

# Thermo physics process peculiarities of modern air breathing engines

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## Abstract

The paper discusses some essential features of thermo physics process for modern high temperature air breathing engines. The first part of paper presents a thermodynamically compatible theory of working heat cycle verified by creation experience of some high temperature jet engines. The thermodynamically compatible theory bases on a weak solutions of initial quasi-linear equations. The second part demonstrates physics of entropy growth and inextricably related loss of total pressure. The analysis bases on conservation laws of mass, momentum and energy, describing the movement working media in the presence of intense heat addition in combustors. Accurate coordination of core and bypass flows is considerate.

## Introduction

Modern high temperature air breathing engines (ABE) have a complex geometry of flow path and require detailed multidimensional modelling throughout a whole engine at the design stage and finishing. The thermodynamic modelling includes into account an accurate coordination of engine basic components (for a turbojet – a compressor, combustor and turbine, for a high-speed air-breathing engine - an air inlet, combustor and nozzle). Coordination of the various components is defined by energy losses directly in the components of the engine.

Experiences gained so far on the development of ABE demonstrate the need to some additional review of the thermodynamics issues. Well known experience of V generation aero engine creation has pointed out the imperfection of modern working process models. The theoretically calculated in design stage engine parameters essentially differ from experimental data for first engine units (for example, X-51A, TP-400 and F-135 [1-6]). Hence, long and expensive creation of new high temperature jet and gas turbine engines is required. Some basic complexities of designing high-temperature ABE relate to origin of so-called "unexpected" heat of working process.

The second part paper presents detail explanation the second law of thermodynamics and shows connection of entropy growth with total pressure losses and dissipation of radiation energy. Additional peculiarities of Brayton's cycle for jet propulsions are detail considered. By that the entropy nature may be clearly demonstrated and estimated. In particular, entropy growth correspond total pressure losses and energy dissipation. In the common case the entropy isn't a state function. The entropy growth closely connects with molecule mass decreasing.

Modern theoretical physics has no answer on entropy growth nature for an isolate thermo dynamical system. In the famous 10-volume course on theoretical physics argues: "The question of the physical basis of monotonic increase of entropy thus remains open" (Landau and Lifshitz. Stat. Phys. 1996, p. 52, [7]). On the problem of increasing entropy the Patriarch of theoretical physics V. L. Ginzburg also puts in first place among the outstanding "three great challenges": "First, we are talking about the increase of entropy, irreversibility and the "arrow of time" [8].

The entropy growth follows from the full law of energy conservation and shows the relationship of the entropy growth with total pressure losses and energy dissipation. The paper presents the one velocity two components model of compressible medium motion (for gaseous and radiation components) [9, 10]. Thermal radiation energy exchange does considerate system as dissipative system and this is one of the dominant mechanisms for energy balance with surroundings for aerospace problems. The high level of gas dynamics models, based on the system of conservation laws with radiation, allow getting accurate simulation of thermodynamic processes for high temperature air breathing engines. The paper contains typical examples for whole turbojet and turbofan engines and their components.

## 1. Closed system of quasi-linear equations with continuous heat

We start with the equations of the first and second laws of phenomenological thermodynamics, which here we write using generally accepted notation

$$\frac{\delta Q}{\delta t} = T \cdot \frac{ds}{dt} = \frac{d\varepsilon}{dt} + p \cdot \frac{dv}{dt} . \quad (1.1)$$

The ratio (1.1) describes quasi-static reversible processes and is the initial energy ratio of thermodynamics. About equation (1.1) S. K. Godunov [11] rightly points out that in the statements of thermodynamics "proof of the existence of universal multiplier integrating equation heat thermally homogeneous systems shall be conducted only based on the study of energy equations, do not represent a closed system".

It is also important to emphasize the issue of principle, namely that the use of ratios (1.1) when solving thermodynamic tasks allows you to obtain the weak solutions of quasi-linear equations involving loss of full pressure. The ratio (1.1) provides the theorems of "alive force", which in our case can be written in the form

$$\frac{d}{dt} \left( \frac{q^2}{2} \right) + \frac{\bar{q}}{\rho} \text{grad} p = 0 . \quad (1.2)$$

The equation theorem of "alive forces" (1.2) displays a simple scalar multiplication of a vector equations of motion

$$\frac{d\bar{q}}{dt} + \frac{1}{\rho} \text{grad} p = 0 \quad (1.3)$$

The vector equation (1.3) includes in the three-dimensional case three scalar conservation laws of momentum, which are easily represented in divergence form. At the same time, the scalar equation (1.2) cannot lead to divergent differential or integral law of conservation

$$\frac{\partial}{\partial t} \left( \rho \frac{q^2}{2} \right) + \text{div} \left( \rho \bar{q} \frac{q^2}{2} \right) + \bar{q} \cdot \text{grad} p = 0 \quad (1.4)$$

Application of theorem "alive forces" (1.2) or equation of specific kinetic energy changes (1.4), as well as the use of ratios (1.1) does not allow you to receive concerted thermodynamically generalized solutions for thermal losses of pressure.

The present report addresses the closed system of quasi-linear equations of thermodynamics, next of integral balance conservation laws of mass, momentum and energy for a fixed time with volume  $\omega$  the boundary  $\gamma$  [12]

$$\begin{aligned} \frac{\partial}{\partial t} \iiint_{\omega} \rho d\omega &= - \iint_{\gamma} \rho \bar{q} \cdot \bar{n} d\gamma \\ \frac{\partial}{\partial t} \iiint_{\omega} \rho \bar{q} d\omega &= - \iint_{\gamma} \left( p \cdot \bar{n} + \rho \bar{q} (\bar{q} \cdot \bar{n}) \right) d\gamma \\ \frac{\partial}{\partial t} \iiint_{\omega} \rho \left( \varepsilon + \frac{q^2}{2} \right) d\omega &= - \iint_{\gamma} \left( p + \rho \left( \varepsilon + \frac{q^2}{2} \right) \right) \bar{q} \cdot \bar{n} d\gamma + \iiint_{\omega} \rho \frac{\delta Q}{\delta t} d\omega \end{aligned} \quad (1.5)$$

where  $\bar{n}$  - the unit vector normal to the exterior boundaries. Determining thermodynamic process of this system is the presence in the volume specific heat supply with speed  $\delta Q/\delta t$ .

From integral system (1.5) we can get differential equations of continuity, momentum and energy

$$\begin{aligned}
\frac{\partial \rho}{\partial t} + \operatorname{div}(\rho \bar{q}) &= 0 \\
\frac{\partial}{\partial t} \rho \bar{q} + \operatorname{div} \Pi &= 0 \\
\frac{\partial}{\partial t} \left[ \rho \left( \varepsilon + \frac{q^2}{2} \right) \right] + \operatorname{div} \left[ \left( p + \rho \left( \varepsilon + \frac{q^2}{2} \right) \right) \bar{q} \right] &= \rho \frac{\delta Q}{\delta t}.
\end{aligned} \tag{1.6}$$

Systems of equations (1.5) and (1.6) close the state equation

$$\varepsilon = \varepsilon(p, \rho). \tag{1.7}$$

Differential ratio (1.6) in divergence form gives the weak solutions of gas dynamics equations in the presence of heat supply. Now we compare these mathematical formulations with the first and second principles of thermodynamics (1.1). From the left part of the energy equation (1.6) we can write

$$\rho \frac{\partial}{\partial t} \left( \varepsilon + \frac{q^2}{2} \right) + \left( \varepsilon + \frac{q^2}{2} \right) \frac{\partial \rho}{\partial t} + \rho \bar{q} \operatorname{grad} \left( \varepsilon + \frac{q^2}{2} \right) + \left( \varepsilon + \frac{q^2}{2} \right) \operatorname{div}(\rho \bar{q}) + \operatorname{div}(p \bar{q}) = \rho \frac{\delta Q}{\delta t}.$$

Using continuity equation we arrive at the ratio

$$\frac{d}{dt} \left( \varepsilon + \frac{q^2}{2} \right) + \frac{1}{\rho} \operatorname{div}(p \bar{q}) = \frac{\delta Q}{\delta t}. \tag{1.8}$$

This form accepts differential law of energy conservation when using closed mathematical formulations of process. Further, taking into account the equation of continuity the ratio (1.8) comes

$$\frac{d\varepsilon}{dt} + p \frac{d}{dt} \left( \frac{1}{\rho} \right) + \frac{d}{dt} \left( \frac{q^2}{2} \right) + \frac{\bar{q}}{\rho} \operatorname{grad} p = \frac{\delta Q}{\delta t}. \tag{1.9}$$

Ratio (1.9) shows that the applied heat to medium volume  $\omega$  moving with speed  $\bar{q}$ , spent on increasing the specific internal energy (with speed  $d\varepsilon/dt$ ), on changing of the work (quasi-static  $pd(1/\rho)/dt$ ), on changing of the specific kinetic energy (with speed  $d(q^2/2)/dt$ ) and the amount of extra power, produced pressure forces  $\bar{q}(\operatorname{grad} p)/\rho$ , missing usually in energy ratios of thermodynamics. Equation (1.8) and (1.9) allow you to simulate the appearance of the full pressure losses at the addition to the heat moving gas stream. The entropy growth closely connects with molecule mass decreasing.

The main conclusion of the carried out transformation is the fact that the process of heat supply to the moving gas flow to smooth solutions runs ratio (1.9), differing from the first laws of thermodynamics for quasi-static reversible processes (1.1). Based on the ratio (1.9) will give response to outfit significant theoretical issues. The first question is in the wording of the conditions of execution of another law of conservation of gas dynamics, namely the law of conservation of entropy. With this major issue are other three important issues: the "mechanical" energy equation (theorem of "alive force"), and the loss of complete pressure and on the arm of the famed "Riemann" errors in thermodynamics.

## 2. Two components model of a gaseous and radiation medium

Further we present the common conservation laws system for the case of the two components model of a gaseous and radiation medium. There are used the index g for gas and the index f for radiation components of medium (for example, for densities  $\rho_g$  and  $\rho_f$ ). For the one velocity model the values of velocity components u, v, w at the axis x, y, z are the same for each medium components. The integral conservation laws are as [12]

$$\begin{aligned}
\frac{d}{dt} \iiint_{\omega(t)} \rho_k d\omega &= \iiint_{\omega(t)} q_k d\omega, \\
\frac{d}{dt} \iiint_{\omega(t)} \rho_k \vec{u} d\omega &= - \iint_{\gamma(t)} p_k \vec{n} d\gamma + \iiint_{\omega(t)} \vec{r}_k d\omega \\
k &= g, f \quad (2.1)
\end{aligned}$$

$$\frac{d}{dt} \iiint_{\omega(t)} \rho_k \left( \frac{1}{2} q^2 + \varepsilon_k \right) d\omega = - \iint_{\gamma(t)} p_k \vec{u} \cdot \vec{n} d\gamma + \iint_{\gamma(t)} K_k \text{grad} T_k \cdot \vec{n} d\gamma + \iiint_{\omega(t)} L_k d\omega.$$

Here  $q^2$  - the square of the velocity vector and

$$L_g = C_{gf}(T_f - T_g) + Q'_g, \quad L_f = C_{gf}(T_g - T_f) + Q'_f.$$

Energy conservation laws are written for heat transfer gas and radiation components (the second terms in the right side of these equations,  $K_g$  and  $K_f$  correspondently thermo transfer coefficients for gas and radiation parts). The last terms in the right side of initial energy equations describe an energy exchange between gas and radiation parts (the space thermostat). The terms  $Q_g$  and  $Q_f$  are an additional energy sources, which include into account energy exchange channels (for example, in the case chemical reactions).

We obtain the summary laws as composition of equations (2.1)

$$\begin{aligned}
\frac{d}{dt} \iiint_{\omega(t)} \rho d\omega &= \iiint_{\omega(t)} q d\omega, \\
\frac{d}{dt} \iiint_{\omega(t)} \rho \vec{u} d\omega &= - \iint_{\gamma(t)} p \vec{n} d\gamma + \iiint_{\omega(t)} \vec{r} d\omega, \\
\frac{d}{dt} \iiint_{\omega(t)} \rho \left( \frac{1}{2} q^2 + \varepsilon \right) d\omega &= - \iint_{\gamma(t)} p \vec{u} \cdot \vec{n} d\gamma + \iint_{\gamma(t)} W d\gamma + \iiint_{\omega(t)} Q d\omega.
\end{aligned} \quad (2.2)$$

Here

$$\begin{aligned}
\rho &= \rho_g + \rho_f, p = p_g + p_f, \varepsilon = \rho_g / \rho \cdot \varepsilon_g + \rho_f / \rho \cdot \varepsilon_f, \\
W &= K_g \text{grad} T_g + K_f \text{grad} T_f, Q = Q_g + Q_f.
\end{aligned} \quad (2.3)$$

The system (2.2) is closing by the state equations

$$\varepsilon_k = \varepsilon_k(\rho_k, T_k), p_k = p_k(\rho_k, T_k), k = g, f.$$

The considered radiation component is the photon compressible gas with non-zero mass of particles [13-15]. Using the simple enough gas kinetic theory we can determine a mass  $m$  of radiation component. Average kinetic energy is

$$E = \frac{m v_{av}^2}{2} = \frac{3}{2} k T_0 = m \frac{3}{2} \frac{R_u T_0}{m N} = \frac{m}{\kappa} \frac{3}{2} \frac{p_0}{\rho_0} = \frac{9}{8} m c^2.$$

Here  $k = R_u / N$  - the Boltzmann's constant,  $R_u$  - the universe gas constant,  $N$  - the Avogadro's number. Therefore we calculate for the radiation component (with  $T_0 = 2.73 \text{ K}$ )

$$m = \frac{4}{3} \frac{k T_0}{c^2} = 5.6 \cdot 10^{-40} \text{ kg}.$$

The gas constant  $R = R_u/mN$  and the specific heat  $c_v$  and  $c_p$  can be found from

$$R = \frac{k}{m} = 0.25 \cdot 10^{17} \text{ J/kgK}, \quad c_v = 0.75 \cdot 10^{17} \text{ J/kgK}, \quad c_p = R + c_v = 1 \cdot 10^{17} \text{ J/kgK}.$$

The radiation component has the state equation  $p = \rho RT$ , which can be written as  $p = (\kappa - 1)\rho\varepsilon$ , where  $\varepsilon = c_v T$  - specific internal energy.

Further we present practice applications of our typical simulations for internal aero physics internal and external problems on the base of the system (2.2). All additional details of our simulation may be found [9-10, 13-15].

### 3. Total pressure losses modelling

We demonstrate now graphically for the one-dimensional case total pressure losses in the plane  $(p, 1/\rho)$ . The figure 1 shows the chart of Brayton's thermal cycle when applying ratios (1.8) and (1.9). The line segment OA shows a line of permanence total pressure at addition of heat Q, the cut EC – the line of the same quantity of heat Q to the moving stream lossless total pressure (equivalent to "Riemann errors"), the cut EG corresponds real heat supply total pressure losses, the cart total pressure line AB meets the constant temperature, the braking curves (e), (f), (g)-isentropic lines.

Total pressure losses appear in heat supply process [16, 17] or in the process of moving gas compression (at the intersection of characteristics in the space-temporal plane). In the latter case, the relationship of pressure with the density is expressed Hugoniot's conditions.

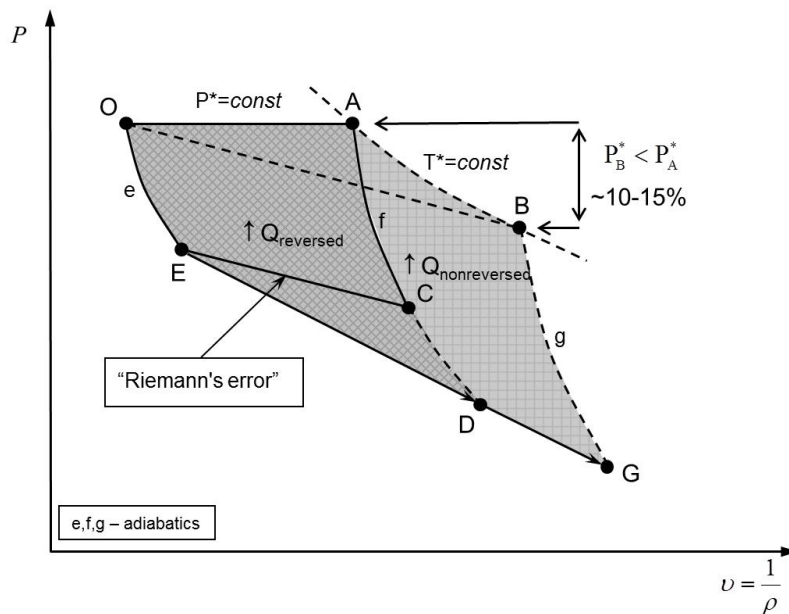


Figure 1: Total pressure losses in the plane  $(p, 1/\rho)$ .

These highly visible losses demand to use increased common compression of modern high-temperature ABE. Currently, the thermodynamic relations described in additional losses in most cases are ignored.

The question of the physical basis of monotonic increase of entropy may be solved by weak solutions of quasy-linear systems. Weak solutions of quasy-linear systems are the solutions with total pressure losses (including a smooth solution with heat addition).

### 4. Some technical applications for turbofan engine

Using our theoretical models we would like to show the possibility of improving the coordination of primary and secondary flows two shaft engine in front of the mixing camera. To optimize the process of mixing the equality condition is desirable for static and total pressure mixing flows. As one of the most effective ways of achieving

optimality conditions of mixing it may be recommend increasing the load of low-pressure turbine (while maintaining speed) by increasing the reactivity of LPT and application of additional stator in diffuser channel after LPT. Developed with this increased LPT power increases the total compression ratio of the fan and total pressure in the secondary flow.

We show preliminary integrated assessment of possible increased load LPT using known Euler's formulas of turbine. Define a volume of gas from the vanes force in case of steady-state flow in the channel formed by the shoulder blades and the control surface, shown in figure 2.

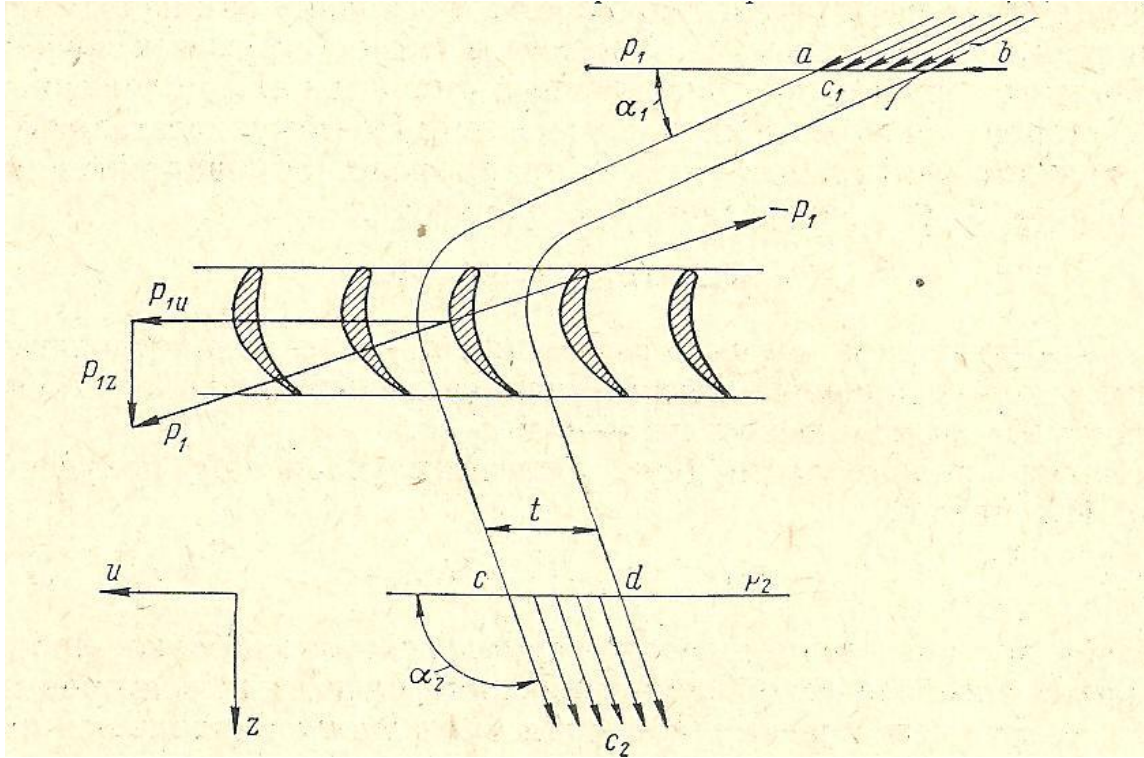


Figure 2: The channel formed by the shoulder blades and the control surface.

According to the law of momentum conservation (2.2) the time derivative equal to the main vector of external forces

$$\frac{d}{dt} \iiint_{\omega(t)} \rho \bar{c} d\omega = \bar{F}. \quad (4.1)$$

The application of this law to the reference surface at the steady flow through the cascade channel (fig. 2) gives

$$\bar{F} = \int_{\omega_1} \rho_1 c_{1n} \bar{c}_1 d\omega + \int_{\omega_2} \rho_2 c_{2n} \bar{c}_2 d\omega, \quad (4.2)$$

where  $\bar{c}$  - velocity vector,  $c_n$  - external velocity vector component. From (4.2) we have

$$\bar{F} = G(\bar{c}_2 - \bar{c}_1), \quad (4.3)$$

where  $G$  – mass flow on unit blade length. In the instant case, the vector of external forces is expressed through mass flow  $G$  and speed. Write an expression for the circumference (along the axis of u)  $P_{1u}$  and axial (axis z)  $P_{1z}$  components of the force on the blade (4.3)

$$P_{1u} = G(c_{1u} - c_{2u}) \quad (4.4)$$

$$P_{1z} = G(c_{1z} - c_{2z}) + (p_1 - p_2)t \quad (4.5)$$

Get the formula for the torque

$$M = G(r_1 c_{1u} - r_2 c_{2u}) \quad (4.6)$$

and for power, developed on the shaft of the turbine

$$\Delta N = \omega \Delta M = \Delta G(u_1 c_{1u} - u_2 c_{2u}), \quad (4.7)$$

here  $r\omega = u$ . The expression for specific work of the 1 kg flowing through the control surface

$$h_u = \frac{dN}{dG} = u_1 c_{1u} - u_2 c_{2u} \quad (4.8)$$

and using of the cosines theorem get to specific work expression

$$h_u = \frac{c_1^2 - c_2^2}{2} + \frac{w_2^2 - w_1^2}{2} + \frac{u_1^2 - u_2^2}{2}. \quad (4.9)$$

Using relations (4.4)-(4.9) we have possibility improving the coordination of primary and secondary flows two shaft engine in front of the mixing camera (in particular, to increase of total pressure in the secondary flows and compensate of above mentioned total pressure losses in the engine core).

For conformation of good efficiency for high load turbine stage with the additional stator cascade there was provided special experimental study for turbine showing on figure 3. The turbine specific work (4.8) was increased on 5% without of marked efficiency losses.

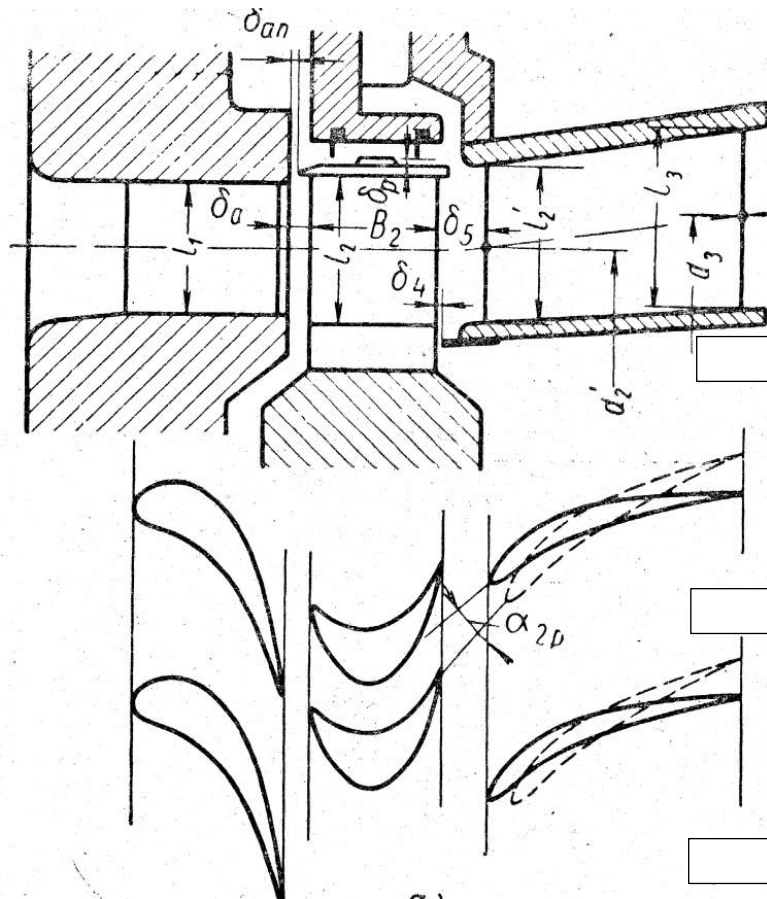


Figure 3: The high load turbine stage with the additional stator cascade.

## 5. Entropy and thermal radiation

Thermal radiation plays the main role in entropy increasing and energy dissipation process. The growth of entropy determines the energy part, which is dissipating in external space as thermal radiation energy. We consider in detail thermal radiation of ABE jets and internal shocks. Radiation of shock waves in the laboratory were performed mainly at low or moderate temperatures (in particular, in experiments on shock tubes). For the theoretical explanation of phenomena observed in previously published works were attracted by the hypothesis of relaxation of no equilibrium in the shock wave front, or a secondary exothermic recombination of the decay products of parent molecules.

With special experiments we have shown that a visible eye glow shock waves and the glow of the jet occurs at a sufficiently high temperature braking of the gas flow - in excess of 1000 K, and this glow can match the traditional equilibrium of a continuous thermal radiation spectrum (the spectrum of blackbody radiation, see figure 4).

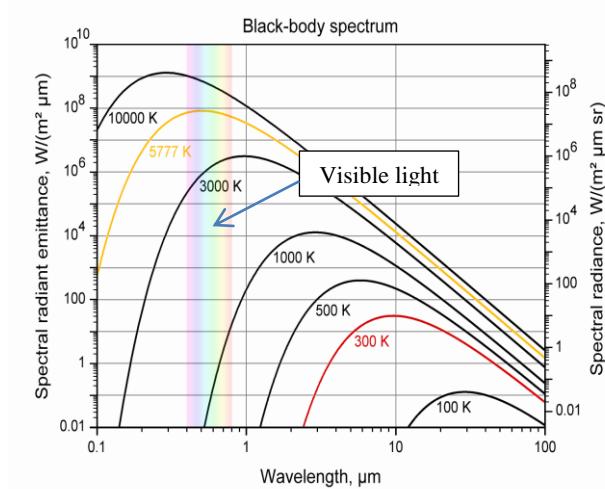


Figure.4: Black-body spectrum for various temperatures.

The experiment was carried out at the high temperature test cell of CIAM in the temperature range 500 – 1800 K and at pressures of several atmospheres. High temperature off-design supersonic jet with the Mach number at the nozzle exit  $M_0=1.3$  in the flooded outer space with the usual atmospheric pressure. When the stagnation temperature of the flow was below 1,000 K in the jet, the jet lighting is not observed. When the stagnation temperature was near 1800 K the luminous jet was in the visible range of well-recorded boundary of radiate hot bodies. Typical photo of a glowing test jet is shown in figure 5. Clearly visible two of the Mach disk, the first three "barrel" of the jet and its external borders. The usual spectrum of the equilibrium thermal radiation meets the surroundings in the visible range at temperatures of about 1800 K and above.



Figure 5: Photo of thermal radiation shock waves in high temperature test jet.

Another photos of visible thermal radiation from shock waves in high temperature test jets we can see on figure 6 and figure 7. The observed glow leaps vividly demonstrates the nature of entropy growth associated with total pressure losses and thermal energy dissipation. The cumulative intensity of this radiation can be approximately estimated by the well-known law by the Stefan-Boltzmann  $U = \varepsilon \sigma T^4$ .

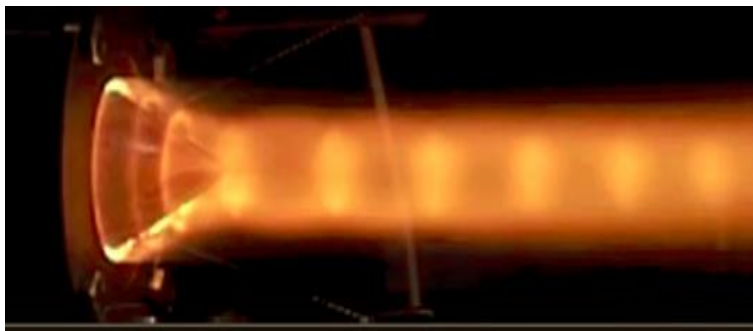


Figure 6: Visible glow of air breathing engine jet.



Figure 7: Visible glow of rocket engine jet.

## 6. Some ABE simulation examples

The described above approaches were used also for investigation of steady and unsteady working points of ABE (figures 8, 9). The power engines were investigated in detail experimentally. Both the whole engine and the core engine were tested. Different design and off-design working points were studied and compared with the test results. The experiments demonstrated significant discrepancy between the tested and design engine parameters for a number of working points in cases if considered radiate effects don't include into account. When we have integrated the system (2.1) the correlation between numerical and experimental data was more satisfactory.

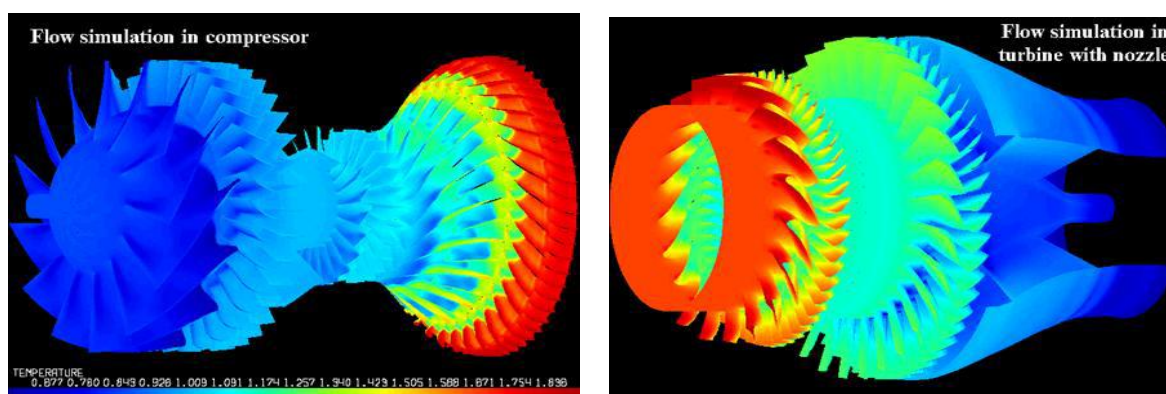


Figure 8. Example of flow simulations in small bypass ABE.

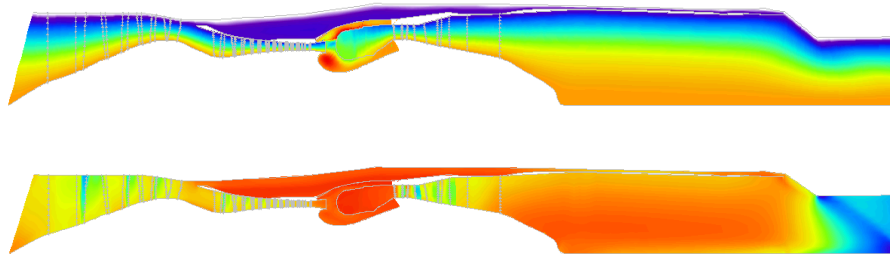


Figure 9: Streamlines and Mach number counters' distributions in whole flow passage of ABE.

These design and computational results were obtained with including of additional “unexpected” radiation losses in combustion chambers. Many additional results of ABE simulation can be found in [13-15].

## Conclusion

ABE modeling should be supported on thermodynamically compatible conservation laws. Thermal radiation includes into account in the design process stage for real coordination of different components of high temperature engines.

The question of the physical basis of monotonic increase of entropy thus remains open in the frame of modern physics and may be solved by week solutions of quasy-linear systems. The entropy growth closely connects with molecule mass decreasing. The entropy growth also determines the total pressure losses.

Week solutions of quasy-linear systems are the solutions with total pressure losses (including a smooth solution with heat addition).

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