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MAX Modeling of Integrally Bladed Rotors with Blends, Mistuning, and Prestress

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Abstract

Integrally bladed rotors (IBRs) are critical components of turbomachinery in the aerospace industry. The use of IBRs has been increasing recently because they provide enhanced performance and reduced weight. One of the most significant challenges posed by IBRs is their low level of damping. The low damping leads to unwanted vibrations that increase the risk of high-cycle fatigue. In addition, IBRs are typically mistuned in real applications, i.e. they present deviations from the nominal geometry due to manufacturing tolerances, damages or wear even though they are designed to be nominally cyclic structures. Mistuning can lead to vibration localization, which further increases the risk of high cycle fatigue. Thus, the geometric design of IBRs is very important. In addition, foreign object damage can affect IBR blades during operation. Distinct from bladed disks with inserted blades, IBRs cannot have their blades replaced because IBRs are manufactured in one piece. As a result, other repair strategies must be used. IBR blades can be repaired by blending, i.e. by removing material in the damaged region to modify the local geometry as to create a smooth surface able to lower stress concentrations while also preserving satisfactory aerodynamic, structural, and vibration characteristics. Such repairs introduce large mistuning in IBRs, which can increase vibration levels. For this reason, it is necessary to optimize blend shapes and locations and to assess their impact on IBR structural dynamics. Finite element (FE) models are traditionally used to study the structural dynamics of IBRs. It is common for industrial models to have several millions of degrees of freedom per sector, making it challenging to obtain results in a timely fashion. Furthermore, the stochastic nature of mistuning requires the use of Monte Carlo simulations to evaluate the expected vibration amplitudes. For this reason, reduced-order models have been used in lieu of FE models to reduce the computational cost of such analyses. A Mode-Accelerated X-Xr (MAX) method was proposed by the authors to study the effects of such blends. In this work, enhancements to the MAX method are presented to accommodate the combined presence of blends and that of other effects, such as blade frequency mistuning and prestress due to centrifugal forces. The use of multiple FE meshes in a single model is also addressed, making it possible to accommodate the use of morphed meshes to better capture the geometry of blended sectors. This enhanced MAX method, referred to as MAX+, is presented. MAX+ is validated by comparing its predictions to high-fidelity FE models, and it is shown to be efficient and effective for simulating blended IBRs in both bench and rotating conditions. Furthermore, the MAX+ approach can be used for the optimization of blend shapes, locations, and patterns when multiple blends are present.

1. Introduction

Integrally bladed rotors (IBRs), also known as blisks, are structures commonly used in gas turbine engines. These components are nominally cyclic symmetric, a feature that can be exploited to significantly reduce the computational cost associated with structural and vibration analysis.[1] However, due to manufacturing tolerances, wear, and damages, the symmetry of the nominal structure is not found in real IBRs. Any deviation from nominal conditions is referred to as mistuning. Due to the presence of mistuning, predictions based on cyclic models are not accurate because localization phenomena can appear, resulting in substantial variations in the forced response.[2] Furthermore, IBRs have very low damping due to the absence of frictional contacts and joints. This can result in large vibration amplitudes in one or more blades, which can lead to high cycle fatigue and ultimately blade failure especially because vibrations are combined with high levels of static stress due to centrifugal forces and blade aerodynamic loading. For this reason,

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it is extremely important to study the vibration characteristics of IBRs taking into account the effects of mistuning. IBRs are commonly studied using highly refined finite element (FE) models, which can provide an excellent level of detail but also a significant computational cost. Due to the stochastic nature of mistuning, it is necessary to perform Monte Carlo simulations to explore the entire range of expected responses.[2] This is unfeasible when large FE models are used. Thus, other modeling techniques are commonly used to reduce the computational cost. The use of reduced order models (ROMs) for the extraction of modal and structural information is common practice both in industry and academia. Researchers have extensively studied in the past the effects of mistuning on the dynamics of IBRs using ROMs.[2] Many of these studies focus in particular on geometric mistuning.[3–5] An interesting type of geometric mistuning is that created by blends,[6] i.e. material removal, for the repair of cracked or damaged IBRs. Such repairs need to be carefully designed so that they have minimal impact on the vibration of the repaired IBRs. For this reason, ROMs[6,7] have been created for the analysis and optimization of such repairs. The Mode-Accelerated X-Xr (MAX) method presented in[6] is particularly adapt to model such effects. MAX accounts for blade frequency mistuning (caused by stiffness or mass mistuning in the blade) using component mode mistuning (CMM).[8] However, other aspects need to be considered, such as the fact that geometric mistuning can present a significant interaction with centrifugal loading and other sources of prestress. Turbomachinery components typically operate in very demanding conditions. As such, blades must withstand significant static and dynamic stresses, due to both aerodynamic loads and centrifugal forces, which can vary depending on the operating conditions. The operating conditions at which the system is excited have a significant impact on the response. For example, rotational effects as well as aeroelastic effects have been shown to have an important role on the vibration characteristics of the system.[9–13] In rotating conditions, the blades experience significant spin softening and stress stiffening, which ultimately change the natural frequencies and modal characteristics. The thermo-mechanical aspects of the system can play a significant role in the IBR life cycle also.[14] The effects of the non-time-varying components of all these phenomena on the IBR can be described using prestress. The introduction of centrifugal effects, a typical source of prestress, can be accommodated in ROMs. The general problem of cyclic rotating structures is tackled in[15,16] using cyclic symmetry together with component modes synthesis.[17] Non-rotating mode shapes are used to project the equations of motions onto a reduced set of coordinates. Some of the effects that arise when mistuning is combined with the effects of rotation are investigated in.[18–20] In[18] the effects of mistuning are studied using a set of basis vectors that adequately spans the interface motion between sectors. That method ensures the compatibility condition at the interface, but it also requires an extra set of modes to be computed. The method was extended also to a range of variable rotation speeds using parametric techniques. In[19,20] a study on cracks and Coriolis effects in rotating disks is presented, without dealing with the compatibility condition at the interface that is of interest here. In[21] a study on the effect of prestress can be found, focusing in particular on the presence of contact forces, but without dealing with geometric mistuning. The effects of rotational speed can be captured as variations in the stiffness matrix, as suggested by previous studies for the blade part only.[10] When it comes to model order reduction, another such solution is to use parametric ROMs.[22,23] However, many parametric ROMs are not the most efficient for IBRs because they require the calculation of multiple sets of modes, and because of possible numerical instabilities of the transformation matrix.[22–24] Even more importantly, the calculation of prestress effects on the stiffness matrix of a mistuned IBR requires full wheel analyses, which are often computationally cumbersome. None of the studies cited above focuses on the efficient extraction of the prestressed matrices for a mistuned system, and little attention is given in the literature to modeling of non-cyclic prestress. The use of FE mesh morphing has become common because it allows for easy changes in the shape of FE models without requiring a new meshing step. This is particularly useful for IBRs with one blended blade, where the blend only affects a small portion of the entire full wheel model. Several studies have been conducted on this topic to understand the implications of FE mesh morphing on ROMs created using the MAX method.[7]

In this paper, the focus is on the creation of a comprehensive method that can be used in the creation of ROMs where blended IBRs need to be analyzed and optimized in the concurrent presence of mistuning and prestress. MAX+ is the result of the integration of previous work from the authors, in particular the MAX method,[6] a numerical conditioning technique[23] and a study on the use of mesh morphing in ROMs.[7] In addition, we present a technique able to capture prestress effects that can be implemented in MAX+. This approach only requires single sector calculations for the extraction of the free-interface sector-level tuned and mistuned matrices at different rotational speeds. The prestress is calculated for every type of mistuned blade at the sector level, thus capturing mistuning effects that could potentially be greatly magnified by the action of centrifugal or steady aeroelastic forces, without the need of a full wheel analysis. The MAX method is used to treat the effects of large blends, and it is extended by introducing prestress effects on the matrices and by treating rotational effects as small mistuning. For further computational savings, MAX+ can be used together with parameterization of the stiffness variation[11] to obtain mistuned results more readily. Numerical results are presented to demonstrate the effectiveness of the approach.

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2. Background

IBRs are structures that are nominally cyclic symmetric, i.e. they are composed of identical sectors that are repeated in space by rotation around a common axis. Figure 1 presents an example of such a structure.

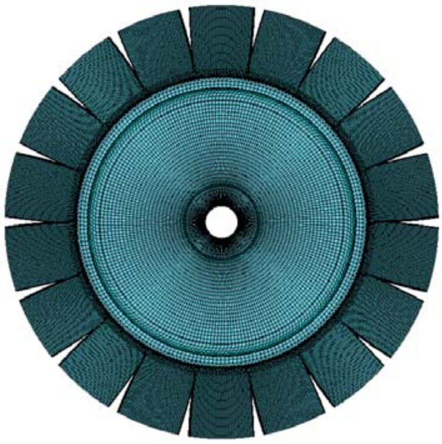

Figure 1: Perfectly symmetric and tuned IBR

The mass and stiffness matrices of an entire cyclic structure assume circulant form.[25] However, for the creation of ROMs it is typically more convenient to manipulate sector level quantities. The free-boundary sector level matrices can be used to describe the system. This requires that the degrees of freedom (DOFs) at the interface between sectors be duplicated, and that interface compatibility be enforced. An IBR sector is shown in Figure 2, with one of the cyclic interfaces highlighted. A more detailed explanation of the concept of cyclic interfaces is provided in Section 3.3.

Figure 2: *Pristine* sector with cyclic interface highlighted in blue

Using a decoupled approach like the one presented in,[3] it is possible to write the system level mass and stiffness matrices as block diagonal, where the diagonal elements are the free-interface matrices of each sector. The coupling is enforced through compatibility equations on the boundary nodes. For a tuned system, the system level matrices can be expressed using a Kronecker product where only sector level quantities are introduced. For example, the mass matrix $\tilde{\mathbf{M}}$ of the entire pristine structure can be written as

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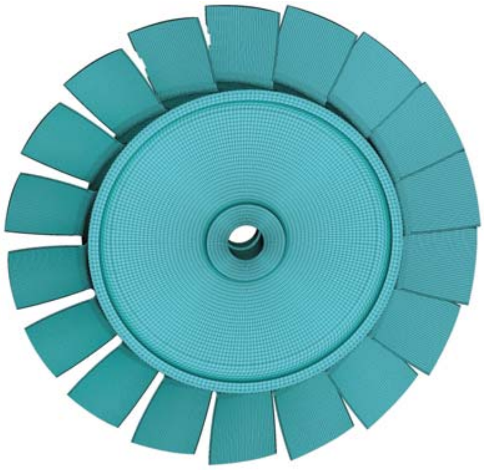

Figure 3: Geometrically mistuned IBR

$$\tilde{\mathbf{M}} = \mathbf{I} \otimes \mathbf{M} = \begin{bmatrix} \mathbf{M} & & \\ & \ddots & \\ & & \mathbf{M} \end{bmatrix}, \tag{1}$$

where the matrix $\mathbf{M}$ represents the free-interface level matrix. In a mistuned system, like the one presented in Figure 3, the sectors are in general different, and thus the Kronecker product cannot be used anymore. The full wheel matrix of a mistuned system becomes more generally

$$\tilde{\mathbf{M}} = \begin{bmatrix} \mathbf{M}_1 & & \\ & \ddots & \\ & & \mathbf{M}_N \end{bmatrix}, \tag{2}$$

where the matrices $\mathbf{M}_i$ of each of the N sectors could potentially be different.

To better explore the impact of this on the ROM creation it is important to recall the concept of *pristine* and *mistuned* sectors. *Pristine* sectors present nominal geometry and properties. This can also include systems subject to prestress that preserve cyclic symmetry. *Mistuned* sectors show instead any deviation from the cyclically symmetric ones. For example, let us consider an IBR composed of pristine sectors. After centrifugal loads are applied, it is still composed of pristine sectors. That is not the case for an IBR showing mistuning in one of its blades. A single mistuned sector is sufficient to convert all pristine sectors into mistuned sectors when prestress is applied. The prestress applied is non-cyclic. Thus, the resulting equilibrium is mistuned in all sectors as well.

Typically, only few sectors are mistuned. Let us now assume that only the first sector is mistuned, with a mistuned mass matrix $\mathbf{M}_m$, while all the other sectors are pristine, with mass matrix $\mathbf{M}$. Equation 2 simplifies to

$$\hat{\mathbf{M}} = \begin{bmatrix} \mathbf{M}_m & & & \\ & \mathbf{M} & & \\ & & \ddots & \\ & & & \mathbf{M} \end{bmatrix}. \tag{3}$$

The sector level matrices, as well as the mode shapes of the system can be readily obtained using commercial FE software. Note that in the absence of prestress even the mistuned sector matrices can be obtained using a single sector calculation, as there is no equilibrium compatibility required.

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3. Methods

3.1 Reduced order model creation

Let us consider the general case of a mistuned IBR (a tuned IBR can be seen as a particular case with zero mistuning). The equation of motion for the structural dynamics of the IBR can be written as

$$\hat{\mathbf{M}}\ddot{\mathbf{x}} + \hat{\mathbf{K}}\mathbf{x} = \mathbf{f}, \tag{4}$$

where $\hat{\mathbf{M}}$ is the mass matrix, $\hat{\mathbf{K}}$ is the stiffness matrix, and $\mathbf{x}$ and $\mathbf{f}$ are the vectors of generalized coordinates and forces respectively for the entire IBR. A ROM can be created by introducing the change of coordinates $\mathbf{x} = \mathbf{Tq}$,where $\mathbf{T}$ is the transformation matrix and $\mathbf{q}$ is the vector of generalized coordinates. The system level matrices $\hat{\mathbf{M}}$ and $\hat{\mathbf{K}}$ presented in Eq. 4 can be prohibitively large to store and to use. For this reason it is often more convenient to use sector level matrices for the calculation of the ROM. The formulation presented in Eq. 2 requires coupling between the sectors. This coupling is provided through the basis vectors included in the transformation matrix $\mathbf{T}$. Using sector level quantities instead of system level ones results in significant savings of computational time and memory. In this case, the system level multiplication with the transformation matrix results in a sum over the sectors of sector level information as in

$$\mathbf{T}^T\hat{\mathbf{M}}\mathbf{T} = \sum_{i=1}^{N} \mathbf{T}_i^T\mathbf{M}_i\mathbf{T}_i, \tag{5}$$

where $\mathbf{T}_i$ is the partition of the transformation matrix corresponding to the i^{th} sector. The MAX transformation matrix matrix can be written as[6]

$$\mathbf{T} = \begin{bmatrix} \mathbf{\Phi}_{MCM} & \mathbf{\Phi}_{BAM} & \mathbf{\Phi} \end{bmatrix}, \tag{6}$$

where, $\mathbf{\Phi}_{MCM}$ represents the constrained modes, $\mathbf{\Phi}_{BAM}$ the acceleration modes, and $\mathbf{\Phi}$ the normal modes. If the number of sectors is large, then it is important to implement this calculation efficiently in the ROM creation process. In MAX+, the transformation matrix includes tuned normal modes, and as a result the properties of cyclic structure can be invoked. Using a technique similar to the one proposed in,[5] it is possible to greatly reduce the number of operations to be carried out. Let us now consider the multiplication of the system level matrices with the normal modes. Equation 5 can be expressed as

$$\mathbf{\Phi}^T\hat{\mathbf{M}}\mathbf{\Phi} = \sum_{i=1}^{N} \mathbf{\Phi}_i^T\mathbf{M}_i\mathbf{\Phi}_i. \tag{7}$$

The tuned normal modes at the system level can be expressed as a linear combination of the vectors $\mathbf{u}$ and $\bar{\mathbf{u}}$ as[3]

$$\mathbf{\Phi}_i = \mathbf{u}\mathbf{f}_{\mathbf{Ai}} + \bar{\mathbf{u}}\mathbf{f}_{\mathbf{Bi}}, \tag{8}$$

where $\mathbf{f}_{\mathbf{A}}$ and $\mathbf{f}_{\mathbf{B}}$ are the coefficients of the real Fourier matrix. Introducing this notation and plugging back into Eq. 8 it can immediately be noticed that only the coefficients change over different sectors, the other quantities remain the same. This feature makes this calculation feasible for highly refined industrial FE models, where the number of DOFs often exceeds several million per sector and the number of blades can be very large.

3.2 Interaction of mistuning and prestress

The focus of this paper is set on the part of MAX+ able to capture prestress effects, i.e. effects that encompass steady state variations in the stiffness and/or the mass matrices. Even though the mass matrix is typically not affected, it could experience small variations if a nonlinear analysis is carried out to obtain the new static equilibrium. The examples presented here refer to centrifugal loads, but the prestress term could come from other sources, such as steady thermal and aeroelastic loads. In particular, this study is concerned with the interaction between mistuning and prestress effects, with a particular focus on geometric mistuning. The presence of a blend is used as an example of geometric mistuning.

Let us now consider the case of the perfectly symmetric IBR shown in Figure 1. When in rotating conditions, all sectors experience the same forcing, as shown in Figure 4, and thus the equilibrium position still preserves its symmetry. As a result, cyclic symmetry analysis can be employed to solve for the equilibrium and extract the sector

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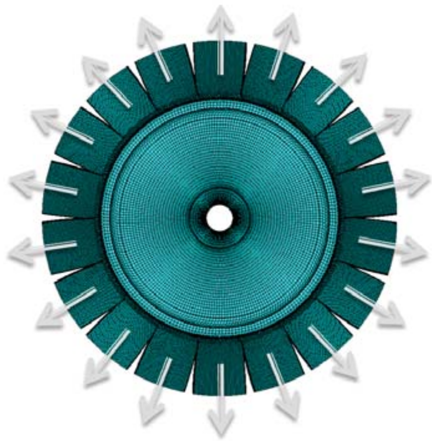

Figure 4: Pristine system subject to preload

level matrices in rotating conditions. The system matrices can still be conveniently written in the same form as Eq. 1. In particular, the stiffness matrix can be written as

$$\tilde{\mathbf{K}}_{\Omega} = \mathbf{I} \otimes \mathbf{K}_{\Omega} = \begin{bmatrix} \mathbf{K}_{\Omega} & & \\ & \ddots & \\ & & \mathbf{K}_{\Omega} \end{bmatrix}, \tag{9}$$

where the subscript Ω is added to represent the presence of prestress. However, these assumptions are not valid when geometric mistuning is present, like in the case shown in Figure 5. In that case, the sectors experience different forces, as shown in Figure 5, resulting in a final equilibrium position that is not cyclic symmetric. The difference between *pristine* and *mistuned* sectors of a mistuned IBR ceases to exist when prestress is applied, as all the sectors become *mistuned* to a certain extent.

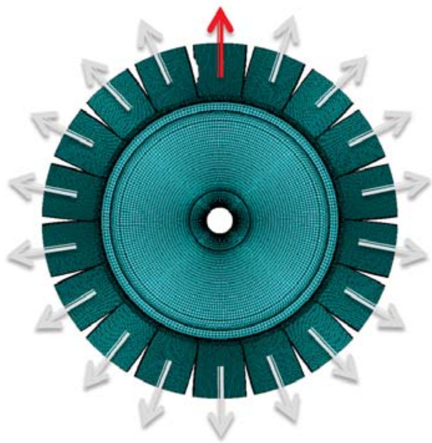

Figure 5: Mistuned system subject to preload

The change in stiffness due to prestress can be expressed as

$$\delta\tilde{\mathbf{K}}_{\Omega} = \begin{bmatrix} \delta\mathbf{K}_{1} & & \\ & \ddots & \\ & & \delta\mathbf{K}_{N} \end{bmatrix}, \tag{10}$$

where $\delta\mathbf{K}_i$ is the prestress contribution to the i^{th} sector. A mistuned IBR composed of both *pristine* and *mistuned* sectors in static conditions becomes instead composed of only *mistuned* sectors when subject to prestress. Thus, the concept

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of *pristine* sector disappears when any form of mistuning and preload are concurrently present. In fact, especially in proximity of the mistuned sector, a significant deviation from cyclic symmetry can be observed. In this paper, a calculation strategy for the extraction of the sector level matrices is presented to re-establish the concept of *pristine* sector, with the aim of reducing the computational cost with little accuracy loss.

3.3 Calculation of the prestressed matrices

When a mistuned IBR like the one shown in Figure 5 is subject to a preload, a full wheel analysis is required for the extraction of the new system matrices. This requires significant computational effort and eliminates properties that are very useful for the ROM creation, such as the existence of *pristine* sectors. In the method proposed here, the full wheel static equilibrium is substituted with single sector calculations. This is based on the assumption that *pristine* sectors remain *pristine*.

First, the static equilibrium for the pristine sector is obtained using cyclic symmetry analysis. Then, the resulting displacements at the cyclic interface highlighted in Figure 2 are imposed as boundary conditions for the *mistuned* case. The prestress loads are then applied to the sector and the equilibrium for the *mistuned* sector calculated. All these analyses are performed using single sector calculations. The underlying assumption is that the deviation from the cyclic prestress is localized in the blend or blade region, which is far from the cyclic interface. Thus, a negligible error is committed by imposing an approximated boundary condition at the cyclic interface.

3.4 Metrics and limitation

The effectiveness of the method is based upon the assumption that there exists a region far enough from the geometrically mistuned area where the deviation from cyclic results is negligible. The presence of such a region is strongly dependent on both *pristine* and *mistuned* geometries, as well as the rotational speed or prestress level. In fact, the magnitude of the error will depend on all these factors.

An error metric is introduced to evaluate the magnitude of the error. This metric is able to quantify the deviation between single sector and full wheel results in the sectors adjacent to the blended one. A full wheel calculation must be carried out only once for a given mistuning type (e.g., type of blend), typically at the highest level of prestress expected. If the error is considered acceptable, it is then possible to use the matrices extracted in the ROM. First, the *pristine* prestressed full wheel stiffness matrix $\tilde{\mathbf{K}}$ is obtained. Then, one of the sectors is substituted with the *mistuned* sector under investigation to obtain the *mistuned* prestressed full wheel stiffness matrix $\hat{\mathbf{K}}$. Once the full wheel matrices are obtained, the prestress deviation matrix $\mathbf{K}_{var}$ can be formulated as follows

$$\mathbf{K}_{var} = |\hat{\mathbf{K}} - \tilde{\mathbf{K}}|. \tag{11}$$

For convenience, the results can be visualized by assigning a measure of the variation in the stiffness matrix entries to every DOF. Recalling that the deviation matrix and the vector of DOFs are defined as $\mathbf{K}_{var} = \left(k_{var_{ij}}\right)$, $\tilde{\mathbf{K}} = \left(\tilde{k}_{ij}\right)$ and $\mathbf{x} = (x_i)$ respectively, one can write

$$x_i = \frac{\sum_{j=1}^{N} k_{var_{ij}}}{\sum_{j=1}^{N} |\tilde{k}_{ij}|}. \tag{12}$$

This approach quantifies the impact that a *mistuned* sector has on a neighboring *pristine* sector.

4. Results and Discussion

Let us consider the IBR shown in Figure 6. The first sector presents a large blend. The MAX method[6] is used to create a ROM. The natural frequencies of the ROM are compared to the ones obtained using the full-order model to validate the method. For each mode, the error is defined as

$$e = \frac{|\omega_{ROM} - \omega_0|}{\omega_0}, \tag{13}$$

where ω_0 represents the frequency obtained using the full-order model, and ω_{ROM} represents the frequency calculated using the ROM.

A baseline set of results can be obtained using a full order model for the IBR shown in Figure 6 and used for validation. Results obtained using the ROM in non-rotating conditions are presented in Figure 7a. Note that the two sets of results are in good agreement, showing an almost negligible error. Establishing a baseline is important because the accuracy of the ROM in rotating conditions is expected to decrease due to the simplifications made during ROM

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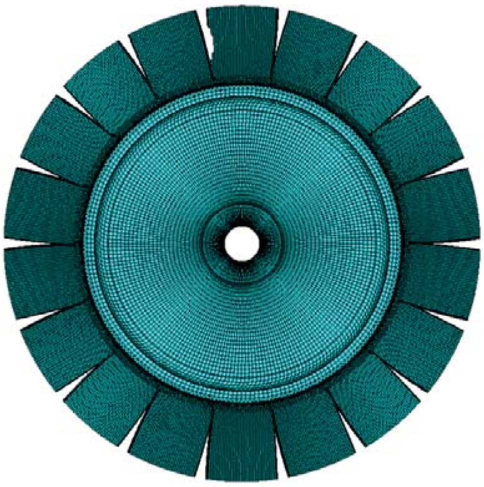

Figure 6: Validation IBR having one blended blade

creation. The error will depend on the blend size and type, and on the magnitude of the prestress. In all the results here presented, the transformation matrix from the MAX method is modified to include the normal modes in prestressed conditions.

Figure 7b shows the frequency error for the case of an IBR spinning at 10000 RPM. Note that the accuracy loss compared to the baseline results is negligible. The accuracy loss becomes noticeable in the case of a rotor spinning at 20000 RPM, as can be seen in Figure 7c. In particular, it is important to note that the blend acceleration modes (BAMs) coming from the MAX method are not able to reduce the error. This is because the mass and stiffness matrices are the sources of error. Nonetheless, MAX+ prediction error is still small.

The error due to the use of single sector calculations can be visualized using the metric presented in Section 3.4. Figure 8 shows a visualization of the stiffness variation matrix $\mathbf{K}_{var}$ for the 20000 RPM case. Figure 8a show that the largest variation is localized in the blended sector. This is expected, and it is appropriate to obtain a deviation in the area adjacent to the blend. Particularly relevant is Figure 8b, which shows the error that is committed by neglecting the stiffness variation in the neighboring sectors, which are assumed to remain *pristine* instead. In this case, the maximum deviation is 0.007%. Please note that a small error is also committed in the blended sector due to the application of approximate boundary conditions. This error is not represented here because it is not captured by this metric.

The case of the IBRs with two blends on neighboring blades[23] shown in Figure 3 is examined next. The presence of two neighboring blends is more challenging to capture because they can mutually influence each other when prestress is applied. The results obtained in rotating conditions at 10000 RPM are shown in Figure 9b, while the baseline results obtained in absence of prestress are shown in Figure 9a. Even in this case, the MAX+ method provides excellent accuracy. Next, the method was applied to the case presented in,[7] which consists of the blended IBR shown in Figure 10a. This case presents different blends on different blades as well as different FE meshes. The results from MAX+ are reported in Figure 10b, and they show excellent agreement between the ROM and the full order model.

Another feature of MAX+ that can be used to save time in the solution of problems with prestress is the use of baseline mode shapes instead of the modes at given prestress levels. The transformation matrix obtained in reference conditions can be used even for those cases when the operating conditions are different. The only variation in that case is in the system matrices. Let us examine the case of Figure 6. The results for the case rotating at 10000 RPM is presented in Figure 11, and they show that even though the error increases significantly compared to the one shown in Figure 7b, the ROM still provides a good approximation. The error increase is in this case due to a change in the mode shapes.

5. Conclusions

In this paper, a novel method, MAX+ was introduced. This method extends the capabilities of MAX and allows the introduction of prestress, finite element mesh morphing, and other system characteristics in a general framework for the analysis of IBR bends. One of the enabling techniques is a method to build ROMs using only sector level matrices of quasi-cyclic structures subject to prestress. The approximation introduced in MAX+ causes little to no accuracy loss. The computational gains are significant, given that full wheel calculations are difficult for large finite element

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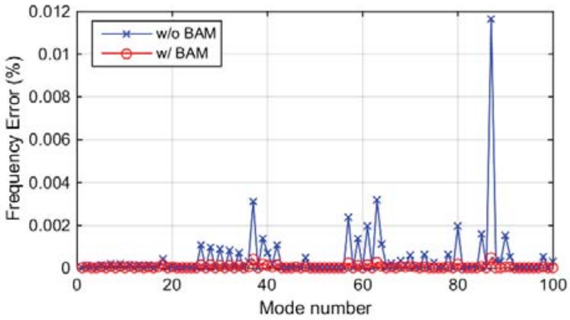
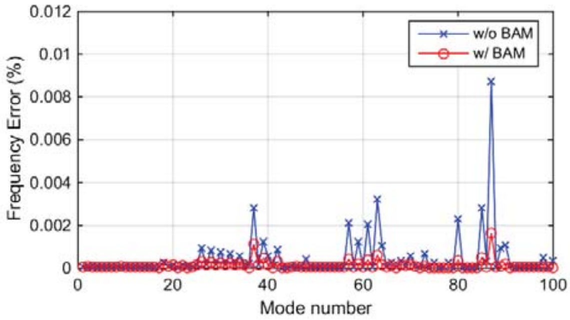
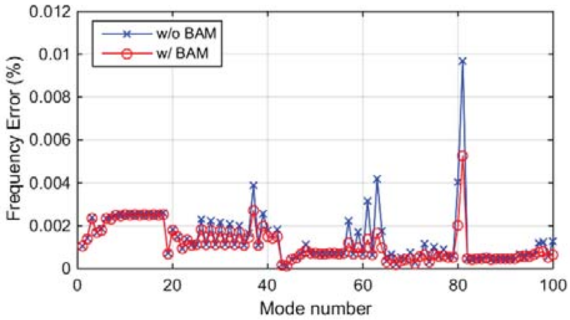


(a) Baseline MAX results without prestress



(b) MAX+ results in rotating conditions at 10000 RPM



(c) MAX+ results in rotating conditions at 20000 RPM

Figure 7: Validation results obtained in different prestress conditions

models. A metric able to quantify the error created by the approximation is proposed also. This metric addresses a significant issue in ROMs, namely knowledge of the modeling error intorduced by model reduction. This error metric was successfully applied to ROM presented in[6] built on the MAX method as well as models built using the MAX+ method. The paper summarized the capabilities of the MAX+ method, which is a framework for the analysis of blended IBRs.[7,23] The MAX+ method can be applied in the design process to identify optimal repair strategies. Results can be obtained quickly for different conditions, without requiring repetitive new finite element studies. The ability to accommodate the use of mesh morphing is also of paramount importance, enabling rapid creation of multiple compelx geometries.

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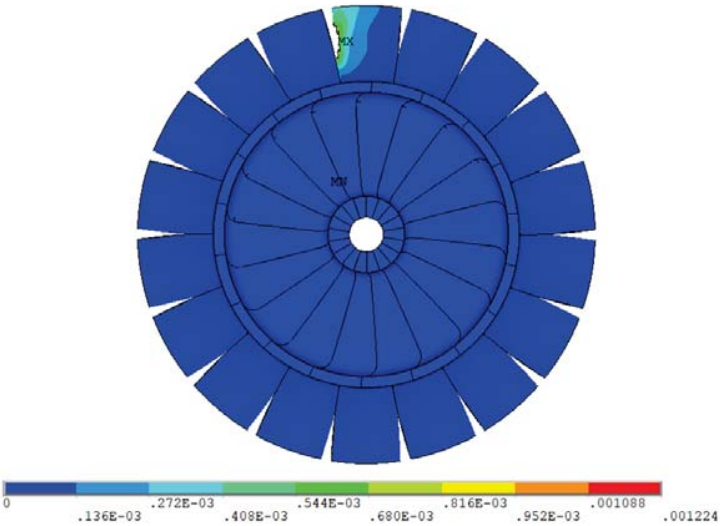
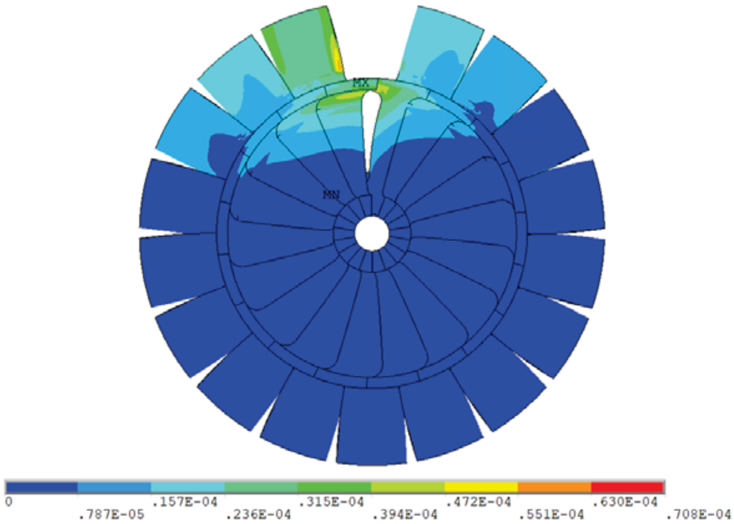


(a)



(b)

Figure 8: Visualization of the error metric between the prestressed *pristine* and *mistuned* stiffness $\mathbf{K}_{var}$

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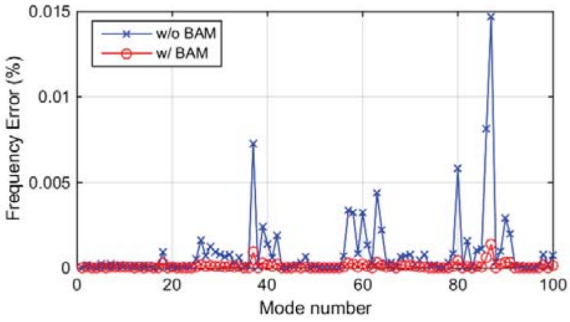
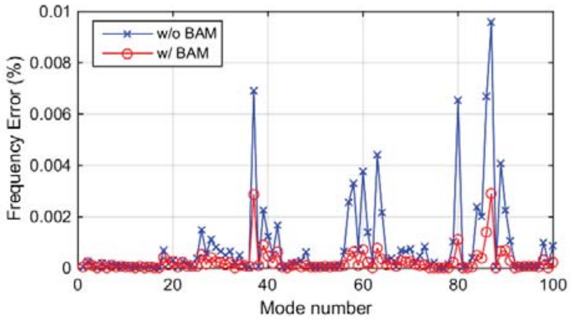


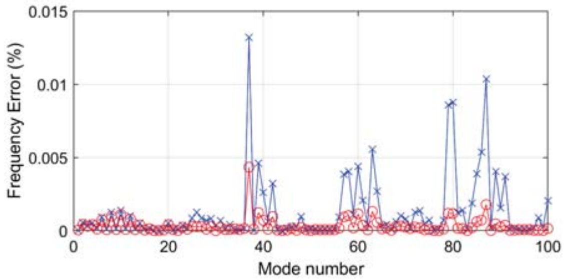
(a) Baseline results without prestress



(b) MAX+ results in rotating conditions at 10000 RPM

Figure 9: Validation results obtained in different prestress conditions for the IBR of Figure 3

(a) Validation case using a morphed mesh[7]



(b) MAX+ results in rotating conditions at 10000 RPM

Figure 10: Validation results obtained for an IBR with two distinct blends

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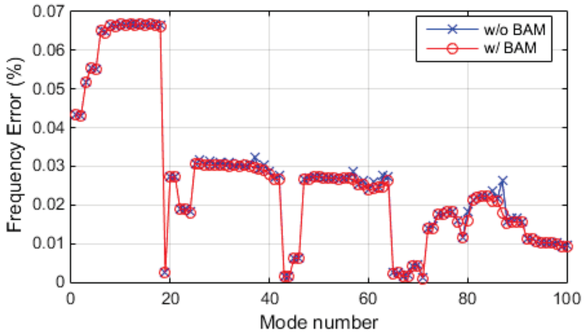

Figure 11: MAX+ results in rotating conditions using baseline modes for the case of Figure 6

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