have several important consequences for the deployment of large constellations in LEO around 700 km or higher:

1. The component satellites should be maneuverable, in order to be able to perform effective collision avoidance maneuvers during the operational phase, to avoid impacts with cataloged objects, and to carry out appropriate end-of-life disposal maneuvers, for significantly reducing the residual lifetime;
2. The satellites should also be extremely reliable, targeting end-of-life disposal success rates close to 99%;
3. This would also imply an appropriate level of protection from the impact damages of micrometeoroids and small untrackable orbital debris, from launch to disposal;

4. Even altitude bands initially relatively empty, from the orbital debris point of view, can reach average collision rates of the order of 10% of the current overall value among cataloged objects in LEO with the addition of a number of uncontrolled constellation satellites in between 650 and 1000;
5. The concomitant deployment and operation of several large constellations in LEO would easily lead to a doubling of the current average collision rate among cataloged objects, even by restraining each system to limit the collision rate growth to around 10%.

In addition to these points, relevant from the orbital debris mitigation perspective, other important aspects, not addressed in this paper, cannot be ignored. For instance, the reduction of the residual lifetime of disposed satellites, of paramount importance in order to preserve the circumterrestrial environment above 650 km, due to the very high number of objects involved in the procedure, would have a huge operational impact on the lower altitude range, typically below 500 km, characterized by crewed missions and permanent human presence. Moreover, without adopting controlled direct de-orbiting on uninhabited oceanic areas, the total casualty expectancy of so many uncontrolled reentries might reach levels considered unacceptable.

Regarding the trade-off between large and small satellites, the substitution one-by-one of large satellites with small ones would be advantageous from a debris mitigation point of view, but, for a fixed total mass deployed in space, fewer more massive satellites would be better than many more less massive ones. The breakeven point would of course depend on the details of the design of the satellites and of the background debris environment, and can be estimated with the following expression:

$$CR_{0-0} + CR_{0-\mathrm{I}} + CR_{0-\mathrm{D}} = \pi V_{\mathrm{Rel}} N_0 \left[\frac{2r_0^2}{V} N_0 + \left(r_0 + r_{\mathrm{I}}\right)^2 \rho_{\mathrm{I}} + \left(r_0 + r_{\mathrm{D}}\right)^2 \rho_{\mathrm{D}} - \frac{2r_0^2}{V} \right] \tag{26}$$

Assuming for the new satellites the area-to-mass relation defined by Eq. (1), and considering the current cataloged debris environment, the number of less massive uncontrolled satellites able to match the additional average collision rate induced by 100 new satellites of 1000 kg was estimated in five altitude bands (Table 5). In the cases analyzed, for a satellite mass reduction of one order of magnitude, substitution ratios in between 1.6 and 3.8 were sufficient to recover the additional collision rate of the more massive objects.

Table 5: Number of new uncontrolled satellites (of 100 kg and 10 kg) needed to produce, in five selected altitude bands, the same additional average collision rate, with cataloged objects, as 100 new satellites of 1000 kg

Mean altitude [km]	**Additional collision rate with cataloged objects [s^{-1}]**	**Number of satellites with M = 1000 kg**	**Number of satellites with M = 100 kg**	**Number of satellites with M = 10 kg**
775	1.06×10^{-9}	100	298	535
825	8.65×10^{-10}	100	376	860
1275	1.48×10^{-10}	100	328	902
1475	4.87×10^{-10}	100	248	407
1975	6.51×10^{-11}	100	278	737

The use of large constellations in LEO will therefore represent a considerable technological, operational and environmental challenge, with the potential of revolutionizing the way satellites are designed, built, tested, launched, used, operated and disposed. The preservation of the LEO environment for future responsible exploitation will probably depend on how successfully all the challenges at stake will be met.

5. Conclusions

In order to evaluate the potential environment criticality of large constellations and plenty of small satellites in LEO, several analytical expressions, based on reasonable simplifying assumptions, were developed and applied to realistic examples. They represent a quite handy and flexible tool, providing preliminary quantitative answers to difficult questions, without the need of resorting to complex models and computations, time consuming, depending from too many competing variables, and affected anyway by considerable uncertainties.

For gauging the environment criticality of new large constellations, a specific figure of merit was introduced: the collision rate percentage increase. Of course, the applicable alert threshold may be open to discussion, but due to the fact that several systems might be operated at the same time, assuming an alert threshold of no more than 10% per single constellation seemed a sensible recommendation.

This suggestion is further supported by the fact that quite accurate long-term simulations of the debris environment, completed in 2000, predicted a doubling of the current (2017) average collision rate in LEO, among objects greater than 10 cm, around 2045, i.e. nearly 30 years in the future, even for the unmitigated business as usual scenario (Table 1), leading in 2100 to about 36 000 objects in LEO greater than 10 cm, to an average collision rate of 1.6 per year among the same objects, and to a total of approximately 70 collisions [2][3]. Being this scenario rather undesirable (much of the debris mitigation efforts, during the last two decades, were carried out worldwide just for avoiding a similar outcome), any effort should be made to avoid an even faster debris growth in LEO compared with that anticipated in the business as usual simulations of nearly twenty years ago. The goal should be to maintain, and possibly stabilize over the long-term, the average collision rate in LEO, among objects larger than 10 cm, below 0.30 collisions per year.

A representative quantitative example allowed the estimation, as a function of the average altitude, of the number of new uncontrolled constellation satellites able to increase by 10% the average collision rate among cataloged objects in LEO. It varied from less than 150, in the 750-800 km height range, to nearly 650, in the almost empty region of space close to 2000 km. These numbers seem high, but, due to the fact that various constellations envisage several thousands of satellites, they imply that each spacecraft should be maneuverable, i.e. able to implement collision avoidance during the operational phase and appropriate end-of-life de-orbiting. Moreover, the success rate of post-mission disposal should be much higher than the current value (around 60%), and possibly quite better than 90%, in order to guarantee the long-term sustainability of space operations in low Earth orbit.

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