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Integrated SVD/EKF for Nano-satellite Attitude Determination in case of Magnetometer Faults

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Abstract
In this study, non-traditional approaches for the nanosatellite attitude and rate estimation are examined for the case of magnetometer faults. For this purpose, two different scenarios for two different approaches are investigated. It is expected that the integrated SVD/EKF is robust against the measurement faults naturally if the measurement noise covariance of the filter used directly from the SVD method. Another attitude and rate estimation algorithm called Robust EKF (REKF) which is also an integrated algorithm but the noise covariance of the filter is adapted by using the multiple measurement noise scale factor (MMNSF). In the first scenario, constant bias is considered additionally to the original magnetometer modeling. Secondly, measurement noise increment as magnetometer fault is added to the system.
Key Words: REKF, SVD, nanosatellite, magnetometer, fault

1. Introduction

Magnetometer and sun sensor are common sensors for attitude determination system of nanosatellite. For attitude determination algorithm, single-frame methods can give an attitude knowledge to the satellite as coarse data. Vectors coming from the selected sensor and developed models can be placed in Wahba's loss function and the loss can be minimized using those methods [1]. Methods depending on only the measurements but not the dynamics of the satellite's motion cannot determine attitude for the period in which there is no vector data coming to the sensor. However, the Kalman filter can adapt its gain for crucial time intervals and estimate the attitude even in the eclipse. For this purpose, Extended Kalman Filter (EKF) has been integrated with a single-frame method as Singular Value Decomposition (SVD).

In general, Kalman filter algorithms can be separated into two types as traditional in which the filter is based on the non-linear measurements and non-traditional in which the filter is based on the linear measurements [2]. In non-traditional approach, coarse attitude angles are found by vector measurements in single-frame algorithms at each step. Those angles are directly used as measurement inputs for EKF, therefore the measurement model is linear. On the other side, measurement models are based on nonlinear models of reference directions for traditional techniques. Thus there is a nonlinear relation between the measurements the states. In this study, integrated SVD/EKF algorithm as non-traditional approach is used.

It is expected that the SVD/EKF is robust against the measurement faults naturally if the measurement noise covariance of the filter used directly from the SVD method. Another method called Robust EKF (REKF) which is also an integrated algorithm but the noise covariance of the filter is adapted by using the multiple measurement noise scale factor (MMNSF) [3]. The adaptation is achieved by reducing the effect of the innovation term of the faulty sensor and eliminating the estimation error caused by the faulty measurements.

In this study, non-traditional approaches for the nanosatellite attitude and rate estimation are examined for the case of magnetometer faults. For this purpose, two different scenarios for two different approaches are investigated. In the first scenario, constant bias is considered additionally to the original magnetometer modeling. Secondly, measurement noise increment as magnetometer fault is added to the system.

If the requirements of the system are performed algorithms for the satellite including more than one sensor can be adapted for the case of a sensor failure. The methods are significant for those cases to make the system reliable all along the mission duration. Integrated SVD/EKF method is based on the SVD's covariance at a single time therefore measurement faults are sensed immediately in the filter right after the SVD's coarse determination algorithm. On the

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other hand, the basis of the REKF method is the covariance of the innovation sequence and their real and theoretical values comparison therefore the mismatch of the models affects the Kalman gain.
SVD/EKF and SVD/REKF methods are compared in two different cases of measurement faults.

2. Mathematical models for vector measurements

2.1 Magnetic field direction vector

IGRF model defines the series in nT seen below which depends on 4 input variables (r,θ,ϕ,t), using numerical Gauss coefficients (g, h) - the global variables in the algorithm [4].

$$B(r,\theta,\phi,t) = -\nabla\left\{a\sum_{n=1}^{N}\sum_{m=0}^{n}\left(\frac{a}{r}\right)^{n+1}[\mathrm{g}_n^m(\mathrm{t})\cos\mathrm{m}\phi + \mathrm{h}_n^m(\mathrm{t})\sin\mathrm{m}\phi]\times P_n^m(\cos\theta)\right\} \quad \textbf{(1)}$$

Here, r is the distance between center of the Earth and satellite (km), a=6371.2 km (magnetic reference spherical radius), θ colatitude (deg), ϕ longitude (deg). The inputs are coming from the LEO satellite which has an orbit propagation algorithm as an input.
For the simulations, orbit of the satellite is propagated. Two Line Element (TLE) set includes the data for the satellite's orbital elements at a specified time. It consists of two lines and 69 character strings of data. The classical orbital elements as inclination (i), right ascension of the ascending node (Ω), eccentricity (e), mean anomaly (M), argument of perigee (ω), and mean motion (n) are the information written in the data set. In this study, SGP4 model for orbit propagation is used [5].
Major axis of the Earth accepted as 6378.137 km. IGRF 12 model makes the calculations at N=13th degree for 5-year intervals. Thus, coefficients of the model are updated at the years of the multiples of five (2010, 2015, etc.). The time dependence of the Gauss coefficients can be denoted as:

$$g_n^m(t) = g_n^m(T_0) + \dot{g}_n^m(T_0)(t-T_0) \quad \textbf{(2)}$$

$$h_n^m(t) = h_n^m(T_0) + \dot{h}_n^m(T_0)(t-T_0) \quad \textbf{(3)}$$

Here, T_0 is the epoch times multiple of five proceeding t and t is in the units of years for the selected time. IGRF-12 model uses predictive secular variation coefficients for 2015-2020 and main field coefficients for 1900-2015.

$$B_o = \begin{bmatrix} B_{x_o} \\ B_{y_o} \\ B_{z_o} \end{bmatrix} = \frac{1}{\sqrt{B_1^2 + B_2^2 + B_3^2}}\begin{bmatrix} B_1 \\ B_2 \\ B_3 \end{bmatrix} \quad \textbf{(4)}$$

Equation (4) shows the direction cosines for magnetic field model changing between -1 and +1, which only aim to determine the direction of the vector.
Three-axis onboard magnetometers of the satellite measure the components of the magnetic field vector in the body frame. Therefore, for the measurement model, which characterizes the measurements in the body frame, gained magnetic field terms must be transformed by the use of direction cosine matrix, A . Overall measurement model may be given as;

$$B_m(k) = A(k)\mathrm{B}_o(k) + v_H(k)\,, \quad \textbf{(5)}$$

where $B_m(k)$ is the measured Earth magnetic field vector as the direction cosines in body frame, $v_H(k)$ is the magnetometer measurement noise.

2.2 Sun direction vector

To determine Sun direction vector in ECI (Earth Centered Inertial) frame, Julian Day (T_{TDB}) should be defined from the satellite's initial data and reference epoch. The first constant is the mean anomaly of the Sun (MSun) at epoch and the second constant is the change of the mean anomaly during Julian Day that generates. After the calculations, the ecliptic longitude of the Sun ($\lambda_{ecliptic}$) and the obliquity of the ecliptic (ε) can be determined by only the input of date in years, months, days and time in hours, minutes, seconds [6].

$$MSun\text{=}357.52772330\text{+}35999.05034TTDB \quad \textbf{(6)}$$

$$\lambda ecliptic\text{=}\lambda MSun\text{+}\sin MSun \quad \textbf{(7)}$$

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$$\varepsilon = 23.4392910 - 0.0130042TTDB \tag{8}$$

Finally, the unit Sun vector ($\boldsymbol{S}_{ECI}$) can be found in the inertial frame.

$$\boldsymbol{S}_{ECI} = \begin{bmatrix} \cos\lambda_{ecliptic} \\ \sin\lambda_{ecliptic}\cos\varepsilon \\ \sin\lambda_{ecliptic}\sin\varepsilon \end{bmatrix} \tag{9}$$

The Sun direction vector measurements can be expressed in the following form:

$$S_m(k) = A(k)S_o(k) + v_S(k)\,, \tag{10}$$

where $S_m(k)$ is the measured Sun direction vector as the direction cosines in body frame, $S_o(k)$ represent the Sun direction vector in the orbit frame as a function of time and orbit parameters, $v_S(k)$ is the sun sensor measurement noise.

3. Single-frame method: SVD

After Wahba's optimization problem definition, two or more vectors can be used in statistical methods to minimize the loss [7]. The loss is the difference between the models and the measurements which are found in unit vectors.

$$L(A) = \frac{1}{2}\sum_i a_i \,|\,\mathrm{b}_i - \mathrm{Ar}_i\,|^2 \tag{11}$$

$$B = \sum a_i b_i r_i^T \tag{12}$$

$$\mathrm{LA} = \lambda_0 - \mathrm{tr(ABT)} \tag{13}$$

where b_i (set of unit vectors in body frame) and r_i (set of unit vectors in reference frame) with their a_i (non-negative weight) are the loss function variables obtained for instant time intervals and λ_0 is the sum of non-negative weights. Also, 'B' matrix is defined to reduce the loss function into the equation (13). Here, maximizing the trace ($tr(\mathrm{AB}^T)$) means minimizing the loss function (L). In this study, Singular Value Decomposition (SVD) Method is chosen to minimize the loss function [8].

$$\mathrm{B=USVT=Udiag|S11\ S22\ S33|VT} \tag{14}$$

$$A_{opt} = Udiag[1 \quad 1 \quad \det(U)\det(V)]V^T \tag{15}$$

The matrices U and V are orthogonal left and right matrices respectively and the primary singular values (S11, S22, S33) can be calculated in the algorithm. To find the rotation angles of the satellite, transformation matrix should be found first with the determinant of one. "diag" operator returns a square diagonal matrix with elements of the vector on the main diagonal.
Rotation angle error covariance matrix (P) is necessary for determining the instant times which gives higher error results than desired.

$$P_{SVD} = Udiag[(\mathrm{s}_2+\mathrm{s}_3)^{-1} \quad (\mathrm{s}_3+\mathrm{s}_1)^{-1} \quad (\mathrm{s}_1+\mathrm{s}_2)^{-1}]U^T \tag{16}$$

where s1 = S11, s2 = S22, s3 = det(U)det(V) S33.
The satellite only has two sensors (e.g. sun and magnetic field sensor), thus the SVD-method fails when the satellite is in eclipse period and when the two observations are parallel.

4. Nontraditional approach for the nanosatellite attitude and rate estimation

In this study, SVD has been used as the observation model in the EKF framework. The SVD and EKF algorithms are combined to estimate the attitude angles and angular velocities.

4.1 SVD/EKF for estimation of satellite attitude and rate in the presence of magnetometer faults

In case of EKF design based on linear Euler angle measurements, determination model of the angles that characterizes satellite's attitude, can be given as [9],

$$\begin{aligned} z_\varphi(k) &= \varphi(k) + \mathrm{v}_\varphi(k)\,, \\ z_\theta(k) &= \theta(k) + \mathrm{v}_\theta(k)\,, \\ z_\psi(k) &= \psi(k) + \mathrm{v}_\psi(k) \end{aligned} \tag{17}$$

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where $\varphi(k)$, $\theta(k)$ and $\psi(k)$ are the attitude angles determined by SVD method, $v_{(\cdot)}(k)$ is the measurement noise of the attitude angles. The mathematical expectations and variances of the measurement noises are $E\left[v_{(\cdot)}(k)\right]=0$, $E\left[v_{\varphi}^2(k)\right]=Var\left(v_{\varphi}(k)\right)$, $E\left[v_{\theta}^2(k)\right]=Var\left(v_{\theta}(k)\right)$ and $E\left[v_{\psi}^2(k)\right]=Var\left(v_{\psi}(k)\right)$.

In this case, measurement vector can be written in the following form, $z^T(k)=\left[z_{\phi}(k) \quad z_{\theta}(k) \quad z_{\psi}(k)\right]$

It is assumed that both measurement and system noise vectors $v^T(k)=\left[v_{\phi}(k) \quad v_{\theta}(k) \quad v_{\psi}(k)\right]$ and $w(k)$ are linearly additive Gaussian, temporally uncorrelated with zero mean and the corresponding covariance:

$$E\left[w(i)w^T(j)\right]=Q(i)\delta(ij),$$

$$E\left[v(i)v^T(j)\right]=R(i)\delta(ij), \qquad \textbf{(18)}$$

It is assumed that process and measurement noises are uncorrelated, i.e.,

$$E\left[w(i)v^T(j)\right]=0,\ \forall i,j. \qquad \textbf{(19)}$$

It is required to design EKF for satellite attitude and rate estimation.

The mathematical model of the LEO satellite's rotational motion about its center of mass, is linearized using quasi-linearization method. We will consider a real-time linear Taylor approximation of the system function at the previous state estimate. The Kalman Filter which is obtained will be called EKF based on linear angle and angle rate measurements. Filter algorithm, in this case as, is given below [10].

Equation of the estimation value,

$$\hat{x}(k+1)=\hat{x}(k+1/k)+K(k+1)\times\left\{z(k+1)-H\hat{x}(k+1/k)\right\} \qquad \textbf{(20)}$$

H is the measurement matrix. Equation of the extrapolation value,

$$\hat{x}(k+1/k)=f\left[\hat{x}(k),k\right] \qquad \textbf{(21)}$$

Filter-gain of EKF

$$K(k+1)=P(k+1/k)H^T(k+1)\times\left[H(k+1)P(k+1/k)H^T(k+1)+R(k)\right]^{-1} \qquad \textbf{(22)}$$

The covariance matrix of the extrapolation error is,

$$P(k+1/k)=\frac{\partial f[\hat{x}(k),k]}{\partial\hat{x}(k)}P(k/k)\frac{\partial f^T[\hat{x}(k),k]}{\partial\hat{x}(k)}+Q(k) \qquad \textbf{(23)}$$

The covariance matrix of the filtering error is,

$$P(k+1/k+1)=\left[I-K(k+1)H(k+1)\right]P(k+1/k) \qquad \textbf{(24)}$$

where $R(k)$ is the covariance matrix of measurement noise, which has diagonal elements built of the variances of angle and angle rate measurement noises and $Q(k)$ is the covariance matrix of the system noises.

Equations given as (20) - (24) represent the EKF, which fulfils recursive estimation of the satellite's rotational motion parameters about its mass center on the linear attitude.

4.2 Robust EKF against Measurement Faults

Systems with multiple sensors can accept sensor failures as long as the system still fulfils the requirements. These methods are extremely important for spacecraft systems, which must be always reliable. In this section, fault tolerant estimation based on Robust Extended Kalman Filter (REKF) will be studied. The basis of the REKF is the comparison of real and theoretical values of the covariance of the innovation sequence. When the operational condition of the measurement system mismatches the models used in the synthesis of the filter, then the Kalman gain changes according to the differentiation in the covariance matrix of the innovation sequence [11].

The innovation sequence of EKF is

$$\Delta(k)=z(k)-Hf\left(\hat{x}(k-1)\right) \qquad \textbf{(25)}$$

If we compare the real and theoretical values of the innovation covariance matrix and add a multiple measurement noise scale matrix, $S(k)$, into the algorithm as,

$$\frac{1}{M}\sum_{j=k-M+1}^{k}\Delta(k)\Delta^T(k)=H(k+1)P(k+1/k)H^T(k+1)+S(k)R(k) \qquad \textbf{(26)}$$

then, we get the definition for the scale matrix as,

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$$S(k)=\left\{\frac{1}{M}\sum_{j=k-M+1}^{k}\Delta(k)\Delta^{T}(k)-H(k+1)P(k+1/k)H^{T}(k+1)\right\}R^{-1}(k) \quad \textbf{(27)}$$

In a normal operation, the scale matrix $S(k)$ will be a unit matrix. In this case, instead of using the scale matrix, a matrix $S^{*}(k)$ has been used. To compose the scale matrix $S^{*}(k)$, the following rule is suggested in [12]:

$$S^{*}(k)=diag(s_1^{*},s_2^{*},\ldots,s_n^{*}) \quad \textbf{(28)}$$

in which

$$s_i^{*}=\max\{1,S_{ii}\} \qquad i=1,n \quad \textbf{(29)}$$

Here S_{ii} represents the i th diagonal element of the matrix $S(k)$.

After correction and diagonalization of the scale matrix via (28) and (29), the Kalman gain can be expressed in the following form as;

$$K(k+1)=P(k+1/k)H^{T}(k+1)\times\left[H(k+1)P(k+1/k)H^{T}(k+1)+S^{*}(k)R(k)\right]^{-1} \quad \textbf{(30)}$$

REKF with MMNSF works by reducing the effect of the innovation term of the faulty sensor and eliminating the estimation error caused by the faulty measurements.

5. Simulation results

In the simulations, nanosatellite is orbiting in low Earth altitudes almost circular. The period of the satellite's orbit is about 6000 sec (see Fig.1).

Only SVD, SVD/EKF and SVD/REKF methods are compared in case of measurement faults. In the first case normal mode without any bias on magnetometer measurements is presented for SVD/EKF in Fig.1. In the first panel, SVD, SVD/EKF estimations and actual values for roll angle can be seen. Second panel shows the error of the integrated method in degrees. The last panel is the cosine of the angle between two measurement vectors. Here, if the value is getting closer to 1 than jump might be expected because of faulty data from the SVD.

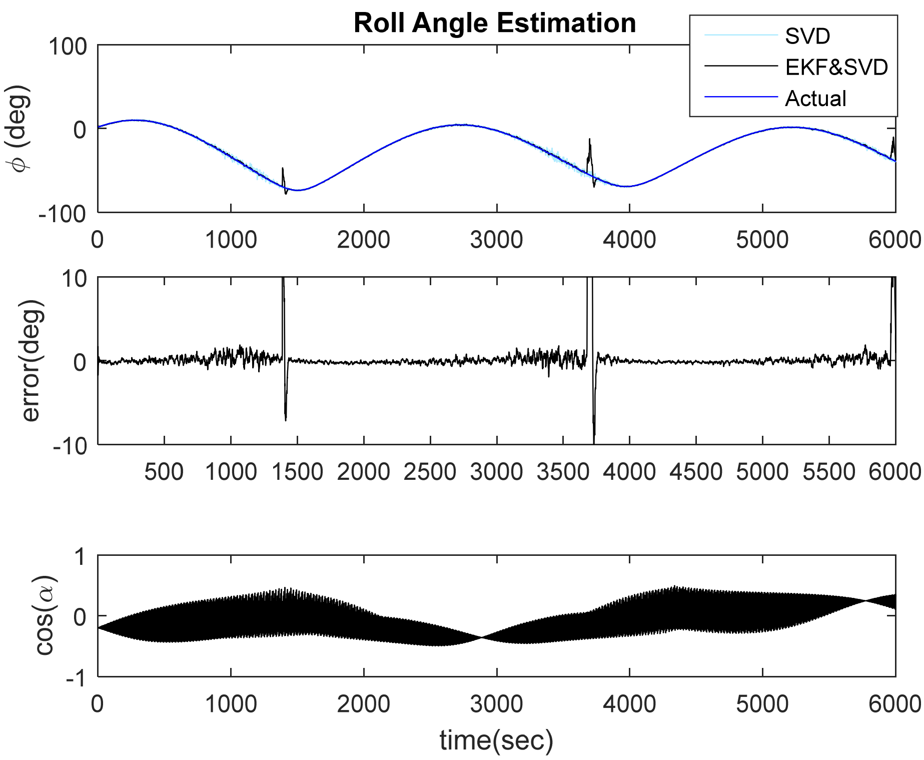

Figure 1: Roll angle estimation of SVD and SVD/EKF in normal mode

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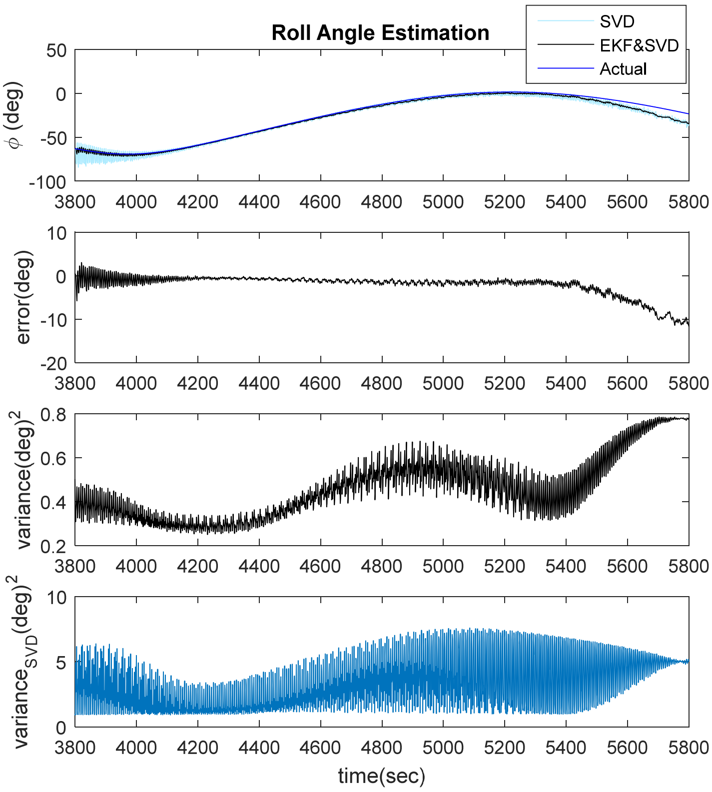

Figure 2: Roll angle estimation with 4 panels: estimation results from SVD and SVD/EKF, error of SVD/EKF, variance of the integrated filter, variance of SVD in bias type faulty case

In the Fig.2, there are four panels showing the SVD/EKF results but in the faulty case. Bias type of fault is added to the measurements for all three axes as 2000 nT between 3800-5800 sec time interval. In that interval, errors from the integrated filter are accumulating in time. Also, variance results can be seen from the third and fourth panels as EKF and SVD respectively.

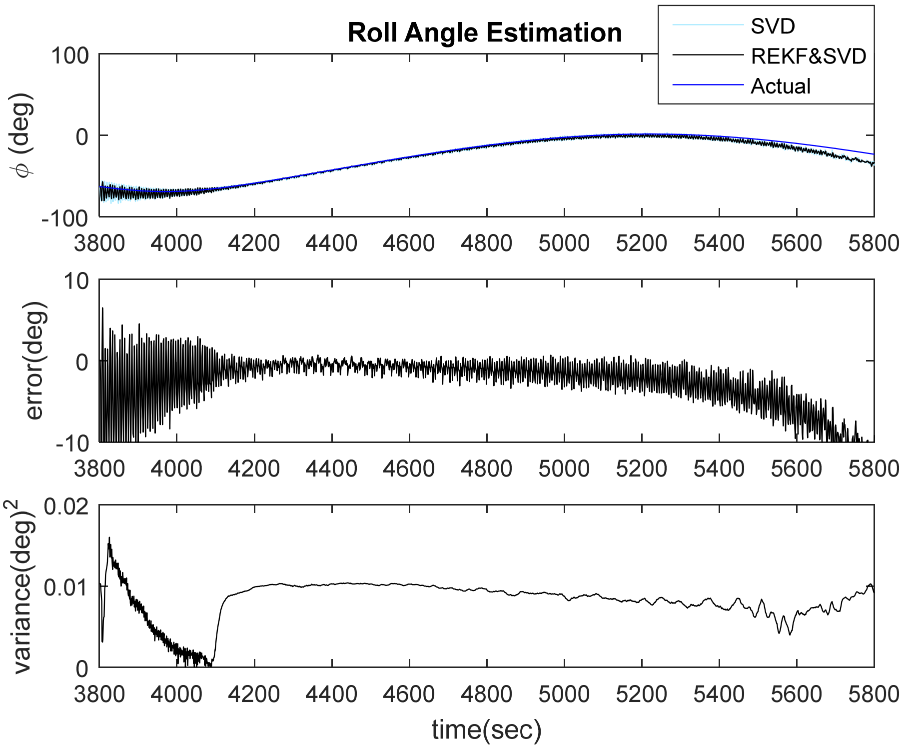

Figure 3: Roll angle estimation with 3 panels: estimation results from SVD and SVD/REKF, error of SVD/REKF, variance of the integrated filter in bias type faulty case

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The same condition with Fig.2 is used in SVD/REKF as seen in Fig.3 (2000 nT bias type of fault). Here, the errors are more oscillated than the before mentioned method which is using the measurement variance directly from the SVD (SVD/EKF). Also, it can be said that the integrated algorithm is also a naturally adaptive filter to the measurement changes.
Bias is applied to magnetometer between 3800-5800 sec of the period of the satellite. The bias (2000 nT for each axis) is sensed less in SVD/EKF method than the only SVD and the robust EKF. Furthermore, either the robust algorithm (SVD/REKF) which takes the R matrix as constant and uses an adaptive scale factor or SVD/EKF cannot eliminate the noise increment type of fault completely. However, SVD/EKF can compensate the errors much more with comparison to the robust filter.

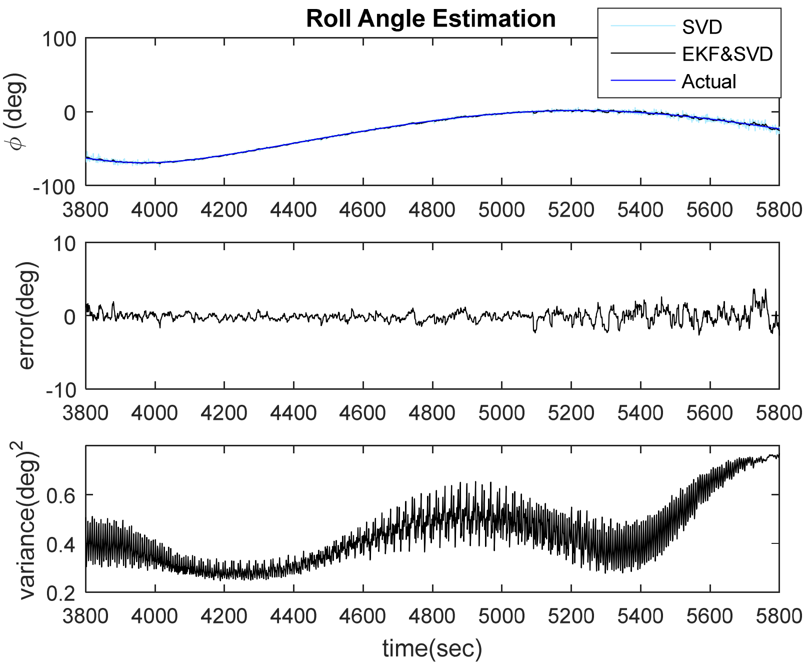

Figure 4: Roll angle estimation with 3 panels: estimation results from SVD and SVD/EKF, error of SVD/EKF, variance of the integrated filter in noise increment type faulty case

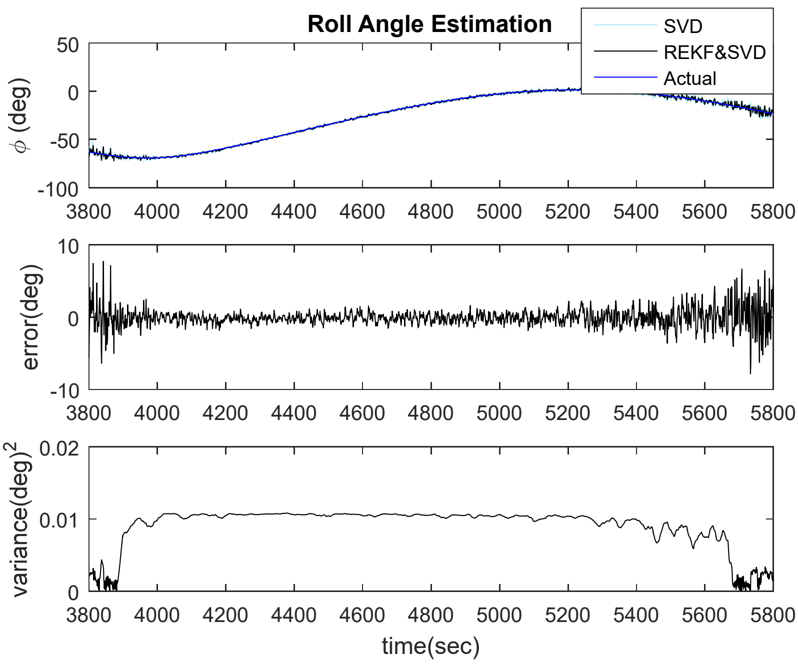

Figure 5: Roll angle estimation with 3 panels: estimation results from SVD and SVD/REKF, error of SVD/REKF, variance of the integrated filter in noise increment type faulty case

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From Table.1, it can be said that the SVD/EKF is more robust to the measurement faults in the sense of both bias and noise increment types than the SVD/REKF.

Table 1: RMS errors for SVD/REKF and SVD/EKF estimation results.

RMS (deg)	**SVD/REKF**			**SVD/EKF**		
	Roll	Pitch	Yaw	Roll	Pitch	Yaw
Normal Mode	0.39	0.29	0.49	0.25	0.27	0.39
Bias	4.05	2.56	4.29	3.25	2.39	3.75
Noise Increment	1.19	0.65	1.20	0.71	0.54	0.80

6. Conclusion

In this study, non-traditional approaches for the nanosatellite attitude and rate estimation are examined for the case of magnetometer faults. For this purpose, two different scenarios for two different approaches are investigated. In the first scenario, constant bias is considered additionally to the original magnetometer modeling. Secondly, measurement noise increment as magnetometer fault is added to the system. Simulation results show that the presented algorithms are robust against measurement noise increment type faults. Methods also can improve the results in the case of constant bias type faults but cannot correct it completely.
SVD/EKF and SVD/REKF methods are compared in two different cases of measurement faults. SVD/EKF method is better in case of measurement faults than the only SVD and REKF which leads that the SVD/EKF is robust against the measurement faults naturally.

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