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Adaptive fast fuzzy integral sliding mode attitude control for reentry vehicle with control input constraints

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Abstract
This work studies the attitude control of reentry vehicle with control input constraints, unknown external disturbances, and modeling inaccuracies. Firstly, a manifold direct adaptive fuzzy control is designed to guarantee the robustness and fast convergence property. Then, the direct adaptive fuzzy control is combined with integral sliding mode control to achieve the higher accuracy. Furthermore, to solve the problem of control input constraints, we design an auxiliary sliding mode manifold variable that can converge to zero in finite time. Finally, numerical simulations are presented to verify the robustness and effectiveness of the proposed attitude control system.

1. Introduction

Reentry vehicle is sensitive to changes in flight conditions, aerodynamic characteristics, and physical parameters because of its broad flight envelope spanning and flight condition of high Mach numbers [1]. As a result, the model of the reentry vehicle is highly nonlinear and time-varying, with modeling inaccuracies and external disturbances [2]. These problems have strong adverse influences on the performance of reentry vehicle and can increase the difficulties of vehicle control system. Therefore, robust control algorithms, such as H∞, sliding mode control(SMC), and adaptive fuzzy control have been used for reentry vehicle control [3-7].

Sliding mode control, as one of the most significant robust control methods, provides a systematic approach to the nonlinear system with modeling inaccuracies and external disturbances under matching conditions [8-9] and has been widely used for the reentry vehicle attitude control [10]. The reusable launch vehicle control problem has been solved by multiple-time-scale sliding mode control [5-6]. And a time-varying sliding mode control is designed for reusable launch vehicle with actuator fault [11]. An integrated sliding mode dynamic inversion control law is proposed to achieve a robust controller for the reentry attitude control [12]. However, these linear sliding mode (LSM) manifolds can only guarantee that the system tracking errors asymptotically converge to zero within infinite time. Compared with asymptotical convergence, however, finite time control is considered as promising control algorithm to offer superior properties, such as rapider response speed, higher tracking accuracy and faster convergence rate. Up to now, finite time control has been applied to the robotic manipulator [13], and motor control [14]. In reference [15], a continuous finite time convergence nonlinear control was developed for non-perturbed arbitrary order systems, which is a class of control algorithm similar to feedback law and has the advantage of the simiplify design structure.

Recently, fuzzy control has found extensive applications for systems that are complex, nonlinear and ill-defined. So far, there are two predominant approaches that have been expatiated in the design of an adaptive fuzzy control system: indirect and direct algorithms. According to the universal approximation theory [16] of the fuzzy logic system, the indirect adaptive fuzzy approach is applied to estimate the system uncertainty terms and then synthesizes a controller based on these estimates [17-18]. In the direct adaptive fuzzy approach, the fuzzy logic system is used to approximate an unknown ideal controller and the existence of the ideal controller can be guaranteed based on the implicit function theory [19-20]. The system governed by direct adaptive fuzzy controller can only be asymptotically convergence even existing ideal finite time controller, however, we can obtain faster convergence rate by choosing ideal finite time controller in this case. Additionally, it should be noted that the direct adaptive fuzzy controller, as the nominal control law, can be combined with other robust control algorithms, such as integral sliding mode design method [21] and H∞, to enhance the robustness and acquire higher tracking accuracy.

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In this paper, an adaptive fast fuzzy integral sliding mode control is proposed for reentry vehicle with control input constraints, unknown external disturbances, and modeling inaccuracies. The adaptive fuzzy system was used to approximate the ideal continuous finite time convergence nonlinear feedback control law. However, different from [23], the direct adaptive fuzzy control is employed to approximate the finite time control law and designed as the nominal scheme of the sliding mode control to offer higher tracking accuracy and faster convergence rate in this paper. A Lyapunov global stability analysis is performed on the closed-loop system. Then the fast fuzzy controller is combined with integral sliding mode control. Additionally, the proposed sliding mode attitude controller is improved for chattering alleviation using a modified super-twisting sliding mode algorithm. Furthermore, to solve the problem of control input constraints, we design an auxiliary sliding mode manifold variable. The auxiliary variable can converge to zero in finite time, and the convergence rate is faster than that of the auxiliary variable designed in reference [24] when the sliding mode manifold is far away from zero. Then the stability of the closed-loop system is proved via Lyapunov approach.

This paper is organized as follows. In section 2, the reentry vehicle mathematical model is described, and the feedback linearization (FBL) model of the vehicle and control objective are presented. Then we review the FLS and propose an adaptive fast fuzzy (AFF) control as the nominal control law of the integral sliding mode control (SMC), in addition, the stability of the closed-loop system is also proved based on Lyapunov Theory in section 3. Numerical simulations are presented in section 4. Finally, the conclusions are summarized in section5.

2. Mathematical Model and Problem Formation

In this paper, we formulate the attitude control target of the reentry vehicle under external distrubances and modeling inaccuracies. And feedback linearization method is applied to cope with the coupling problem of the reentry vehicle model.

2.1. Reentry vehicle model

According to the reentry vehicle model presented in [26], the dynamic equations of the vehicle model under unpower during reentry phase can be described as nonaffine nonlinear system:

$$\begin{aligned} \dot{\bar{x}} &= f(\bar{x}) + \sum_{k=1}^{3} g_k(\bar{x}) u_k + \Delta f \\ y_i &= h_i(\bar{x}) \qquad i = 1,2,3 \end{aligned} \tag{1}$$

where $f(\bar{x})$, $g_k(\bar{x})$ are smooth functions in R^n :

$$f(\bar{x}) = \begin{bmatrix} f_1(\bar{x}) & f_2(\bar{x}) & f_3(\bar{x}) & f_4(\bar{x}) & f_5(\bar{x}) & f_6(\bar{x}) \end{bmatrix}^T$$

$$= \begin{bmatrix} -p\cos\alpha\tan\beta + q - r\sin\alpha\tan\beta \\ p\sin\alpha - r\cos\alpha \\ -p\cos\alpha\cos\beta - q\sin\beta - r\sin\alpha\cos\beta \\ \dfrac{(I_{xx} - I_{yy} + I_{zz})I_{xz}}{I^*} pq + \dfrac{(I_{yy} - I_{zz})I_{zz} - I_{xz}^2}{I^*} qr \\ \dfrac{I_{xz}}{I_{yy}}(r^2 - p^2) + \dfrac{I_{zz} - I_{xx}}{I_{yy}} pr \\ \dfrac{(I_{xx} - I_{yy})I_{xx} + I_{xz}^2}{I^*} pq + \dfrac{(-I_{xx} + I_{yy} - I_{zz})I_{xz}}{I^*} qr \end{bmatrix}$$

$$g_1(\bar{x}) = [0, 0, 0, \frac{I_{zz}}{I^*}, 0, \frac{I_{xz}}{I^*}]^T$$

DOI: 10.13009/EUCASS2019-45

$$g_2(\bar{x}) = [0,0,0,0,\frac{1}{I_{yy}},0]^T$$
$$g_3(\bar{x}) = [0,0,0,\frac{I_{xz}}{I^*},0,\frac{I_{xx}}{I^*}]^T$$

where $\bar{x} = [\alpha,\beta,\mu,p,q,r]^T$ is the state vector, $u = [u_1,u_2,u_3]^T = [M_x,M_y,M_z]^T$ is the control input and M_x, M_y, M_z are the roll, pitch and yaw moments respectively. $y = [y_1,y_2,y_3]^T = h(\bar{x}) = [\alpha,\beta,\mu]^T$ denote the system output. α, β, μ stand for attack angle, sideslip angle, and bank angle respectively. p, q, r are the angular rate vector defined as roll, pitch, and yaw rates respectively. I_{xx}, I_{yy}, I_{zz} and I_{xz} are the vehicle inertia, and $I^* = I_{xx}I_{zz} - I_{xz}^2$. The vector Δf denotes the modeling inaccuracies term and unknown external disturbances term.

2.2. Feedback Linearization

According to the Lie derivative notation, the derivative of y_i in system (1) can be expressed as follows:

$$y_i^{(j)} = L_f^j(h_i) + \sum_{k=1}^{3} L_{gk}(L_f^{j-1}(h_i))u_k \quad j = 1,2,\cdots,l_i \tag{2}$$

where the Lie derivatives are defined as follows:

$$\begin{aligned} &L_f(h_i) = \frac{\partial h_i}{\partial \bar{x}_1} f_1 + \frac{\partial h_i}{\partial \bar{x}_2} f_2 + \cdots + \frac{\partial h_i}{\partial \bar{x}_n} f_n, \\ &L_f^j(h_i) = L_f(L_f^{j-1}(h_i)), \\ &L_{g_k}(h_i) = \frac{\partial h_i(\bar{x})}{\partial \bar{x}} g_k. \end{aligned}$$

and $L_{g_k}(L_f^{j-1}(h_i))$ meets the following conditions:

$$\begin{aligned} &L_{g_k}(L_f^{m-1}(h_i)) = 0, \quad m = 1,2,\cdots l_i - 1, \\ &L_{g_k}(L_f^{l_i-1}(h_i)) \neq 0 \end{aligned} \tag{3}$$

where (l_1, l_2, l_3) is the relative degree of the nonlinear system (1). Only when $l = l_1 + l_2 + l_3 = n$ (n denotes the dimension of the nonlinear system), the system can be completely transformed to a linear system. Otherwise, the nonlinear system can be only partially linearized or not be linearized.

Applying the feedback linearization technique to the model of reentry vehicle, we can calculate the time derivative of the each output y_i until the control input u_k appears in the final equations. The final equations can be obtained by differentiating double times to each output y_i. The output dynamic for y is described by:

DOI: 10.13009/EUCASS2019-45

$$\begin{bmatrix}\ddot{y}_1\\ \ddot{y}_2\\ \ddot{y}_3\end{bmatrix}=\begin{bmatrix}L_f^2(h_1)\\ L_f^2(h_2)\\ L_f^2(h_3)\end{bmatrix}+\begin{bmatrix}L_{g_1}(L_f h_1) & L_{g_2}(L_f h_1) & L_{g_3}(L_f h_1)\\ L_{g_1}(L_f h_2) & L_{g_2}(L_f h_2) & L_{g_3}(L_f h_2)\\ L_{g_1}(L_f h_3) & L_{g_2}(L_f h_3) & L_{g_3}(L_f h_3)\end{bmatrix}M+\Delta v$$
$$=F+EM+\Delta v \tag{4}$$

where $\Delta v=[\Delta v_1,\Delta v_2,\Delta v_3]$ denotes the aggregate disturbance. From equation (4), we can know that the relative degree of the reentry vehicle ($l=2+2+2=6$) is equal with the dimension of the nonlinear system ($n=6$). So the system (1) can be completely linearized.
By calculating, we can know

$$\det(E)=\frac{\cos\beta}{I*I_{yy}}-\frac{\sin\beta\tan\beta}{I*I_{yy}}\approx\frac{\cos\beta}{I*I_{yy}}\neq 0$$

where sideslip angle β is considered as 0 during the reentry phase.
Now, we define the control moment as follows:

$$M=E^{-1}(-F+v) \tag{5}$$

Then, substituting the expression (5) into (4):

$$\ddot{y}=v+\Delta v \tag{6}$$

where $v=[v_1,v_2,v_3]^T$ denotes the assistant control input.

2.3. Problem description

In this paper, the attitude control design target of reentry vehicle is to determine the control moment M vector to guarantee that the output angle $y=[\alpha,\beta,\mu]^T$ can track the desired commanded values $y_c=[\alpha_c,\beta_c,\mu_c]^T$ asymptotically under external disturbances and modeling inaccuracies, i.e.

$$\lim_{t\to\infty}e=\lim_{t\to\infty}(y-y_c)=0 \tag{7}$$

where $e=[e_1,e_2,e_3]^T=[y-y_c]^T=[\alpha-\alpha_c,\beta-\beta_c,\mu-\mu_c]^T$ is the tracking error vector.

3. The controller design

In this section, an adaptive fast fuzzy integral sliding mode controller is designed for the reentry vehicle. The controller includes two parts: 1) design a nominal controller; 2) put forward a desired integral sliding mode manifold and obtain the new adaptive control law, and the problem of control input constraints is also considered.
Before giving the sliding mode manifold and controller design, some lemmas to be used in the followingsection are p resented.

Lemma 1. [27] Consider the following system:

DOI: 10.13009/EUCASS2019-45

$$\dot{z} = f(z), \quad f(0) = 0, \quad z \in R$$

Suppose there exists a continuous function $V_f(z): U \to R$ and the following conditions are met:

1) $V_f(z)$ is positive define;

2) There exist real numbers $\rho, \quad \rho' > 0$ and $\lambda \in (0,1)$, the following inequalities can be obtained

$$\dot{V}_f + \rho V_f^{\lambda} \le 0$$
$$\dot{V}_f + \rho' V_f + \rho V_f^{\lambda} \le 0$$

Then, the origin of the system is a finite-time stable equilibrium.

Lemma 2. [30] For $z_i \in R$, $i = 1, 2, \cdots n$, $\rho'' \in (0,1]$ is a real number, and then the following inequality holds:

$$(|z_1| + |z_2| + \cdots |z_n|)^{\rho''} \le |z_1|^{\rho''} + |z_2|^{\rho''} + \cdots |z_n|^{\rho''}$$

3.1. Description of adaptive fuzzy logic system (FLS)

A FLS is composed of four principal components: fuzzifier, fuzzy rule base, inference engine, defuzzifier. The four parts of a fuzzy system decide a MISO structure: $\Omega \in R^n \to R$, where Ω is a compact set. In this paper, e and $\dot{e}$, which characterize the degree of the departure from the equilibrium point $(e, \dot{e}) = (0, 0)$, are the input of the FLS. The parameter v_{eq} is the output of the FLS.

The fuzzifier maps the detected crisp input space Ω into the fuzzy sets defined in Ω. And the defuzzifier performs a reverse function to map fuzzy sets in R into a crisp point in R [29]. The fuzzy rule base consists of a set of fuzzy rules based upon "IF-THEN". In this paper, the fuzzier maps an observed crisp point $(e, \dot{e})$ into the fuzzy sets $F_1^j, \quad F_2^j$ in Ω. The fuzzy rule base and inference engine perform a mapping from fuzzy sets of input $F_1^j, \quad F_2^j$ to fuzzy sets of output A^j. The defuzzifier maps the fuzzy sets A^j to a crisp point v_{eq}.

The fuzzy rule base consists of M rules in the following form:

$$R^j: \text{IF } e \text{ is } F_1^j \text{ and } \dot{e} \text{ is } F_2^j \text{, THEN } v_{eq} \text{ is } A^j. \quad j = 1, 2, \cdots M. \tag{8}$$

where F_1^j, F_2^j, A^j are fuzzy sets corresponding to e, $\dot{e}$, v_{eq}.

Each fuzzy rule IF-THEN of (8) defines two fuzzy application $F_1^j \times F_2^j \to A^j$. And the membership functions of F_1^j, F_2^j, A^j are expressed by $\mu_{F_1}^j, \quad \mu_{F_2}^j, \quad \mu_A^j$.

In this paper, the FLS designed by using singleton fuzzifier, product inference, center average defuzzifier, so v_{eq} can be written as linear combinations of fuzzy basis functions as follows [29]:

$$v_{eq} = f(e, \dot{e}) = \frac{\sum_{j=1}^{M} \overline{v}_{eq}^j \mu_{F_1^j}(e) \mu_{F_2^j}(\dot{e})}{\sum_{j=1}^{M} \mu_{F_1^j}(e) \mu_{F_2^j}(\dot{e})} \tag{9}$$

DOI: 10.13009/EUCASS2019-45

where $\bar{v}_{eq}^{j}$ is the point at which $\mu_{A_1^j}(e,\dot{e},v_{eq})$ achieves their maximum value.

Denote by

$$\upsilon=(\bar{v}_{eq}^{1},\bar{v}_{eq}^{2},\cdots,\bar{v}_{eq}^{M}),\ \varsigma^{j}(e,\dot{e})=\frac{\mu_{F_1^j}(e)\mu_{F_2^j}(\dot{e})}{\sum_{j=1}^{M}\mu_{F_1^j}(e)\mu_{F_2^j}(\dot{e})} \tag{10}$$

hence the formula (9) can be rewritten as follows:

$$v_{eq}=f(e,\dot{e})=\upsilon\varsigma(e,\dot{e}) \tag{11}$$

where υ are adaptive parameter vectors, and $\varsigma(e,\dot{e})$ is the fuzzy basis function vector with its element showed as:

3.2. Direct adaptive fast fuzzy (AFF) controller

In this section, the direct adaptive fast fuzzy controller is design for nonlinear system with matching external disturbances. The fuzzy controller can approximate the unknown implicit ideal controller as follows:

$$v_{eq}=\upsilon\varsigma(e,\dot{e}) \tag{12}$$

Then , the next step should be the design of an adaptive law for the parameters υ to guarantee that the control law v_{eq} can approximate the ideal controller.

Theorem 1: Consider the reentry vehicle system (6), the adaptive fast fuzzy (AFF) control (12) with the adaptive law (23) can guarantee that the attitude tracking error e asymptotically converges to zero in a fast convergence rate.

Proof: According to the equation (6) of the controlled system, the dynamic equation of the error e can be obtained as follows:

$$\ddot{e}=v_{eq}-\ddot{y}_c+\Delta v \tag{13}$$

In the work of [21], a continuous finite time convergence nonlinear control was developed for non-perturbed arbitrary order systems. Motivated by this, the origin is a globally finite time stable equilibrium for the system (13) under the feedback law

$$v_{eq}^{*}=\ddot{y}_c-\Delta v-k_1|e|^{\alpha_1}\operatorname{sgn}(e)-k_2|\dot{e}|^{\alpha_2}\operatorname{sgn}(\dot{e}) \tag{14}$$

where k_1, k_2 are positive constants, v_{eq}^{*} is ideal control output, α_1, α_2 are given as follows:

$$\alpha_1=\frac{\alpha_2\alpha_3}{2\alpha_3-\alpha_2},\quad \alpha_2\in(0,1),\quad \alpha_3=1$$

Since, the external disturbances Δv can be expressed by:

DOI: 10.13009/EUCASS2019-45

$$\Delta v = \ddot{y}_c - v_{eq}^* - k_1 |e|^{\alpha_1} \operatorname{sgn}(e) - k_2 |\dot{e}|^{\alpha_2} \operatorname{sgn}(\dot{e}) \tag{15}$$

Substituting the expression (15) into (13) , we can obtain:

$$\ddot{e} = v_{eq} - v_{eq}^* - k_1 |e|^{\alpha_1} \operatorname{sgn}(e) - k_2 |\dot{e}|^{\alpha_2} \operatorname{sgn}(\dot{e}) \tag{16}$$

Now, the optimal parameter υ^* and the minimum approximation error ψ are defined as follows:

$$\upsilon^* = \arg\min[\sup_{e,\dot{e}\in R} | v_{eq}^* - v_{eq}(e,\dot{e} \,|\, \upsilon^*) |]$$
$$\psi = v_{eq}(e,\dot{e} \,|\, \upsilon^*) - v_{eq}^*$$

Then, the equation (16) can be rewritten as:

$$\begin{aligned} \ddot{e} &= v_{eq} - v_{eq}^* - k_1 |e|^{\alpha_1} \operatorname{sgn}(e) - k_2 |\dot{e}|^{\alpha_2} \operatorname{sgn}(\dot{e}) \\ &= v_{eq}(e,\dot{e} \,|\, \upsilon) - v_{eq}(e,\dot{e} \,|\, \upsilon^*) + v_{eq}(e,\dot{e} \,|\, \upsilon^*) - v_{eq}^* - k_1 |e|^{\alpha_1} \operatorname{sgn}(e) - k_2 |\dot{e}|^{\alpha_2} \operatorname{sgn}(\dot{e}) \\ &= (\upsilon - \upsilon^*)^T \varsigma(e,\dot{e}) + \psi - k_1 |e|^{\alpha_1} \operatorname{sgn}(e) - k_2 |\dot{e}|^{\alpha_2} \operatorname{sgn}(\dot{e}) \end{aligned} \tag{17}$$

Now, the equation (17) can be expressed as a state space form

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \Psi - k_1 |x_1|^{\alpha_1} \operatorname{sgn}(x_1) - k_2 |x_2|^{\alpha_2} \operatorname{sgn}(x_2) \end{cases} \tag{18}$$

where $\Psi = (\upsilon - \upsilon^*)^T \varsigma(e,\dot{e}) + \psi$, $x_1 = e$, $x_2 = \dot{e}$.
Next, a Lyapunov global stability analysis is performed on the closed-loop system (18).
The Lyapunov function V consists of two parts: the one part is the Lyapunov function V_1 that will be generated by the Variable Gradient Method [30], the other is V_2 that is used for generating the adaptive law of the fuzzy logic system.
The idea is to choose a ∇V_1 so that it would be the gradient of a positive definite function V_1, at the same time, $\dot{V}_1$ would be negative definite or non-negative definite. In order to satisfy the requirement that V_1 be a scalar function , ∇V_1 must satisfy the following curl conditions:

$$\frac{\partial(\nabla V_1)_i}{\partial x_j} = \frac{\partial(\nabla V_1)_j}{\partial x_i}, \quad \forall i, j = 1, 2$$

Considering the closed-loop equations (18) for the most general case, ∇V_1 is assumed to be of the form:

$$\nabla V_1 = \begin{bmatrix} a_{11}x_1 + a_{12}x_2 \\ a_{21}x_1 + a_{22}x_2 \end{bmatrix} \tag{19}$$

DOI: 10.13009/EUCASS2019-45

Combining (18) and (19), $\dot{V}_1$ is found as:

$$\begin{aligned}
\dot{V}_1(x) &= (grad\ V_1)^T\,\dot{x} \\
&= \begin{bmatrix} a_{11}x_1 + a_{12}x_2 & a_{21}x_1 + a_{22}x_2 \end{bmatrix} \begin{bmatrix} x_2 \\ \Psi - k_1\left|x_1\right|^{\alpha_1}\mathrm{sgn}(x_1) - k_2\left|x_2\right|^{\alpha_2}\mathrm{sgn}(x_2) \end{bmatrix} \\
&= a_{11}x_1x_2 + a_{12}x_2^2 + \Psi(a_{21}x_1 + a_{22}x_2) - a_{21}k_1|x_1|^{1+\alpha_1} \\
&\quad - a_{22}k_1|x_1|^{\alpha_1}\mathrm{sgn}(x_1)x_2 - a_{21}k_2x_1\left|x_2\right|^{\alpha_2}\mathrm{sgn}(x_2) - a_{22}k_2\left|x_2\right|^{\alpha_2+1} \\
&= x_1x_2(a_{11} - a_{22}k_1\,|\,x_1\,|^{\alpha_1-1} - a_{21}k_2\,|\,x_2\,|^{\alpha_2-1}) + x_2^2(a_{12} - a_{22}k_2\left|x_2\right|^{\alpha_2-1}) \\
&\quad + \Psi(a_{21}x_1 + a_{22}x_2) - a_{21}k_1|x_1|^{1+\alpha_1}
\end{aligned} \tag{20}$$

When

$$\begin{cases} a_{11} - a_{22}k_1\,|\,x_1\,|^{\alpha_1-1} - a_{21}k_2\,|\,x_2\,|^{\alpha_2-1} = 0 \\ a_{12} - a_{22}k_2\left|x_2\right|^{\alpha_2-1} < 0 \\ a_{21} = a_{12} = 0,\quad a_{22} = \left|x_2\right|^{1-\alpha_2} \end{cases} \tag{21}$$

the equation (20) can be rewritten as follows:

$$\dot{V}_1(x) = -k_2x_2^2 + \Psi(a_{21}x_1 + a_{22}x_2)$$

The Lyapunov function V_2 is designed as follows:

$$V_2(x) = \frac{1}{2r}(\upsilon - \upsilon^*)^T(\upsilon - \upsilon^*)$$

where $r > 0$ is a constant.
Then the derivative $\dot{V}_2$ is given by

$$\dot{V}_2(x) = \frac{1}{r}(\upsilon - \upsilon^*)^T\,\dot{\upsilon}$$

Therefore, the derivative of the Lyapunov function V can be obtained as follows:

DOI: 10.13009/EUCASS2019-45

$$\begin{aligned}\dot{V} &= \dot{V}_1(x)+\dot{V}_2(x)\\ &= -k_2x_2^2+\Psi(a_{21}x_1+a_{22}x_2)+\frac{1}{r}(\upsilon-\upsilon^*)^T\dot{\upsilon}\\ &= -k_2x_2^2+(a_{21}x_1+a_{22}x_2)\{(\upsilon-\upsilon^*)^T\varsigma(e,\dot{e})+\psi\}+\frac{1}{r}(\upsilon-\upsilon^*)^T\dot{\upsilon}\\ &= -k_2x_2^2+\frac{1}{r}(\upsilon-\upsilon^*)^T\{\dot{\upsilon}+r(a_{21}x_1+a_{22}x_2)\varsigma(e,\dot{e})\}+(a_{21}x_1+a_{22}x_2)\psi\end{aligned} \tag{22}$$

Then the adaptive update law of υ is given by:

$$\dot{\upsilon}=-r(a_{21}e+a_{22}\dot{e})\frac{\mu_{F_1^j}(e)\mu_{F_2^j}(\dot{e})}{\sum_{j=1}^{M}\mu_{F_1^j}(e)\mu_{F_2^j}(\dot{e})}=-r(a_{21}e+a_{22}\dot{e})\varsigma(e,\dot{e}) \tag{23}$$

Substituting the adaptive law (23) into (22), $\dot{V}$ is found as:

$$\dot{V}(x)=-k_2x_2^2+a_{22}x_2\psi$$

The parameter ψ is the minimum approximation error. According to the universal approximation theory of the fuzzy logic system, the condition that ψ is small enough by designing the direct fuzzy controller can be guaranteed, so the condition of $k_2x_2^2>|a_{22}x_2\psi|$ is satisfied.
Then

$$\dot{V}\le 0 \tag{24}$$

Therefore, the expression of the Lyapunov function V is found as:

$$V=V_1+V_2\ge 0 \tag{25}$$

Thus, from the equation (24) and (25), it can be known that $V(t)$ is positive definite and $\dot{V}(t)$ is negative definite. Using the Lyapunov Theory, the attitude tracking error e can converge to zero.
Furthermore, we can know that the condition of $\dot{V}=0$ is satisfied only when the trajectory of the state equation (19) is at the equilibrium point $x_2\equiv 0, x_1=\dot{x}_2=0$.
Since, the attitude tracking error e can asymptotically converge to zero.

Remark 1. It should be noted that the new adaptive law (13) is different from the current adaptive law of the direct adaptive fuzzy (AF) control [33] due to existing the parameters a_{21} and a_{22}. The parameters a_{21} and a_{22} must satisfy the condition (22) to ensure the stability of the closed-loop system. According to the condition, the adaptive law includes the term $x_2^{1-\alpha_2}$ that has a critical influence, which can ensure that the adaptive parameter has faster regulative rate. So the attitude tracking error e can quickly converge to zero.

3.3. Realization of adaptive fast fuzzy integral sliding mode control (AFFSMC)

The sliding mode control method includes two parts: the nominal control law and robustness term. The direct adaptive fast fuzzy controller that has been designed in previous subsection, is employed as nominal control law of

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sliding mode control in this subsection. And the robustness term will be proposed to guarantee the performance of the controller.
According to the concept of the integral sliding mode in [12], the proposed sliding mode manifold is designed as follows:

$$S = \dot{e} - \int_0^t v_{eq} dt \tag{26}$$

Sliding mode manifold is established with a proper controller, we can obtain that:

$$\dot{S} = \ddot{e} - v_{eq} = 0 \tag{27}$$

And according to equation (7), we can know that

$$\ddot{e} = \ddot{y} - \ddot{y}_c \tag{28}$$

Take $v = v_{eq} + v_s$ into consideration and substitute (6), (28) into (27), the dynamic of the proposed sliding mode can be expressed as:

$$\dot{S} = \ddot{y} - \ddot{y}_c - v_{eq} = v + \Delta v - \ddot{y}_c - v_{eq} = v_s + \Delta v - \ddot{y}_c = v_s + \varphi \tag{29}$$

Assumption 1. The function φ and its derivative are assumed to be bounded and satisfies

$$\|\varphi\| \le \eta_1, \quad \|\dot{\varphi}\| \le \eta_2$$

where η_1, $\eta_2 > 0$ is a constant.
To achieve strong robusrness and chattering free, inspired by [24], a new super-twisting (STW) second order sliding mode algorithm is applied as follows:

$$\begin{aligned} v_s &= -\alpha |S|^{1/2} \operatorname{sgn}(S) - \frac{\mu_0}{\mu_1}(\exp(\mu_1 |S|) - 1)\operatorname{sgn}(S) + \omega \\ \dot{\omega} &= -\beta \operatorname{sgn}(S) - \frac{\beta_0}{\beta_1}(\exp(\beta_1 |S|) - 1)\operatorname{sgn}(S) \end{aligned} \tag{30}$$

where μ_0, μ_1 and β_0 are positive constants.
Remark 2. The new proposed reaching law has faster convergence rate than that of [22] by similar analysis as the reference [32]. The reaching law can also ensure that the sliding mode manifold is established in finite time, and eliminating the chattering phenomenon.
Now, substituting (30) into (29), we can obtain that

$$\dot{S} = -\alpha |S|^{1/2} \operatorname{sgn}(S) - \frac{\mu_0}{\mu_1}(\exp(\mu_1 |S|) - 1)\operatorname{sgn}(S) + \omega + \varphi \tag{31}$$

DOI: 10.13009/EUCASS2019-45

Theorem 2. Consider the reentry vehicle system (6) with the sliding mode manifold defined by equation (26). The adaptive fuzzy integral sliding mode control law (32) with the adaptive law (13) can ensure that the sliding mode manifold S will converge to the corresponding region in finite time, and the attitude tracking error e will converge to zero.

$$\begin{aligned} v_{sat} &= sat(v) = sat(v_{eq} + v_s) \\ &= sat(\upsilon\varsigma(e,\dot{e}) - \alpha\left|\tilde{S}\right|^{1/2}\operatorname{sgn}(\tilde{S}) - \frac{\mu_0}{\mu_1}(\exp(\mu_1\left|\tilde{S}\right|)-1)\operatorname{sgn}(\tilde{S}) + \omega) \end{aligned} \tag{32}$$

$$\tilde{S} = S - \hat{S} \tag{33}$$

$$\dot{\hat{S}} = -k_{s1}\left|\hat{S}\right|^{\gamma} sgn(\hat{S}) - k_{s2}\hat{S}^{\tau} + v_{sat} - v \tag{34}$$

$$v_{sat} = sat(v) = \begin{cases} v_{\max}, & if\ v > v_{\max} \\ v, & if\ v_{\min} \le v \le v_{\max} \\ v_{\min}, & if\ v < v_{\min} \end{cases} \tag{35}$$

where γ, τ, k_{s1} and k_{s2} are positive constants.
Because of introducing the auxiliary variable $\tilde{S}$ in equation (29), the sliding mode manifold S is replaced with auxiliary variable $\tilde{S}$ in new monified STW.

Proof. The following Lyapunov function candidate is considered

$$V_3 = \frac{1}{2}\tilde{S}^T\tilde{S} \tag{36}$$

Then the time derivative of Lyapunov function V_3 is given by:

$$\begin{aligned} \dot{V}_3 &= \tilde{S}^T\dot{\tilde{S}} = \tilde{S}^T(\ddot{e} - v_{eq}) = \tilde{S}^T(v_s + \varphi) \\ &= \tilde{S}^T[-\alpha\left|\tilde{S}\right|^{1/2}\operatorname{sgn}(\tilde{S}) - \frac{\mu_0}{\mu_1}(\exp(\mu_1\left|\tilde{S}\right|)-1)\operatorname{sgn}(\tilde{S}) + \omega + \varphi] \end{aligned} \tag{37}$$

According to the inequality of arithmetic and geometric means

$$\tilde{S}(\varphi+\omega) \le a\left\|\tilde{S}\right\|^2 + \frac{\left\|\varphi+\omega\right\|^2}{a} \tag{38}$$

The equation (37) satisfies the following inequality

$$\begin{aligned} \dot{V}_3 &= \tilde{S}^T[-\alpha\left|\tilde{S}\right|^{1/2}\operatorname{sgn}(\tilde{S}) - \frac{\mu_0}{\mu_1}(\exp(\mu_1\left|\tilde{S}\right|)-1)\operatorname{sgn}(\tilde{S}) + \omega + \varphi] \\ &\le -\tilde{S}^T\frac{\mu_0}{\mu_1}(\exp(\mu_1\left|\tilde{S}\right|)-1)\operatorname{sgn}(\tilde{S}) - \tilde{S}^T\alpha\left|\tilde{S}\right|^{1/2}\operatorname{sgn}(\tilde{S}) + \vartheta \end{aligned} \tag{39}$$

DOI: 10.13009/EUCASS2019-45

where $\vartheta = \|\varphi + \omega\|^2 / a$, a is a positive constant, and the inequality (38) is applied in (39).
According to the Lemma 2, the following facts are found as:

$$-\tilde{S}^T \frac{\mu_0}{\mu_1}(\exp(\mu_1 |\tilde{S}|) - 1)\,\mathrm{sgn}(\tilde{S}) \le -\mu_2 \tilde{S}^T \tilde{S} \le \frac{-2\mu_2}{\mu_3} V \qquad \textbf{(40)}$$

$$\begin{aligned} -\tilde{S}^T \alpha |\tilde{S}|^{1/2}\,\mathrm{sgn}(\tilde{S}) &\le \alpha(\frac{2}{\mu_3})^{\frac{1+0.5}{2}}(\frac{\tilde{S}^T\tilde{S}}{2})^{\frac{1+0.5}{2}} \\ &\le \alpha(\frac{2}{\mu_3})^{\frac{3}{4}} V^{\frac{3}{4}} \quad (\mu_3 \ge 1) \end{aligned} \qquad \textbf{(41)}$$

Therefore, we can obtain that

$$\dot{V}_3 \le -\mu_4 V_3 - \mu_5 V_3^{\frac{3}{4}} + \vartheta \qquad \textbf{(42)}$$

where $\mu_2 = \mu_0 / \mu_1,\ \ \mu_4 = -2\mu_2 / \mu_3,\ \ \mu_5 = \alpha(2/\mu_3)^{3/4}$.
The inequality (42) can be rewritten as follows:

$$\dot{V}_3 \le -(\mu_4 - \frac{\vartheta}{V_3})V_3 - \mu_5 V_3^{\frac{3}{4}}$$

If $\mu_4 - \vartheta / V_3 > 0$, the condition that the finite time convergence to region can be satisfied by Lemma 1. Thus, the sliding mode manifold $\tilde{S}$ can reach the region

$$\|\tilde{S}\| \le \sqrt{\frac{\mu_3 \vartheta}{\mu_2}}$$

in finite time.
Now, we will prove that the attitude tracking errors e can converge to zero.
The fuzzy controller, as the nominal control law, is design as continuous finite time convergence nonlinear control law, so the proof can refer to Theorem 1.
Remark 3. Based on the fuzzy integral sliding mode control, the control input constraints problem is considered. To solve the problem, we design an auxiliary sliding mode manifold variable. The auxiliary variable $\tilde{S}$, motivated by the reference [33], is applied to amend the sliding mode manifold in the control system. But the saturation controller of the reference [33] can only guarantee that the auxiliary variable converge to zero asymptotically, not in finite time. By Lemma 1, the auxiliary variable $\tilde{S}$ can converge to zero in finite time. And the convergence rate is faster than that of the auxiliary variable designed in reference [24] when the sliding mode manifold is far away from zero.

4. Simulation results

In this section, the simulations of the reentry vehicle in conjunction with the proposed controller are carried to validate the effectiveness. The aerodynamic model and parameters of 6-DOF mathematic model of reentry vehicle given in [30] is used.
Case A: Fast convergence rate and robustness illustration of the AFF.

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The attitude angle tracking responses are shown in Figure 1, which shows that the attitude angles can track the command informations. The attitude angle that controlled by AFF converge to commands in a short time less than 3s, but AF fails to this condition. Thus, compared with the current adaptive fuzzy (AF) controller, the new AFF shows a faster response and convergence speed.

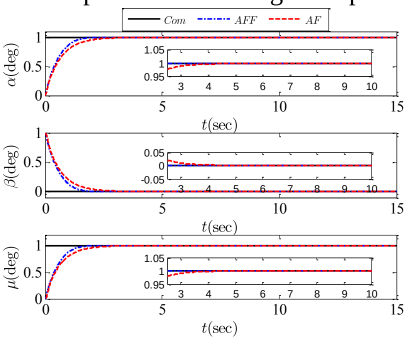

Figure 1: Simulation results of AFF and AF

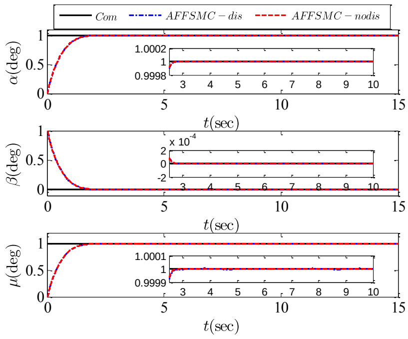

Figure 2: Simulation results of AFFSMC

Case B: Comparison of robust performance of the nonlinear system with matching disturbance Δv governed by direct fuzzy controller and AFFSMC.

(1) Robustness illustration of the AFFSMC

The control law (12) is applied when the uncertainty disturbance problems are considered. The comparisons of the attitude tracking are shown for this case in Figure 2. The outputs of the system governed by AFFSMC can converge to 10^{-4} times of their desired, which varifies the AFFSMC robustness against modeling uncertainties and external disturbances.

(2) Comparison of robust performance of the AFF and AFFSMC

To compare the robust performance of AFF and AFFSMC, about 30 percent of bias condition for inertia and 30 percent of bias condition for air density are considered in the simulation scenario. And the external disturbance torque ΔM becomes 2 times original.

Figure 3 presents the results of the attitude angle tracking of the system controlled by AFFSMC and AFF. The attitude tracking of the system controlled by AFFSMC, that even converging to 10^{-4}, are smaller than that of system controlled by AFF. Therefore, AFFSMC possesses strong robustness against model uncertainties and external disturbances.

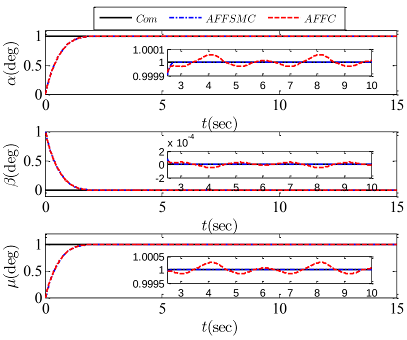

Figure 3: Simulation results of AFFSMC and AFF

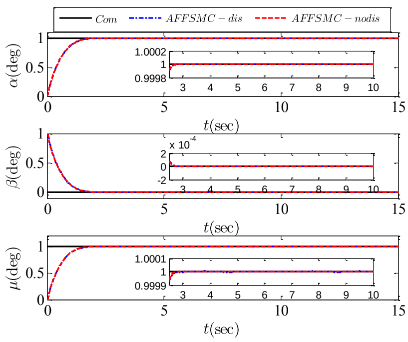

Figure 4: Simulation results of of AFFSMC

Case C: Performance illustration of the proposed controller in the presence of control input constraints

In this case, control input constraints are $v_{\max,\min} = \pm 0.3$, and other parameters of the simulation is not changed.

The results of the attitude angle tracking, sliding mode manifold and control input are presented in Figure 4, 5, 6 respectively. From Figure 4, we can know that the attitude angles output still converge to command information in a fast response speed even existing control input constraints, model uncertainties and external disturbances. The sliding mode manifold responses are shown in Figure 5. It is clear that the sliding mode manifold values of the AFFSMC do not converge to zero but are kept around the zero in the presence of external disturbances and control input constraints. The results of the control input variables are shown in Figure 6. From Figure 6, we can know that: 1) there exist the control input saturation in the initial stage of the system response; 2) the control inputs are smooth continuous; 3) the chattering is alleviated by super twisting algorithm.

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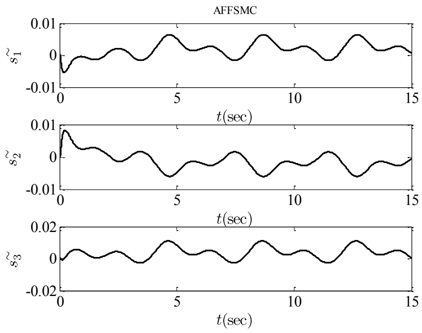

Figure 5: Simulation results of the sliding mode manifold of AFFSMC

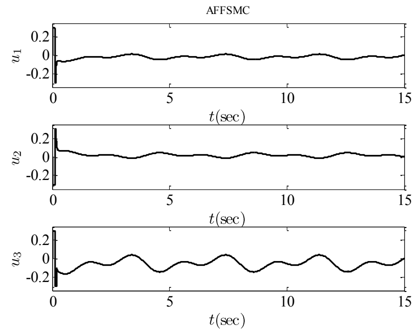

Figure 6: Simulation results of the control variables of AFFSMC

5. Conclusions

This work proposes a adaptive fast fuzzy integral sliding mode control for the attitude control of reentry vehicle with external disturbances, model inaccuracies, and control input constraints. This paper has three major aspects of contributions:

(1) The AFFSMC attitude control adopts adaptive fuzzy algorithm as the nominal controller. By designing suitable fuzzy logic system and the adaptive law, it can guarantee that the attitude angle tracking converge to reference signals in a fast convergence rate. Meanwhile, because of the direct adaptive fuzzy algorithm designed as nominal control law, the robustness of the reentry vehicle is improved. The performances of the proposed controller have been examined via numerical simulations of the governing nonlinear system.

(2) The proposed sliding mode attitude controller is improved for chattering alleviation using a modified super-twisting sliding mode algorithm.

(3) The problem of control input constraints has been solved. We design an auxiliary sliding mode manifold variable that can converge to zero in finite time when the sliding mode manifold is far away from zero.

The tracking errors remain low even under external disturbances, model inaccuracies and control input constraints. Thus, the proposed controller is not only robustness, but also able to accommodate actuator failures under control input constraints.

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