DOI: 10.13009/EUCASS2017-29

Automatic estimation of modal parameters using a hybrid method of MOPSO and k-means clustering in the time-frequency domain

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Abstract
A hybrid method of MOPSO (Multi-objective Particle Swarm Optimization) and k-means clustering is proposed to extract time-frequency (TF) ridges for automatically estimating time-varying modal parameters. In the hybrid method, the particle swarm is partitioned into sub-swarms by k-means clustering to search local best resolutions and the external archive is used to store the local best resolutions. Moreover, the external archive is inputted as the exclusive centroids of k-means clustering for k-means clustering is sensitive to the centroids, and is applied to guide particles of sub-swarms for MOPSO has limitations in handling multi-objective optimization problems with more than three objectives. Comparisons with an original MOPSO and DBSCAN (Density-Based Spatial Clustering of Applications with Noise) show that the hybrid method is effective in handling multi-mode modal parameter extraction by a synthetic example.

1. Introduction

Modal parameters, including modal frequency, damping ratio, mode shape, modal mass and modal stiffness, are essential in structure dynamic analysis. And literatures on modal parameter extraction are frequently raised in the last decades[1-9]. Methods of modal parameter extraction in the frequency domain[1-3] are early proposed and widely used in engineering practice. However, Fourier Transform (FT) limits the methods to stationary cases, because FT can only process stationary signals and FT is a must in the frequency domain based methods. The time-domain based methods[4], in which time series are directly dealt with, can be extended to process nonstationary cases[5-6], while nonstationary signals are often brought forward in the recent years. However, drawbacks of mode missing and false modes limit the time domain based methods to experienced engineers. Time-frequency (TF) domain based methods for extracting modal parameters[7-9] are a hot issue in the engineering field for good noise-immune performance and no requirements on stationarity of signals. And this study focuses on the topic of extracting modal parameters in TF domain.

In the TF domain based methods for modal parameter extraction, modal frequencies have a close relationship with TF ridges, and the mode shapes are related to the amplitude and phase of TF ridges. Consequently, TF ridge extraction is significant in the TF domain based methods[9]. In most cases, TF ridges are manually extracted because the TF domain based methods are frequently used to extract time-invariant modal parameters and a small number of points are enough to fit the straight TF ridges. When time-varying modal parameters are focused on and time series signals to be processed are very long, especially accurate time-varying modal parameters are required, manually extracting TF ridges is bound to be a huge burden. Fortunately, TF ridges are defined as local peaks in TF spectrogram. Extracting TF ridges can be converted to a multi-objective optimization problem. So this study focuses on a hybrid method for automatically extracting TF ridges in the TF domain.

Particle swarm optimization (PSO), proposed by Kennedy and Eberhart[10] that is inspired by the social behavior of bird flocks when searching for food, is a population based stochastic optimization approach to solve single-objective optimization problems and each particle represents a candidate solution to the optimization problem. For its high efficiency and easy implement, PSO is extended to handle multi-objective optimization problems[11-13], which is referred to as Multi-Objective PSO (MOPSO). Most MOPSO methods use an external archive to store the nondominated solutions found in the search process[11-12]. And some strategies, e.g. the ε-dominance method[13],

DOI: 10.13009/EUCASS2017-29

are applied to retain its diversity of the external archive. In MOPSO, that how to assign the global best position for each particle is an important problem, and the crowding-distance method[12] serves to determine neighboring candidates in the external archive with Roulette-Wheel method being used to choose the global best position from the neighboring candidates. However, it is reported that MOPSO has some limitations when handling problems with more than three objectives[14]. Analogous to that particle swarm is divided into sub-swarms for solving multi-objective problems in MOPSO, nonhierarchical clustering methods, such as k-means clustering[15] and fuzzy c-means clustering methods[16], partition a group of objects into a number of clusters on the basis of a similarity measure[17]. However, k-means clustering method is sensitive to initial centroids, resulting in a false clustering result. To this end, a hybrid method of MOPSO and k-means clustering is proposed to extract TF ridges for automatic estimation of time-varying modal parameters.

The correspondence is organized as follows. Section 2 presents the hybrid method of MOSPO and k-means for extracting TF ridges. In Section 3, a simulated example is applied to show how the proposed hybrid method works. And performance comparisons are conducted with MOPSO proposed in Ref.[11], k-means clustering method and a potential clustering method DBSCAN[18], respectively. Conclusions are drawn in Section 4.

2. Estimation of modal parameters using MOPSO and *k*-means clustering

2.1 Definition of TF ridges

TF ridges contain crucial information on the characteristics of multi-component signals[19-20]. In structural dynamics, TF ridges correspond directly to modal frequencies. Herein a brief formulation is presented on TF ridges. Considering the signal in the following form

$$f(t)=\sum_{n=1}^{N}A_n(t)\cos(\phi_n(t)) \tag{1}$$

where the amplitudes $A_n(t)$ are continuously differentiable and the phase $\phi_n(t)$ are twice continuously differentiable. Since the hybrid method proposed in this study is a post-processing algorithm for TF ridge extraction, other TF analysis method, e.g. wavelet transform, plays an equal role to STFT. Without loss of generality, STFT (Short-Time Fourier Transform) is chosen to convert time series into TF representation. The STFT of $f(t)$ is defined as

$$F(t,\omega)=\int_{-\infty}^{+\infty}f(\tau)g(\tau-t)e^{-i\omega(\tau-t)}d\tau \tag{2}$$

where $g(t)$ is a sliding Gaussian or Hamming window function. Then the continuous STFT of Eq.(1) can be written as

$$F(t,\omega)=\frac{1}{2}\sum_{n=1}^{N}A_n(t)e^{i\phi_n(t)}\overline{\hat{G}(\phi_n^{'}(t)-\omega)}+o(\left|A_n^{'}(t)\right|,\left|\phi_n^{''}(t)\right|) \tag{3}$$

where $\hat{G}(\omega)=\int_{-\infty}^{+\infty}g(\tau)e^{-i\omega\tau}d\tau$, and $\overline{G(\omega)}$ is the conjugate of $G(\omega)$. Given that the sliding window function $g(t)$ has the form of e^{-t^2} in the time domain, $\overline{\hat{G}(\phi_n^{'}(t)-\omega)}$ has a peak when $\omega=\phi_n^{'}(t)$ and a fast decay when $\omega\neq\phi_n^{'}(t)$ for that e^{-t^2} is transformed into $\sqrt{\pi}e^{-(\omega-\phi_n^{'}(t))^2/4}$ in the frequency domain. Namely, STFT square modulus $\left|F(t,\omega)\right|^2$ is centralized near N curves with $\omega=\phi_n^{'}(t)$, $n=1,...,N$ in the TF domain. The N curves are defined as TF ridges[19]. As formulated in Eq.(3), TF ridge extraction can be treated as a multi-objective problem and the hybrid method of MOPSO and k-means clustering is developed to handle such a multi-objective problem.

2.2 Hybrid method of MOPSO and k-means clustering

Given the particle swarm $\mathbf{X}=\left\{\mathbf{x}_1,...,\mathbf{x}_n\right\}$, the k-means clustering method is used to partition $\mathbf{X}$ into k sub-swarms, $\mathbf{C}=\left\{\mathbf{C}_1,...,\mathbf{C}_k\right\}$ with $k\le n$ and k being known in advance, such that

1) $\mathbf{C}_j\neq\mathbf{\Phi}, j=1,...,k$;

2) $\bigcup_{j=1}^{k}\mathbf{C}_j=\mathbf{X}$;

3) $\mathbf{C}_j\bigcap\mathbf{C}_i=\mathbf{\Phi}, j,i=1,...,k, j\neq i$ [21].

The k-means clustering is a two-stage method: first randomly choose k points $\mathbf{c}=\{\mathbf{c}_1,...,\mathbf{c}_k\}$ as the initial centroids from the given particles $\mathbf{X}=\{\mathbf{x}_1,...,\mathbf{x}_n\}$ and roughly divide $\mathbf{X}=\{\mathbf{x}_1,...,\mathbf{x}_n\}$ into k clusters $\mathbf{C}=\{\mathbf{C}_1,...,\mathbf{C}_k\}$ with $\mathbf{c}_j$ being the centroid of the cluster $\mathbf{C}_j$; second minimize the objective function $P=\sum_{j=1}^{k}\sum_{i=1}^{n}u_{i,j}dis(\mathbf{x}_i-\mathbf{c}_j)$ by iteratively determining the centroids and reassigning the particles to clusters until the end condition is satisfied, and the objective function is subjected to $\sum_{j=1}^{k}u_{i,j}=1, 1\leq i\leq n$ with $dis(\mathbf{x}_i-\mathbf{c}_j)$ representing a distance or dissimilarity measure between the cluster $\mathbf{C}_j$ and the particle $\mathbf{x}_i$. In addition, if the particle $\mathbf{x}_i$ belongs to the cluster $\mathbf{C}_j$, $u_{i,j}=1$, or else $u_{i,j}=0$ [15].

In practice, k-means clustering method is sensitive to the initial centroids, resulting in a false clustering result. As mentioned above, most MOPSO methods, which use an external archive to store nondominate solutions found in the searching procedure, have some limitations when dealing with multi-objective optimization problems with more than three objectives. Moreover, the external archive is redundant to reserve the diversity of non-dominate solutions, namely it is difficult and time consuming to choose the global solution for each particle[13]. In this study, a hybrid method of MOPSO and k-means clustering is proposed to extract TF ridges for automatic estimation of time-varying modal parameters. In the hybrid method, the particle swarm is partitioned into k sub-swarms by k-means clustering and the external archive element $\mathbf{c}_j\in\{\mathbf{c}_1,...,\mathbf{c}_k\}, j=1,...,k$ is used as the global best position to guide each particle of corresponding sub-swarms. Simultaneously, the external archive $\mathbf{c}=\{\mathbf{c}_1,...,\mathbf{c}_k\}$ is the exclusive centroids of k-means clustering method in order to de-sensitize to the initial centroids and avoid null cluster. The procedure of the proposed hybrid method is illustrated in Fig.1. As shown in Fig.1, the particles are uniformly distributed in the initializing procedure and the particles are updated by Eq.(4) in MOPSO.

$$\begin{cases}\mathbf{v}_{ij,d}^{m+1}=\xi\cdot\mathbf{v}_{ij,d}^{m}+\alpha\cdot r_1\cdot(\mathbf{p}_{j,d}^{m}-\mathbf{x}_{ij,d}^{m})+\beta\cdot r_2\cdot(\mathbf{c}_{j,d}^{m}-\mathbf{x}_{ij,d}^{m})\\ \mathbf{x}_{ij,d}^{m+1}=\mathbf{x}_{ij,d}^{m}+\mathbf{v}_{ij,d}^{m+1}\end{cases}\tag{4}$$

where ξ is a weighting value with $\xi=0.9-0.4\cdot(m-1)/M$, M denotes the maximum iteration times in MOPSO, r_1 and r_2 are two random numbers in the range [0,1], α and β are two constant numbers that indicate how much the personal and social components influence on individual velocity of each particle, $\mathbf{p}_{j,d}^{m}$ is the local best position that particle $\mathbf{x}_i$ could find so far, $\mathbf{c}_{j,d}^{m}$ is the global best position in the jth sub-swarm. The superscript "m" represents the iteration time with the subscript "d" suggesting the dimension of particle $\mathbf{x}_i$. The external archive is updated by the ε-dominance strategy[12] as formulated in Eq.(5)

$$\begin{cases}f(\mathbf{c}_{j,d}^{m+1})\big/(1+\varepsilon)\geq f(\mathbf{c}_{j,d}^{m}), \forall j=1,...,k\\ f(\mathbf{c}_{j,d}^{m+1})\big/(1+\varepsilon)> f(\mathbf{c}_{j,d}^{m}), \exists j=1,...,k\end{cases}\tag{5}$$

with $f(\bullet)$ denoting the objective function and $\mathbf{c}_{j,d}^{m+1}=\{\mathbf{x}_{ij,d}^{m+1}\mid\max(f(\mathbf{x}_{ij,d}^{m+1})),\mathbf{x}_i\in\mathbf{C}_j\}$. If the condition formulated in Eq.(5) is satisfied, the global best position is updated to be $\mathbf{c}_{j,d}^{m+1}$ in the jth sub-swarm, or else remains unchanged. In addition, the external archive is initialized as follows:

1) $\mathbf{c}_1=\{\mathbf{x}_i\mid\max(f(\mathbf{x}_i)), i=1,...,n\}$;

2) $\mathbf{c}_j=\left\{\mathbf{x}_i\mid\max(f(\mathbf{x}_i)),\mathbf{x}_i\in\bigcap_{m=1}^{j-1}\{\mathbf{x}_i\mid|\mathbf{x}_i-\mathbf{c}_m|\geq\rho\}, i=1,...,n\right\}, j=2,..,k$, where ρ is a constant.

DOI: 10.13009/EUCASS2017-29

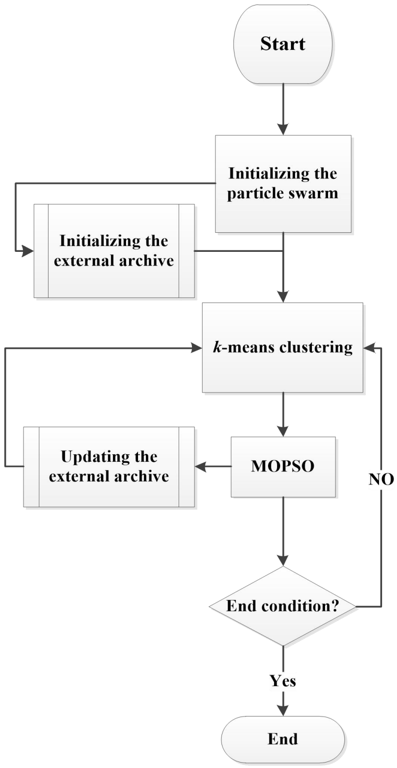

Fig.1 the hybrid method of MOPSO and k-means clustering

2.3 Modal parameter extraction in TF domain

Since the hybrid method is developed for modal parameter extraction in TF domain, it is necessary to briefly describe the modal parameter extraction algorithm in the TF domain[9]. The following description reveals that extracting TF ridges is essential in the TF domain based modal parameter extraction algorithm.

① Extract TF ridges of $q_i(t,\omega),\ i=1,..,N$ by the hybrid method, and the TF ridges are represented as $q_i^j(t,\ \omega_j(t)),\ i=1,..,N,\ j\le k$. It should be noted that $q_i(t,\omega),\ i=1,..,N$ is the TF representation of time series $q_i(t),\ i=1,..,N$ with the aid of STFT or wavelet transform, and i is the sequence number of sensors with j denoting the mode number;

② Estimate mode shape vectors. Assuming $q_1^j(t,\ \omega_j(t))\ne 0,\ \forall j=1,..,k$, choose $q_1(t,\omega)$ to be the reference in this study. The coefficient of corresponding mode shape vector is estimated as

$$c_i^j(t)=\left|\frac{q_i^j(t,\ \omega_j(t))}{q_1^j(t,\ \omega_j(t))}\right|\cdot\cos(\theta_i^j),\quad j=1,..,k,\ \ i=1,..,N \tag{6}$$

where θ_i^j is the phase difference between $q_i^j(t,\ \omega_j(t))$ and $q_1^j(t,\ \omega_j(t))$, which is defined in Eq.(7)

$$\theta_i^j=\begin{cases}0, & \left|\angle q_i^j(t,\ \omega_j(t))-\angle q_1^j(t,\omega_j(t))\right|\approx 0,2\pi\\ \pi, & \left|\angle q_i^j(t,\ \omega_j(t))-\angle q_1^j(t,\omega_j(t))\right|\approx \pi\end{cases} \tag{7}$$

As shown in Fig.2, TF ridges correspond directly to modal frequencies, and mode shape vectors are estimated with the aid of the extracted TF ridges as formulated in Eqs.(6-7). In addition, if damping ratios are focused on, singular value decomposition (SVD) can be introduced to process single-mode signals[22] and TF ridges are usable in reconstructing single-mode signals[19].

DOI: 10.13009/EUCASS2017-29

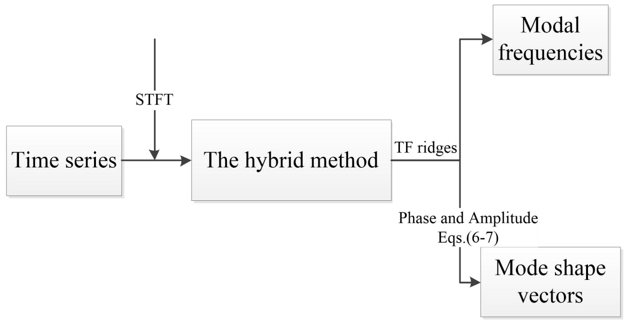

Fig.2 Modal parameter estimation using the hybrid method

3. A synthetic example

An artificial multi-component signal is used to show how the hybrid method of MOPSO and k-means clustering works. And the artificial multi-component signal is formulated in Eq.(8)

$$
\begin{aligned}
y_1(t) &= \begin{bmatrix} 1.5 & -1 & 1 & -1 & 1 \end{bmatrix} \cdot \sin\left(\begin{bmatrix} 2\pi(35-5t) \\ 2\pi(210-30t) \\ 690\pi t \\ 2\pi(760-55t^2) \\ 2\pi(1115-33t^2) \end{bmatrix} \right) \\
&= 1.5\sin(2\pi(35-5t)) - \sin(2\pi(210-30t)) + \sin(690\pi t) \ldots \\
&\quad - \sin(2\pi(760-55t^2)) + \sin(2\pi(1115-33t^2))
\end{aligned} \tag{8}
$$

The synthetic signal is converted into TF formulation by STFT, and TF spectrum is illustrated in Fig.3. As shown in Fig.3, there are five TF ridges and the five TF ridges shown in Fig.4(d) are extracted by the hybrid method proposed in this study. Moreover, Fig.4(a-c) shows how the hybrid method extracts the five peak points of the frequency spectrum line at 0.58s. As shown in Fig.4(a), the particles are uniformly distributed in the searching space when the hybrid method starts; after one iteration, most of particles converge at their own objective points as seen in Fig.4(b); Fig.4(c) shows the final result when the end condition is satisfied and majority of the particles converge at the ideal points.

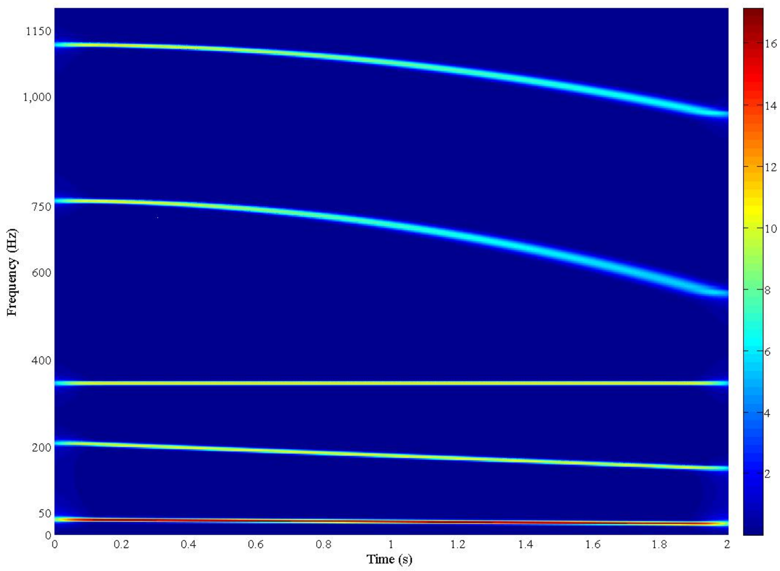

Fig.3 TF formulation of the synthetic multi-component signal

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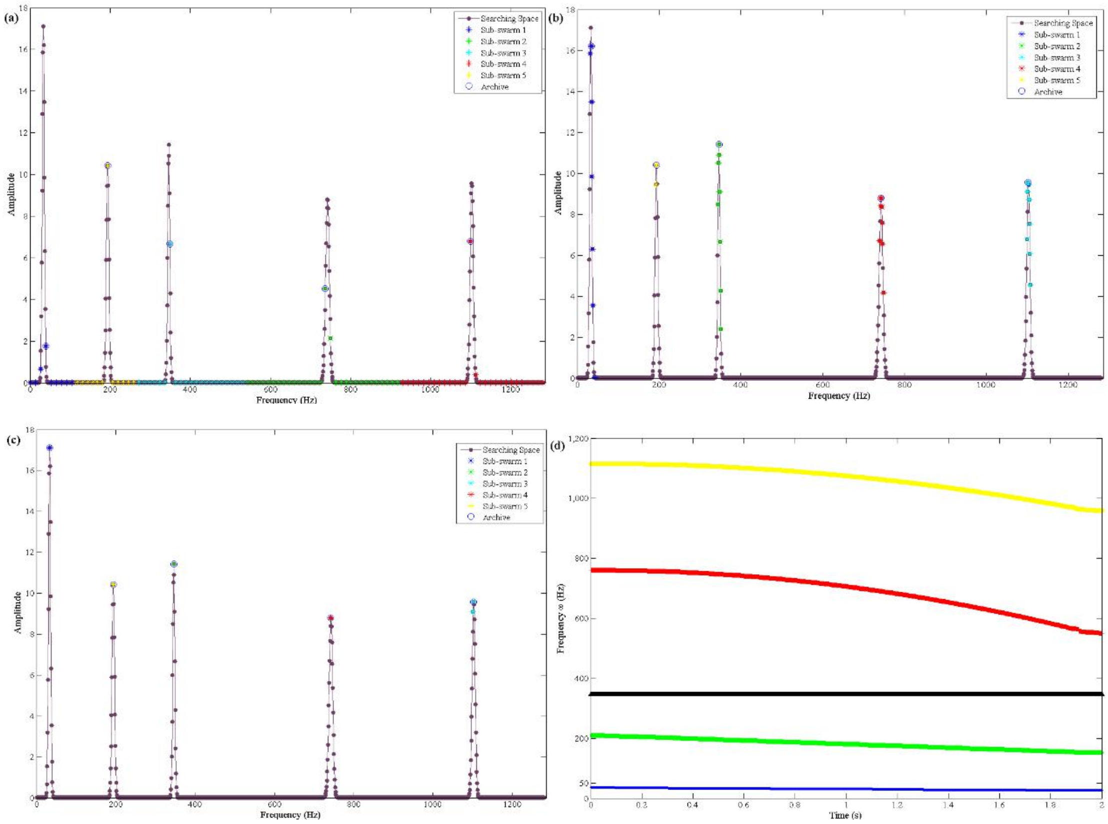

Fig.4 TF ridges of the artificial multi-component signal extracted by the hybrid method of MOPSO and *k*-means clustering; (a) the initializing step; (b)after the first iteration; (c) when the end condition is satisfied; (d) the extracted TF ridges

Fig.5 shows the TF ridges extracted by *k*-means clustering method and MOPSO in Ref.[11], respectively. Compared with the extracted TF ridges shown in Fig.4(d), the well extracted TF ridges shown in Fig.4(d) reveal that the hybrid method of MOPSO and *k*-means clustering is effective and accurate in solving such a multi-objective optimization problem of extracting TF ridges. Fig.6 shows TF ridges extracted by DBSCAN, which can discover clusters of arbitrary shapes and size in large spatial databases[18]. In this study, DBSCAN are used twice in one iteration for extracting TF ridges. As shown in Fig.6(a), frequency spectrum points are clustered into two clusters when DBSCAN is first used. Then as shown in Fig.6(b), the black points are clustered into five clusters when DBSCAN twice used. Compared with the hybrid method, TF ridges are well extracted by both of the methods. However, DBSCAN is time-consuming while DBSCAN costs 2.18s every iteration and the hybrid method needs 0.11s every iteration.

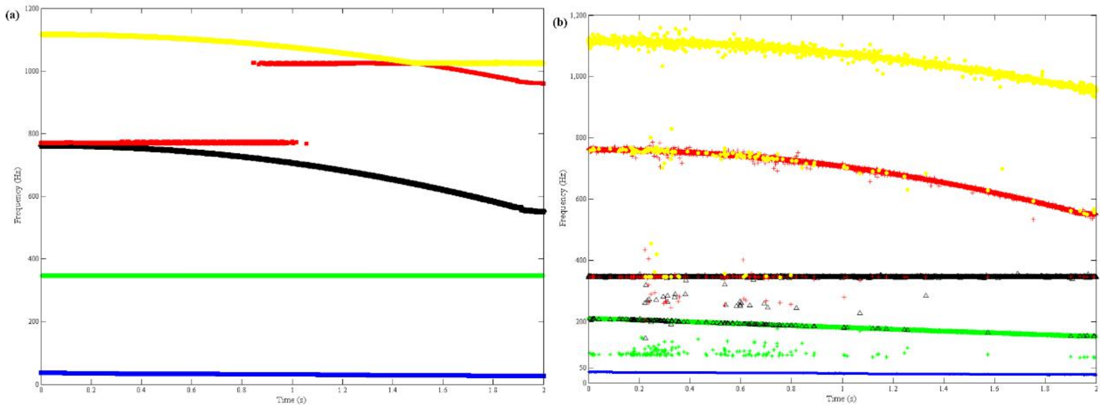

Fig.5 TF ridges of the artificial multi-component signal extracted by *k*-means clustering and MOPSO, respectively; (a) by *k*-means clustering method; (b) MOPSO

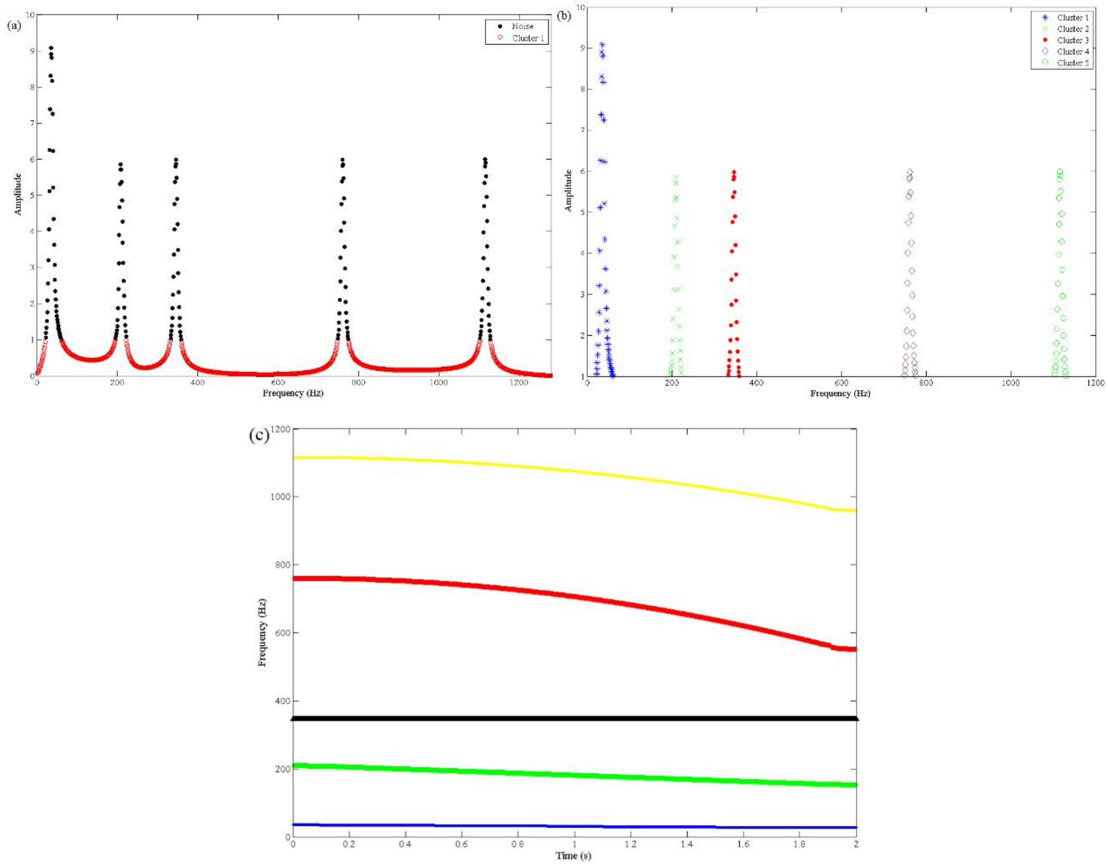

Fig.6 TF ridges extracted by DBSCAN; (a) the first step; (b) the second step

Hereinbefore, modal frequencies are focused on and mode shapes are essential in modal analysis. To this end, another artificial signal is introduced to show how mode shape vectors are estimated. And such an artificial signal is formulated in Eq.(9).

$$\begin{aligned} \mathrm{y}_2(t) &= \begin{bmatrix} 0.5 & 1 & -1 & 1 & -1 \end{bmatrix} \cdot \sin\left(\begin{bmatrix} 2\pi(35-5t) \\ 2\pi(210-30t) \\ 690\pi t \\ 2\pi(760-55t^2) \\ 2\pi(1115-33t^2) \end{bmatrix} \right) \\ &= \begin{bmatrix} 1/3 & -1 & -1 & -1 & -1 \end{bmatrix}^T \begin{bmatrix} 1.5 & -1 & 1 & -1 & 1 \end{bmatrix} \cdot \sin\left(\begin{bmatrix} 2\pi(35-5t) \\ 2\pi(210-30t) \\ 690\pi t \\ 2\pi(760-55t^2) \\ 2\pi(1115-33t^2) \end{bmatrix} \right) \end{aligned} \tag{9}$$

From Eq.(9), the mode shape vector can be defined as $\begin{bmatrix} 3 & -1 & -1 & -1 & -1 \end{bmatrix}$. Fig.7 shows how Eqs.(6-7) are used to estimate one element of the mode shape vector. As shown in Fig.7, the element is estimated to be $3\times\cos(0)=3$. In addition, it is necessary to announce parameters used in the hybrid method of MOPSO and *k*-means clustering. The maximum iteration of the hybrid method is set to be 2 and the particles are updated 50 times with Eq.(4) in MOPSO in one iteration. $\alpha=\beta=1.49$ indicates that the personal and social components play the same importance on updating individual velocity of each particle. The particle swarm has 100 particles, which is divided into five sub-swarms by *k*-means clustering, and the searching space contains 1280 points.

DOI: 10.13009/EUCASS2017-29

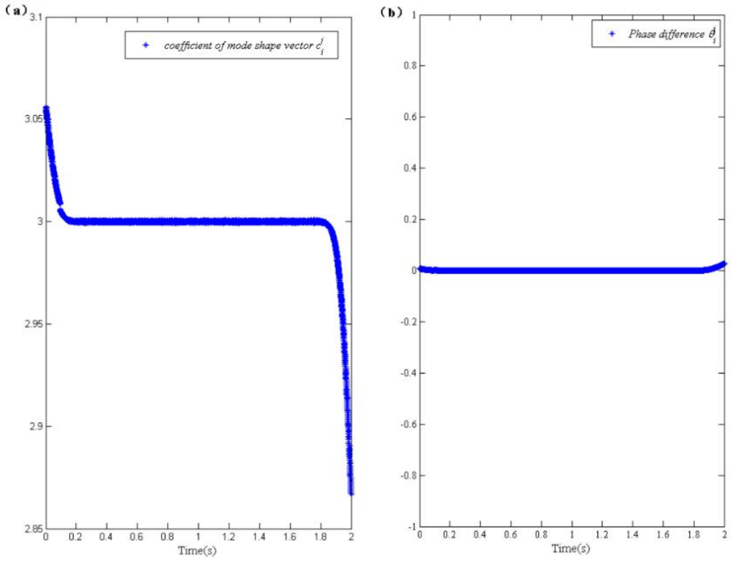

Fig.7 A element of mode shape vector

(a) coefficient of mode shape vector $c_i^j(t)$; (b) phase difference θ_i^j

4. Conclusions

A hybrid method of MOPSO and k-means clustering is proposed in this study and applied as a post-processing method to extract TF ridges for automatically estimating time-varying modal parameters in the TF domain. A synthetic signal is used to show how the hybrid method extracts TF ridges and is used in estimating modal parameters. Comparisons with k-means clustering, one MOPSO method and original DBSCAN confirm that the hybrid method is effective and accurate. With aid of the hybrid method, modal parameters are automatically estimated in the TF domain.

Acknowledgment

The research is supported by the Institutive Foundation of Hypervelocity Aerodynamics Institute, CARDC. And the authors are grateful to Carlos A. C. Coello for sharing MOPSO codes.

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