TWO POTENTIAL BREAKTHROUGHS ANSWERING THE BALLISTIC MISSILE DEFENSE CHALLENGE

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ABSTRACT

The assumption that the reentry vehicle (RV) of a future Tactical Ballistic Missile, carrying a non conventional warhead, is capable of maneuvering made the challenge of intercepting extremely serious. Against such threats a zero leakage rate is needed, thus successful interception requires very small miss distances that cannot be guaranteed by the classical guidance and estimation methods used in the past against manned aircraft. An innovative interceptor guidance strategy based on two not yet used design approaches, namely pursuit-evasion game concepts and integrated design of the estimator and the guidance law, is proposed. Extensive simulation results show that the combination of these approaches has the potential achieving a hit-to-kill accuracy against randomly maneuvering threats.

INTRODUCTION

Ballistic Missile Defense (BMD) became one of the most challenging issues in the free world in the last two decades due to the concern of non conventional ballistic missile warheads. Against such threats a zero leakage rate is needed. The majority of the BMD interceptors are designed for hit-to-kill, assuming that the required guidance precision can be achieved. Such performance was demonstrated so far against non maneuvering targets or against presumed maneuver structures. The demonstrated "direct hit" interceptions of such targets has been the consequence of the great progress made in missile guidance technology in the last decades. Such technology has been necessary, but not sufficient, to meet the challenge of intercepting eventually maneuvering ballistic missiles. In contrast to the impressive progress in missile guidance technology, the concepts of guidance law development remained (unfortunately) conservative.

It has been demonstrated in numerous simulation studies that currently used guidance laws and estimation techniques cannot guarantee a "hit-to-kill" accuracy against maneuvering targets. The paper addresses two directions that have the potential correcting the deficiencies of the conservative common practice in the estimation/guidance law design, namely the inadequate mathematical formulation of maneuverable target interception and the separate design of the estimator and the optimal guidance law.

The "classical" approach of formulating the interception of maneuvering targets as a (one-sided) optimal control problem assuming the target maneuver is unjustified and can yield misleading results. Target maneuvers are independently controlled and cannot be anticipated. The interception scenario of a maneuvering target has to be formulated as a zero-sum pursuit-evasion game that guarantees robust homing performance against all feasible target maneuvers. In the last decades several guidance laws were derived based on such formulation [1-7], but have not yet been implemented in BMD interceptor designs, in spite of the proof of their clear advantage [8].

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Due to noisy measurements a realistic interception is a stochastic control problem. Therefore, an estimator (filter) is an indispensable component of the guidance loop and estimation accuracy limits the homing performance. The difficulties solving such problem can be circumvented by assuming the validity of the *Certainty Equivalence Principle* and the associated *Separation Theorem.* The *Certainty Equivalence Principle,* formulated for control problems in the sixties [9-10] states that the optimal control function of a stochastic problem is the same as the optimal control function of the associated (*certainty equivalent)* deterministic one. If the validity of this principle can be proven, then the estimator and the controller can be designed separately. This is the *Separation Theorem*.

Maneuvering targets introduce another difficulty. Even if the guidance law itself does not require the knowledge of the target maneuver, for the estimator design a target maneuver model is needed. An incorrect model leads to a poor estimation, limiting the achievable guidance accuracy. Identification of the target maneuver model is a prerequisite for accurate estimation. Nevertheless, correct identification and estimation are only necessary, but not sufficient for accurate guidance. Since the acceleration of a moving object from another platform cannot be measured, it has to be reconstructed by an observer in the noise free case, or by an estimator using the available noisy measurements. Thus, in a guidance system the estimator performs a dual role, the role of a noise filter and the role of an observer.

The generic future BMD threat may carry a nonconventional warhead and can perform random maneuvers during the reentry. In the lower atmosphere aerodynamic maneuvers can be created either intentionally or obtained as a “natural” maneuver of a spin stabilized aerodynamically stable reentry vehicle (RV). In higher altitudes, where the air density is too low, propulsive devices can be used. The RV designer may spare 15-20% of the weight and the volume for such a devise and the necessary propellant. The maneuverability advantage of an interceptor against a maneuvering RV is unknown and can be in some cases only marginal.

Unfortunately, a real world BMD interception does not belong to the class for which the validity of the *Certainty Equivalence Principle* and the associated *Separation Theorem* was proven. In spite of that, it has been of common practice to design the estimators and missile guidance laws independently. In most cases this comfortable design approach satisfied the requirements, due to the great maneuverability advantage of the missiles over their manned aircraft targets. Thus, for future BMD separate design of the estimator and the guidance law can be, at the best, suboptimal.

Based on the above stated observations the two directions for improving critical BMD interceptor design has been identified: (i) adopting differential game concepts instead of the one-sided optimal control and (ii) designing the estimator and the guidance law using an integrated system approach. In this paper a description of differential game based guidance laws and an integrated estimation/guidance design is presented. Both elements were developed at university research level, where their effectiveness was demonstrated by extensive Monte Carlo simulations. The structure of this paper is the following. In the next section the three-dimensional BMD problem is formulated. Descriptions of the differential game based guidance laws and an algorithm for integrated estimation/guidance design are presented in the subsequent sections. This is followed by a demonstrative example including the scenario data, the generic target and interceptor models used, as well as the results of extensive Monte Carlo simulations. Conclusions are offered at the end.

Problem Formulation

Scenario Description

Two scenarios of intercepting maneuvering targets are considered. The first one is a generic 3D endo-atmospheric BMD scenario with time-varying velocities and acceleration limits. The second is a horizontal, constant speed model used for the sake of research efficiency. The more complex 3D scenario is used mainly for validation. In both, the homing endgame starts as soon as the seeker of the interceptor locks on the target. The relative geometry is near to a head-on engagement. It is assumed that at this moment the initial heading error, with respect to a collision course, is small and neither the interceptor nor the target is maneuvering. These assumptions allow the linearization of the geometry and the decoupling of the 3D equations of motion to two identical sets in perpendicular planes [11].

Information Pattern

The interceptor measures range and range rate with good accuracy that allows computing the time-to-go. The measurements of the line of sight angle are corrupted by a zero mean white Gaussian angular noise. The interceptor's acceleration is accurately measured, but the target acceleration has to be estimated. The target has no information on the interceptor, but can start evasive maneuvers at any time and change the direction of the maneuver randomly.

Lethality Function

The objective of the interception is the destruction of the TBM. The probability of destroying the target is determined by the following simplified function

$$P_d = \begin{cases} 1 & M \le R_k \\ 0 & M > R_k \end{cases} \tag{1}$$

where R_k is the lethal (kill) radius of the warhead and M is the miss distance.

Cost Function

The natural cost function of the interception is the miss distance. Due to the noisy measurements and the random target maneuvers, the miss distance is a random variable. Based on (1), the efficiency of a missile depends on the lethal radius R_k of its warhead. One figure of merit is the single shot kill probability (SSKP) of a given warhead, defined by

$$SSKP = E\,\{P_d\,(R_k)\} \tag{2}$$

where $E\{\cdot\}$ is the expectation over the noise samples against any feasible target maneuver. The guidance objective is to maximize this value. An alternative figure of merit is the smallest lethal radius R_k that guarantees a predetermined probability of kill. In several recent studies [10-12] the required probability has been assumed as 0.95, yielding

$$J = R_k = \arg\,\{SSKP = 0.95\} \tag{3}$$

Interception Dynamics

The analysis of a planar interception endgame is based on the following set of simplifying assumptions:

(i) The engagement between the interceptor (*pursuer*) and the maneuvering target (*evader*) takes place in a plane.

(ii) Both have constant speeds V_j and bounded lateral accelerations $|a_j| \le (a_j)_{max}$ (j = E, P).

(iii) The maneuvering dynamics are approximated by first order transfer functions with the time constants τ_P and τ_E .

(iv) The relative interception trajectory can be linearized with respect to the initial line of sight.

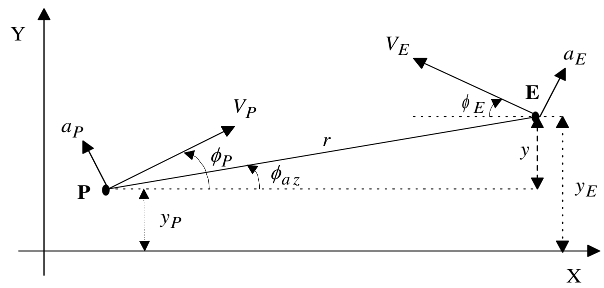


Figure 1. Planar interception geometry

Fig. 1 shows a schematic view of the endgame geometry. The two velocity vectors are generally not aligned with the reference line of sight. The aspect angles ϕ_P and ϕ_E are, however, small. Thus, the approximations $\cos(\phi_i) \approx 1$ and $\sin(\phi_i) \approx \phi_i$, (i = P, E), are coherent with assumption (iv). Based on assumptions (ii) and (iv) the final time of the interception can be computed for any initial range R_0

$$t_f = R_0 / (V_P + V_E) \tag{4}$$

allowing to define the time-to-go by

$$t_{go} = t_f - t \tag{5}$$

The state vector in the equations of relative motion normal to the reference line is

$$X^T = (x_1, x_2, x_3, x_4) = (y, dy/dt, a_E, a_P) \tag{6}$$

where

$$y(t) \triangleq y_E(t) - y_P(t) \tag{7}$$

The corresponding equations of motion and the respective initial conditions are

$$\dot{x}_1 = x_2 ; \qquad x_1(0) = 0 \tag{8}$$

$$\dot{x}_2 = x_3 - x_4; \qquad x_2(0) = V_E \phi_{E0} - V_P \phi_{P0} \tag{9}$$

$$\dot{x}_3 = (a_E^c - x_3)/\tau_E ; \qquad x_3(0) = 0 \tag{10}$$

$$\dot{x}_4 = (a_P^c - x_4)/\tau_P ; \qquad x_4(0) = 0 \tag{11}$$

where a_E^c and a_P^c are the commanded lateral accelerations of the *evader* and the *pursuer,*

$$a_E^c = a_E^{max}\, v; \qquad |v| \leq 1 \tag{12}$$

$$a_P^c = a_P^{max}\, u; \qquad |u| \leq 1 \tag{13}$$

v and u being the respective normalized controls. The non-zero initial conditions $V_E\phi_{E0}$, $V_P\phi_{P0}$ represent the initial velocity components not aligned with the reference line of sight. By assumption (iv) these components are small. The set of equations (8)-(11) can be written in a compact form as a linear, time invariant, vector differential equation

$$\dot{X} = AX + Bu + Cv \tag{14}$$

The problem involves two non-dimensional parameters of physical significance: the pursuer/evader maneuverability ratio

$$\mu = a_P^{max} / a_E^{max} \tag{15}$$

and the ratio of the evader/pursuer time constants

$$\varepsilon \triangleq \tau_E/\tau_P \tag{16}$$

The miss distance can be written as

$$M = |DX(t_f)| = |x_1(t_f)| \tag{17}$$

where

$$D = (1, 0, 0, 0) \tag{18}$$

DG GUIDANCE LAWS

In this subsection several perfect information differential game based guidance laws using this model are described. Since the future target maneuver (or strategy) is not known, the interception of a maneuverable target is formulated as a *zero-sum differential game*. Such formulation of an aerial interception was first suggested by Isaacs [12]. Due to the nonlinear nature of the scenario only very few reduced dimensional pursuit-evasion games, based on oversimplified assumptions, could be solved. Therefore, much effort has been devoted using linear interception models, as in the previous section, in order to obtain closed form solutions.

In linear pursuit-evasion games two types of cost function are the most common. In the first one the cost is the absolute value of the miss distance, assuming bounded controls, as in (12) and (13).

$$J = |y(t_f)| \tag{19}$$

The second type is a quadratic form including penalties on the control efforts instead of assuming hard bounds on the controls.

$$J = \frac{1}{2}y^2(t_f) + \frac{1}{2}\int_{t_0}^{t_f} [\alpha u^2(t) - \beta v^2(t)]dt \tag{20}$$

This paper concentrates on the first type. Details of the second type can be found in [5] and [7]. Linear pursuit-evasion games of both types have closed form solutions based on the notion of the *zero effort miss distance* defined by

$$Z(t) = D\,\Phi(t_f, t)\,X(t) \tag{21}$$

where $\Phi(t_f, t)$ is the transition matrix of the homogeneous system $\dot{X} = AX$. This is the miss distance that will be created if both *players* don't use any control until the final time. Due to (18) Z(t) is a scalar variable. Obviously for any model

$$Z(t_f) = y(t_f) \tag{22}$$

The explicit form of Z(t) depends on (14), which can be replaced by a scalar differential equation

$$\dot{Z}(t) = \tilde{B}(t_f, t)u(t) + \tilde{C}(t_f, t)v(t) \tag{23}$$

where

$$\tilde{B}(t_f, t) = D\Phi(t_f, t)B\,; \quad \tilde{C}(t_f, t) = D\Phi(t_f, t)C \tag{24}$$

The necessary conditions of (min-max) game optimality for the cost function (19) provide the candidate optimal strategies

$$u^*(t) = \text{sign}\,\{Z(t_f)\}\,\tilde{B}(t_f, t) \tag{25}$$

$$v^*(t) = \text{sign}\,\{Z(t_f)\}\,\tilde{C}(t_f, t) \tag{26}$$

and the guaranteed miss distance

$$Z(t_f) = Z(t) - \text{sign}Z(t_f)\int_t^{t_f}\left\{\left|\tilde{B}(t_f, t)\right| - \left|\tilde{C}(t_f, t)\right|\right\}dt \tag{27}$$

Assuming that Z(t) does not change sign, a <u>candidate</u> *optimal* trajectory that terminates with $Z(t_f)$ can be constructed by backward integration using (23) and one can test whether the family of such (*regular*) trajectories fills the entire game space. Regions that are left empty by such construction are *singular* and within them another pair of *optimal strategies* has to be found.

Game Models

The simplest game model is of an *ideal* pursuer and an *ideal* evader where the accelerations are directly controlled [2], but this model, being not realistic, cannot provide a reliable basis in guided missile design. The model used in [3] recognized the effect of interceptor dynamics on the homing performance and approximated it by a first order transfer function with time constant τ_P, but assumed ideal *evader* dynamics (τ_E=0), the "worst case" for the *pursuer.* Although the guidance law, based on this model and denoted DGL/0, is pessimistic and cannot guarantee zero miss distance, it has an important advantage not requiring the knowledge of the target acceleration.

A more realistic and balanced game model is where both the *pursuer* and the *evader* dynamics are approximated by first order transfer functions with the time constants τ_P and τ_E respectively [4]. For such a model, denoted as DGL/1 the expression for the *zero-effort miss distance* becomes

$$Z(t_{go}) = y + \dot{y}\, t_{go} + a_E\, \tau_E^2 \psi(\theta/\varepsilon) - a_P\, \tau_P^2 \psi(\theta) \tag{28}$$

where θ is the normalized time-to-go $\theta = t_{go}/\tau_P$ and

$$\psi(\varsigma) = \left[e^{-\varsigma} + \varsigma - 1\right] \tag{29}$$

$$\tilde{B}(t_f,t) = a_P^{max}\ \tau_P \psi(\theta)\ ; \qquad \tilde{C}(t_f,t) = a_E^{max}\ \tau_E \psi(\theta/\varepsilon) \tag{30}$$

The *normalized zero-effort miss distance* can be defined by

$$z(t) = Z(t) \big/ \tau_P^2\, a_E^{max} \tag{31}$$

and its terminal value becomes

$$z(0) = z(\theta) - \text{sign}\{z(\theta)\} \int_0^{\theta} h(\theta) d\theta \tag{32}$$

For the model of DGL/1

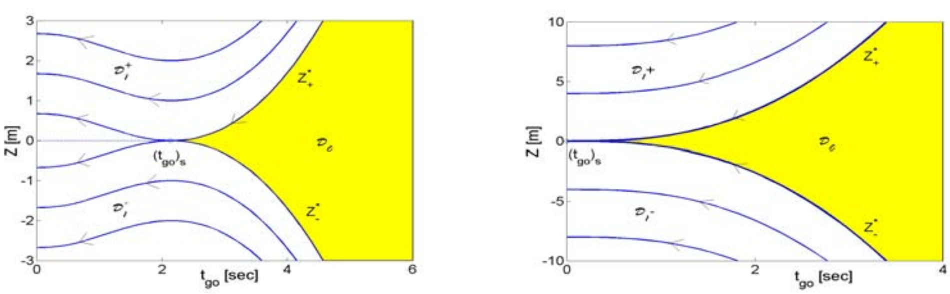
$$\int_0^{\theta} h(\theta)\, d\theta = \int_0^{\theta} \{\mu\psi(\theta) - \varepsilon\psi(\theta/\varepsilon)\}\, d\theta \tag{33}$$

where μ and ε are two parameters of physical significance defined in (15) and (16).

Depending on the values of these parameters, $h(\theta)$ can be either positive or negative. If $\mu > 1$ and $\mu\varepsilon < 1$, the function has a minimum at $\theta = \theta_s$, where θ_s is the non-zero solution of the equation

$$\mu\psi(\theta) - \varepsilon\psi(\theta/\varepsilon) = 0 \tag{34}$$

and the game space decomposition is shown in Fig. 2a.



2a. $\mu\varepsilon < 1$ 2b. $\mu\varepsilon \geq 1$

Figure 2. DGL/1 game space decomposition for $\mu > 1$.

The two *limiting* trajectories (Z_+^*, Z_-^*), where $Z(t_{go})$ does not change sign, reach the t_{go} axis tangentially at $t_{go} = (t_{go})_s = \tau_P\theta_s$. The reduced game space is decomposed into a *singular* region $\mathcal{D}_0$, which is between these trajectories for $t_{go} > (t_{go})_s$ and the *regular* region $\mathcal{D}_1$. In $\mathcal{D}_1$ the *optimal strategies* are given by (25) and (26), while the non-zero *value* of the game depends on the initial conditions.. In the *singular* region the *optimal strategies* are arbitrary and the *value* of the game is a non-zero constant J_s. Every trajectory starting in $\mathcal{D}_0$, must go through the *throat* $[Z((t_{go})_s) = 0]$. This is a *dispersal point* for the *evader* to decide on the maneuver direction for $t_{go} < (t_{go})_s$. The guidance law DGL/0 is a special case of this family.

If $\mu > 1$ and $\mu\varepsilon \geq 1$, the only solution of (34) is $\theta = 0$, $h(\theta)$ is always positive and the game space decomposition is shown in Fig. 2b. From any initial condition in the *singular* region $\mathcal{D}_0$ the *value* of the game, i. e. the guaranteed zero miss distance, is zero.

In most practical situations the speeds are not constant, thus the longitudinal accelerations have a component normal to the line of sight affecting the homing process. Moreover, in a vertical plane interception the maneuvering capabilities vary also with altitude. If these variables are known as a function of the time-to-go, the resulting pursuit-evasion game can still be solved using a linear time varying model as proposed in [6]. The state vector of this problem includes also the aspect angles ϕ_E and ϕ_P. These aspect angles are assumed to be small and the approximations $\cos(\phi_i) \approx 1$, $\sin(\phi_i) \approx \phi_i$, $(i=P,E)$, are uniformly valid, suitable for linear analysis.

Computation of the interception's final time for a given initial range, needed for the time-to-go, is obtained by taking into account the known nominal speed profiles. From the known velocity profiles the longitudinal accelerations can be computed and used in the equations of motion. The expression of zero effort miss distance of this time varying problem is more complex than in the cases discussed earlier. However, this linear pursuit-evasion game with time varying speeds and control bounds is solved similarly as the game with constant parameters. The only difference is that the pursuer/evader maneuverability ratio μ is not constant. The decomposition of the game space is similar to those shown in Figs 2a and 2b. The pursuer guidance law based on the solution of this game (merely an extension of DGL/1) is denoted DGL/E. The details can be found in [6].

In linear differential game solutions with bounded control, such as DGL/0, DGL/1 and DGL/E, the optimal pursuer strategy in the *singular* regions is arbitrary. Any strategy that keeps the state of the game in $\mathcal{D}_0$ is optimal, guaranteeing the same *value*. Thus the implementation of the interceptor guidance laws based on such game solution is not unique. One option is to use the bang-bang type control of (25) everywhere. Another alternative is to adopt in $\mathcal{D}_0$ a (preferably) linear guidance law reaching the maximum admissible level at the boundaries (Z^*_+, Z^*_-), i. e. respecting the acceleration limits of (13). In [2] and [3] such linear guidance laws are suggested. The advantage of the linear guidance law within $\mathcal{D}_0$ is eliminating the unnecessary chattering due to (25).

INTEGRATED ESTIMATOR/GUIDANCE LAW DESIGN

Since the validity of the *Certainty Equivalence Principle* and the associated *Separation Theorem* has not been proven for a realistic homing guidance endgame, in this paper an engineering approach towards an integrated estimation and guidance algorithm is introduced. The essential concept in such integrated design is the parallel development of both elements that can alleviate the limitations of the classical estimation approach in short duration end-games. The requirements of the estimator's two tasks are contradictory: a narrow bandwidth for good filtering and a wide bandwidth to follow the eventual changes. Since no single estimator can satisfy both, the different tasks performed by a classical estimator have to be separated and assigned to different elements within a corporate estimation system. The main ideas involved in this non conventional design concept were developed and tested by using a simplified linearized planar (horizontal) constant speed model of the interception scenario [13]. The integrated estimation and guidance algorithm was extended and validated in generic 3D BMD scenarios [14].

In the planar scenario [13] only a single maneuver, a randomly switched “bang-bang” type evasion, was considered. The estimation of the state variables (including target acceleration) used in the guidance law was performed by a Kalman filter augmented with a narrow bandwidth shaping filter, using an exponentially correlated acceleration (ECA) model [15]. If the target changes the maneuver direction during the endgame at an early phase, the slow estimator identifies it, has time to converge and provides accurate information for guidance. However, if the change occurs nearer to the end there is no time for this process and a large miss distance is created. This problem was alleviated by using a set of "tuned" estimators, assumes the timing of the direction reversal (switch) of the “bang-bang” maneuver. An estimator “tuned” to the correct switch eliminates the delay and yields excellent homing performance. Even if the switch occurs shortly after the time anticipated by the estimator, good performance is obtained. Due to this robustness property a few adequately “tuned” estimators cover the range of interest. These estimators should have a wide bandwidth allowing fast convergence.

During the development of the algorithm in [13] it was found that if the switch occurs near to the end the interceptor is unable to reach its maximum acceleration needed to correct the guidance error due to the inherent detection delay. This deficiency was alleviated (for small values of time-to-go) by increasing the lateral acceleration command while the actual acceleration limits are still respected. Further homing improvement was achieved by introducing a time varying dead zone version of the *signum* function in DGL/1 for the period when the “tuned” estimators were used. This modification, used until the switch is detected, reduced the error created during the detection delay.

In dealing with three-dimensional endo-atmospheric BMD scenarios [14], the study considered two target maneuver models. The first one, assuming a roll stabilized target, is a slowly varying, planar "bang-bang" maneuver in a horizontal plane. The amplitude of the maneuver is monotonically increasing as the target descends to lower altitudes. The second type of maneuver assumes a rolling target with a fixed angle of attack in body coordinates, creating a spiral maneuver with monotonically increasing amplitude. A spinning, aerodynamically stable, reentry vehicle will perform a similar maneuver. Each type of maneuver requires a different type of estimator. The distinction between the two different types is the first step in the estimation process. The group of estimators for the "bang-bang" type maneuver is similar to the one used in [13], but instead of a constant acceleration command a monotonically increasing one is anticipated.

The second type of evasive target maneuver creating a spiral trajectory requires a different estimator. This is a typical three-dimensional maneuver and two planar estimators are used to estimate the projections of the motion in perpendicular planes. Within each plane it is a random phase periodical motion, thus, the shaping filter is of the second order [16], assuming a known maneuver frequency. Such Kalman filter estimates not only the target acceleration, but also its time derivative (the "jerk"). If the maneuver frequency is correctly predicted, the output of such estimator converges well to the actual maneuver even if its amplitude is slowly varying. As a consequence the homing accuracy is satisfactory. If the frequency is incorrect, the estimation is degraded and the homing performance is poor. The bandwidth of such a periodical estimator can be tuned to allow a reasonable frequency error without compromising the homing accuracy.

The first task carried out at the beginning of the endgame is to distinguish between the two different maneuver types by using a multiple model structure. Once the decision between the two types is made, the second phase of the target model identification for each one becomes different. For the “bang-bang” type target maneuver the direction of the maneuver has to be found. For a "spiral" maneuver the frequency range of the actual target maneuver has to be identified with a reasonable accuracy. After the model identification the appropriate narrow bandwidth state estimator is selected to forward information to the guidance law. Continuous computation of the a-posteriori probabilities are used to confirm the correctness of the selection. For a "spiral" maneuver no dramatic changes in the model are expected. For the “bang-bang” type target maneuver the eventual change of direction is expected to be detected by a sufficiently fast detector leading to the use of the nearest "tuned" estimator as in [13].

Until the target maneuver is identified, DGL/0 [3] not requiring the target maneuver and a simple narrow bandwidth estimator are used. After the maneuver identification DGL/E [6], derived for time varying parameters, is used. This requires the velocity and maneuverability profiles in the endgame that can be precalculated along a nominal trajectory. If the target maneuver is non periodical the guidance law modifications used in [13] and stated earlier are applied.

DEMONSTRATIVE EXAMPLE

In this section the results of extensive Monte Carlo simulations of BMD scenarios are summarized demonstrating the potential of the new approach in achieving excellent homing performance against randomly maneuvering threats. Endo-atmospheric interceptions terminating in the altitude band of 20-30 km with an initial range of 20 km are considered. The target is a generic RV with aerodynamic control, performing either spiral or horizontal bang-bang evasive maneuvers. It is launched from the distance of 600 km on a minimum energy trajectory and characterized by a ballistic coefficient β=5000 kg/m^2 and a trimmed lift-to-drag ratio Λ = 2.6. Its velocity at reentry of an altitude of 150 km is V_{e0} = 1720 m/s with a flight path angle of $\gamma_{e0} = -18^o$ and a horizontal distance of 210 km from its surface target.

The interceptor is generic roll stabilized two-stage solid rocket missile that has two identical guidance channels for aerodynamic control (skid to turn). The second stage rocket motor is ignited with a delay in order to guarantee that for any interception altitude the endgame terminates with a positive longitudinal acceleration and non decreasing maneuverability. The dynamics of the interceptor and the target are approximated by first order transfer functions with equal time constants $\tau_P = \tau_E$ = 0.2 sec. The seeker of the interceptor provides angular measurements at a sampling rate of 100 Hz. The measurements are corrupted by zero-mean white Gaussian angular noise with a standard deviation of 0.1 mrd.

The test of the combined estimation/guidance scheme against a very large set of randomly selected target maneuvers yielded encouraging results. The duration of the endgame was slightly above 4 sec. Against both types of maneuvers, namely "bang-bang" maneuvers with random switch and random phase periodical maneuvers with frequencies between 0.05-2.0 Hz, the target maneuver identification was performed in the first part of a short duration BMD endgame, allowing sufficient time for precise estimation needed for accurate homing guidance. Homing accuracy statistics indicate that 95% of the miss distances were less than 32 cm, with an average miss distance of less than 20 cm.

CONCLUSIONS

An innovative guidance strategy is generated by integrating the design of a multiple model adaptive estimator and a differential game based guidance law. Extensive simulation results demonstrate that the combination of these two new approaches provides a substantial homing accuracy improvement compared to earlier results. Moreover, the new guidance strategy has the potential satisfying the "hit-to-kill" requirement against two types of stressing evasive target maneuvers. This achievement, reached at university research level, represents a scientific breakthrough, but its acceptance and application in future interceptor design requires a revolutionary change of mind set within the missile development community.

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