ROBUST SUBOPTIMAL FEEDBACK CONTROL FOR LOW-THRUST TRANSFERS BETWEEN NONCOPLANAR ELLIPTICAL AND CIRCULAR ORBITS

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It is considered the synthesis of robust suboptimal feedback control based on minimum-time multi-revolution transfer between non-coplanar elliptical and circular orbits in the inverse square gravity field. The problem is solved using asymptotic nature and symmetries of optimal control on the unperturbed trajectory. It is shown stability of the feedback control with respect to perturbations and deviation of initial conditions. Derived suboptimal feedback control can be used for providing of spacecraft autonomy and for mission design purpose.

INTRODUCTION

There are considered a minimum-time multi-orbit low thrust trajectories. It is used approach based on usage of equinoctial orbital elements, maximum principle, and numerical averaging of equations of optimal motion [1]. In contrast to [1], it is used continuation method [2, 3] to solve boundary value problem of maximum principle. The continuation method allows to widen region on convergence substantially and to compute optimal solutions for different boundary conditions and spacecraft parameters by stable way.

General case of optimal multi-orbit low thrust transfer between non-coplanar elliptical orbits was considered in [3]. This paper, using numerical methods from [2, 3], presents particular but practically important problem of spacecraft insertion into a circular orbit from an initial elliptical orbit. More precisely, it is considered minimum-time multi-orbit low thrust transfer between non-coplanar elliptical and circular orbits in the inverse square gravity field. It is considered constant ejection velocity problem, i.e. thrust and specific impulse assume to be fixed during thrusting. Thrust direction is unconstrained and it is chosen from optimality conditions. It is supposed that initial line of apsides belongs to plane of final orbit. Such kind of boundary conditions is typical for spacecraft insertion into high circular orbits, in particularity into geostationary orbit (GEO). Of course, considered problem is invertible, so obtained results are applicable for transfer from an initial circular orbit into a final elliptical orbit.

Considered problems are characterized by long transfer duration, high sensitivity with respect to errors in the initial conditions determination, to errors in the thrust steering, and to perturbations due to external forces. So, it is required periodical correction of thrust steering program to compensate trajectory drift due to various perturbations. Of course, such kind of correction may be realized by periodical repeated calculation of optimal trajectory using current estimation of spacecraft phase vector as initial conditions and current estimation of thrust and specific impulse values. But this approach requires a large computational capability; therefore it is hard problem for onboard realization. These computations can be carried at a ground facility and then transmitted to the spacecraft as renewed control program but this leads to decreasing of spacecraft autonomy and to additional work load of mission control center. Therefore, it is actual the development of onboard feedback control algorithm providing close to optimal trajectory and stability with respect to errors in orbit determination, thrust vector deviations, and orbit perturbations.

Main purpose of the study is synthesis of feedback control providing closeness to minimum-time trajectory for insertion into the final circular orbit not only in case of inverse square gravity field, but in case of perturbed motion too.

1. MATHEMATICAL STATEMENT OF OPTIMAL CONTROL PROBLEM

Let write equations of spacecraft controlled motion in the inverse square gravity field using equinoctial elements in the following form [1, 3]:

$$
\left.
\begin{aligned}
\frac{dh}{dt} &= \delta \frac{P}{m}\frac{h}{\xi}\cdot h\cos\vartheta\cos\psi,\\
\frac{de_x}{dt} &= \delta \frac{P}{m}\frac{h}{\xi}\left\{\xi\sin F\sin\vartheta\cos\psi + \left[(\xi+1)\cos F + e_x\right]\cos\vartheta\cos\psi - e_y\eta\sin\psi\right\},\\
\frac{de_y}{dt} &= \delta \frac{P}{m}\frac{h}{\xi}\left\{-\xi\cos F\sin\vartheta\cos\psi + \left[(\xi+1)\sin F + e_y\right]\cos\vartheta\cos\psi + e_x\eta\sin\psi\right\},\\
\frac{di_x}{dt} &= \delta \frac{P}{m}\frac{h}{\xi}\cdot\frac{1}{2}\tilde{\varphi}\cos F\sin\psi,\\
\frac{di_y}{dt} &= \delta \frac{P}{m}\frac{h}{\xi}\cdot\frac{1}{2}\tilde{\varphi}\sin F\sin\psi,\\
\frac{dF}{dt} &= \frac{\xi^2}{h^3} + \delta \frac{P}{m}\frac{h}{\xi}\cdot\xi\eta\sin\psi,\\
\frac{dm}{dt} &= -\delta\frac{P}{w},
\end{aligned}
\right\}
\qquad (1)
$$

where δ - thrusting function (δ=1 during thrusting and δ=0 during coasting), P – thrust, m – spacecraft mass, ϑ - pitch (angle between projection of thrust vector onto current orbital plane and circumferential direction), ψ - yaw (angle between thrust vector and current orbital plane), $h = \sqrt{\frac{p}{\mu}}$, $e_x = e\cos(\Omega+\omega)$, $e_y = e\sin(\Omega+\omega)$, $i_x = \tan\frac{i}{2}\cos\Omega$, $i_y = \tan\frac{i}{2}\sin\Omega$, and $F = \nu+\omega+\Omega$ - equinoctial elements, p – focal parameter, e – eccentricity, ω - argument of pericenter, i – inclination, Ω - right ascension of ascending node (RAAN), ν - true anomaly, $\xi = 1 + e_x\cos F + e_y\sin F$, $\eta = i_x\sin F - i_y\cos F$, $\tilde{\varphi} = 1 + i_x^2 + i_y^2$, w – exhaust velocity, μ - gravity parameter of primary. Values of thrust P and exhaust velocity w are fixed in the considered problem.

It is required to transfer spacecraft having initial mass m_0 from initial orbit

$$h=h_0,\ e_x=e_{x0},\ e_y=e_{y0},\ i_x=i_{x0},\ i_y=i_{y0} \qquad (2)$$

into final orbit

$$h=h_k,\ e_x=e_y=0,\ i_x=i_y=0 \qquad (3)$$

for minimal time T, i.e. it is considered minimization of performance index

$$J = \int_0^T dt \to \min . \qquad (4)$$

Nullification of final inclination in (3) is achieved by rotation of reference frame base plane to provide its aligning with plane of final orbit.

Let write Hamiltonian of optimal control problem in following form [3]:

$$H = -1 + \frac{\xi^2}{h^3}p_F + \delta\frac{P}{m}\frac{h}{\xi}\left(A_\tau\cos\vartheta\cos\psi + A_r\sin\vartheta\cos\psi + A_n\sin\psi\right), \qquad (5)$$

where $A_\tau = hp_h + \left[(\xi+1)\cos F + e_x\right]p_{ex} + \left[(\xi+1)\sin F + e_y\right]p_{ey}$, $A_r = \xi\left(\sin F\cdot p_{ex} - \cos F\cdot p_{ey}\right)$, $A_n = \eta\left(-e_y p_{ex} + e_x p_{ey}\right) + \frac{1}{2}\tilde{\varphi}\left(\cos F\cdot p_{ix} + \sin F\cdot p_{iy}\right) + \xi\eta\cdot p_F$, and p_h, p_{ex}, p_{ey}, p_{ix}, p_{iy}, p_F are co-state variables.

Optimal control $\delta(t)$, $\vartheta(t)$, $\psi(t)$ is determined from the Hamiltonian (5) maximization:

$$\cos\vartheta = \frac{A_\tau}{\sqrt{A_\tau^2 + A_r^2}},\ \sin\vartheta = \frac{A_r}{\sqrt{A_\tau^2 + A_r^2}}, \qquad (6)$$

$$\cos\psi = \frac{\sqrt{A_\tau^2 + A_r^2}}{\sqrt{A_\tau^2 + A_r^2 + A_n^2}},\ \sin\psi = \frac{A_n^2}{\sqrt{A_\tau^2 + A_r^2 + A_n^2}}, \tag{7}$$

$$\delta \equiv 1. \tag{8}$$

Identity (8) means the optimal trajectory does not include coast arcs, therefore spacecraft mass can be considered as following function of time:

$$m = m_0 - (P/w)\cdot t. \tag{9}$$

Substituting of optimal control (6-8) into (5) leads to the following expression for optimal Hamiltonian:

$$H = -1 + \frac{P}{m}\frac{h}{\xi}\left(A_\tau^2 + A_r^2 + A_n^2\right)^{1/2} + \frac{\xi^2}{h^3} p_F = -1 + kPA + H_F, \tag{10}$$

where $k = \frac{1}{m}\frac{h}{\xi}$, $A = \left(A_\tau^2 + A_r^2 + A_n^2\right)^{1/2}$, $H_F = \frac{\xi^2}{h^3} p_F$.

Equations of optimal motion becomes following:

$$\left.\begin{aligned}
\frac{d\mathbf{x}}{dt} &= \frac{\partial H}{\partial \mathbf{p}} = P\left[k\left(A_\tau \frac{\partial A_\tau}{\partial \mathbf{p}} + A_r \frac{\partial A_r}{\partial \mathbf{p}} + A_n \frac{\partial A_n}{\partial \mathbf{p}}\right)A^{-1}\right],\\
\frac{dF}{dt} &= \frac{\partial H}{\partial p_F} = P\left[k\left(A_\tau \frac{\partial A_\tau}{\partial p_F} + A_r \frac{\partial A_r}{\partial p_F} + A_n \frac{\partial A_n}{\partial p_F}\right)A^{-1}\right] + \frac{\partial H_F}{\partial p_F}, \quad ,\\
\frac{d\mathbf{p}}{dt} &= -\frac{\partial H}{\partial \mathbf{x}} = -P\left[\frac{\partial k}{\partial \mathbf{x}}A + k\left(A_\tau \frac{\partial A_\tau}{\partial \mathbf{x}} + A_r \frac{\partial A_r}{\partial \mathbf{x}} + A_n \frac{\partial A_n}{\partial \mathbf{x}}\right)A^{-1}\right] - \frac{\partial H_F}{\partial \mathbf{x}}\\
\frac{dp_F}{dt} &= -\frac{\partial H}{\partial F} = -P\left[\frac{\partial k}{\partial F}A + k\left(A_\tau \frac{\partial A_\tau}{\partial F} + A_r \frac{\partial A_r}{\partial F} + A_n \frac{\partial A_n}{\partial F}\right)A^{-1}\right] - \frac{\partial H_F}{\partial F},
\end{aligned}\right\} \tag{11}$$

where $\mathbf{x} = \left(h, e_x, e_y, i_x, i_y\right)^{\mathrm{T}}$, $\mathbf{p} = \left(p_h, p_{ex}, p_{ey}, p_{ix}, p_{iy}\right)^{\mathrm{T}}$.

As it is considered transfer between orbits, the final value of true longitude F is not fixed, so $p_F(T)=0$. Optimal Hamiltonian does not depend on F after averaging, therefore $\frac{dp_F}{dt} = -\frac{\partial H}{\partial F} = 0$. Therefore, $p_F \equiv 0$ on an optimal solution. Taking into account future averaging, optimal Hamiltonian (12) becomes following:

$$H = -1 + kPA, \tag{12}$$

and equations of motion (13) can be rewritten in the form:

$$\left.\begin{aligned}
\frac{d\mathbf{x}}{dt} &= \frac{\partial H}{\partial \mathbf{p}} = \delta P\left[k\left(A_\tau \frac{\partial A_\tau}{\partial \mathbf{p}} + A_r \frac{\partial A_r}{\partial \mathbf{p}} + A_n \frac{\partial A_n}{\partial \mathbf{p}}\right)A^{-1}\right],\\
\frac{d\mathbf{p}}{dt} &= -\frac{\partial H}{\partial \mathbf{x}} = -\delta P\left[\frac{\partial k}{\partial \mathbf{x}}A + k\left(A_\tau \frac{\partial A_\tau}{\partial \mathbf{x}} + A_r \frac{\partial A_r}{\partial \mathbf{x}} + A_n \frac{\partial A_n}{\partial \mathbf{x}}\right)A^{-1}\right]
\end{aligned}\right\} \tag{13}$$

Equations (13) are numerically averaged during their integration using following averaging scheme:

$$\frac{d\mathbf{y}}{dt} = \frac{1}{T}\int_{t_0}^{t_0+T} \mathbf{f}_e(\mathbf{y}, F, t)dt = \frac{n}{2\pi}\int_0^{2\pi} \mathbf{f}_e(\mathbf{y}, F, t)\frac{dt}{dF}dF, \tag{14}$$

where $\mathbf{y}=(\mathbf{x}^{\mathrm{T}}, \mathbf{p}^{\mathrm{T}})^{\mathrm{T}}$, $\mathbf{f}_e(\mathbf{y},F,t)$ – right hands of differential equations (13), $n = \frac{1}{\mu}\left[\sqrt{1 - e_x^2 - e_y^2}\Big/ h\right]^3$ - mean motion, $dt/dF = h^3/\xi^2$.

Final values of state vector **x**, co-state vector **p**, and residual vector **f** at T are computed as a result of integration of (13) using averaging (14). Residual vector has following form:

$$\mathbf{f} = \begin{pmatrix} h(T) - h_k \\ e_x(T) - e_{xk} \\ e_y(T) - e_{yk} \\ i_x(T) - i_{xk} \\ i_y(T) - i_{yk} \\ H(T) \end{pmatrix} = 0 \tag{15}$$

Equation (15) should be solved with respect to vector of two-point boundary value problem parameters z :

$$\mathbf{z} = \begin{pmatrix} \mathbf{p}(0) \\ T \end{pmatrix} \tag{16}$$

Such a way, the optimal control problem is reduced to the boundary value problem (13-16), which is solved using continuation method (see detailed description of the continuation method in [2-4]).

2. FEATURES OF MINIMUM-TIME TRANSFERS BETWEEN NON-COPLANAR ELLIPTICAL AND CIRCULAR ORBITS WHEN INITIAL POSITION OF LINE OF APSIDES IS FREE

In case if initial position of line of apsides is free, this line should be aligned with line of crossing of initial and final orbits plane as it is resulted from transversality conditions [3].

Let use non-dimensional variables choosing radius of final orbit r_k as a length unit and inverse mean motion $(r_k^3/\mu)^{1/2}$ as a time unit. As it follows from problem statement there are only three essential parameters of initial orbit fully determining boundary value problem of maximum principle correct to rotation of reference frame. There are non-dimensional perigee radius r_p, non-dimensional apogee radius r_a, and inclination i of initial orbit. Indeed, so as line of apsides coincides with line of crossing initial and final orbits plane, it is always can be defined the reference frame providing nullification of initial RAAN and argument of perigee: it is necessary to align X axis to the initial perigee direction when Y axis should belong to the final orbit plane.

Due to problem symmetry optimal control is defined by three essential parameters too. There are initial co-states p_h, p_{ex}, p_{ix}. Initial co-states p_{ey} and p_{iy} equal to 0. Under this conditions optimal control is symmetrical with respect of line of apsides.

Application of averaging leads to asymptotic nature of the optimal solution. It means the averaged optimal solution is applicable for any thrust, specific impulse, and spacecraft mass while it keeps an assumption of smallness of relative orbital parameters change during one orbit. In this case while “slow” orbital parameters are close, there are close dependency of thrust steering angles versus true anomaly for trajectories with different thrust acceleration and specific inpulse.

Finally, the optimal solution is scalable on distance from primary and gravity parameters. Indeed, let consider transfer in the inverse square gravity field having gravity parameter μ from the elliptical orbit having pericenter radius r_{p0}, apocenter radius r_{a0}, and inclination i_0 to the final circular orbit having radius r_k and inclination i_k. Let non-dimensional initial co-states p_{h0}*, p_{ex0}*, p_{ix0}* and non-dimensional transfer duration T* are solution of the considered optimal control problem. Obtained solution can be scaled to transfer in the inverse square gravity field having another gravity parameter μ_1 from the elliptical orbit having pericenter radius r_{p01}, apocenter radius r_{a01}, and inclination i_{01} to the final circular orbit having radius r_{k1} and inclination i_{k1} if $r_{p01} = r_{p0}r_{k1}/r_k$, $r_{a01} = r_{a0}r_{k1}/r_k$, $i_{k1} = i_{01} + (i_k - i_0)$. In this case transfer duration becomes equal to $T_1 = (r_{k1}^3/\mu_1)^{1/2}T^*$.

3. OPTIMAL TRAJECTORIES COMPUTATION ON THE GRID OF INITIAL PERIGEE RADIUS, APOGEE RADIUS, AND INCLINATION

It was carried out numerical analysis of minimum-time multi-orbit transfer between non-coplanar elliptical orbit and GEO in the wide range of initial orbit parameters having purpose to use these results for development an engineering technique for easy low-thrust mission design and for feedback control synthesis. It was considered three-dimensional grid of initial perigee radius, apogee radius, and inclination. Minimal value of perigee and apogee radii on the grid is r_0=6571 km, j-th node of grid has value $r_j = \exp\left[\ln r_0 + \frac{j}{20}\ln\frac{r_{GEO}}{r_0}\right]$, where j is varying from 0 to 39, and r_{GEO}=42164 km. So, it is determined 40×40 grid having maximal perigee and apogee radii ~246539.565 km. It was considered range of initial inclinations from 0 to 90 deqrees with step 5 degrees. Total number of nodes on the grid is 40×40×19. Taking into account constraints $r_p \le r_a$, there are 40×(40+1)×19/2=15580 different initial orbits defined on the considered grid.

It was solved optimal control problem for each initial orbit from the grid. As a result? There were computed and stored values of required characteristic velocity, minimal and maximal geocentric distances during the transfer, and initial values of co-states. Required characteristic velocity was used for fast estimation of spacecraft final mass [4, 5] and initial co-states were used for synthesis of suboptimal feedback control.

4. SYNTHESIS OF SUBOPTIMAL FEEDBACK CONTROL

Defined on three-dimensional grid of initial orbit parameters values of initial co-states p_h, p_{ex}, p_{ix} can be used in the linear interpolation procedure to estimate the initial values of co-states for any given initial elliptical orbit having line of apsides aligned along line of crossing initial and final orbit planes. These initial co-states p_h, p_{ex}, p_{ix}, supplemented by null values of other co-states, can be used for integration of unaveraged equation of motion. Numerical simulation shows a good results when this approach is used for unperturbed trajectory in the inverse square gravity field. In this case osculating elements of unaveraged trajectory perform short-period oscillations around averaged solution.

Situation becomes more complex if there are taken into account perturbations including accelerations, errors in the initial conditions, and errors in the thrust vector implementation. In this case errors are increasing with time, leading to inadmissible final residuals. For example, let consider low-thrust transfer from initial elliptical orbit having perigee altitude 500 km, apogee altitude 30000 km, and inclination 62.8° to GEO (altitudes are referenced to the mean earth radius 6371 km). Initial spacecraft mass is 1000 kg, thrust – 0.2 N, specific impulse – 1500 s. Fig. 1 shows time history of semi-major axis, apogee and perigee radii for following cases: (1) averaged trajectory (dashed line); (2) unperturbed unaveraged trajectory (solid thin line); (3) unaveraged trajectory taking into account perturbations from geopotential up to 10^{th} degree and 10^{th} order and lunisolar perturbations (blue line); (4) unaveraged trajectory having perturbations from (3) and coasting arcs during eclipses (red line). One can see, considered perturbations leads to deviation in the final perigee and apogee radii up to 4-5 thousands km. Final deviation in inclination reaches to 13-14°.

Next approach of using pre-computed initial co-states is calculating of current co-states values for current values of inclination, apogee and perigee radii by linear interpolation over the grid. Again, there are assigned zero values to p_F, p_{ey}, p_{iy}. Calculated values of co-states are substituting to equations (6), (7) for thrust steering angles computation. Such kind of approach provides stability in semi-major axis, eccentricity, and inclination, but it remains unstable with respect to perturbation of argument of perigee and RAAN. As a result, there are accumulated errors in all orbital elements due to rotation of line of apsides and line of nodes though total error become less than in previous case.

It was used following transformation of current co-states to provide stability of control with respect to perturbations in all orbital elements:

$$\begin{aligned}
p_h^* &= p_h(r_a, r_p, i),\\
p_{ex}^* &= p_{ex}(r_a, r_p, i)\cos(\omega+\Omega),\\
p_{ey}^* &= p_{ex}(r_a, r_p, i)\sin(\omega+\Omega),\\
p_{ix}^* &= p_{ix}(r_a, r_p, i)\cos(\Omega),\\
p_{iy}^* &= p_{ix}(r_a, r_p, i)\sin(\Omega),
\end{aligned} \tag{17}$$

where p_h, p_{ex}, p_{ix} again are calculated using linear interpolation over the three-dimensional grid at current values of r_p, r_a, and i. Control (17) does not depend on time explicitly, explicitly depends on all 6 orbital elements and it has form of feedback control. It is easy to show the yaw has local minimum or maximum in the orbital nodes and pitch has value 0 or 180 degrees in apsides due to usage of transformed value (17) of current co-state vector in the equations (6), (7). As a result, even if lines of apsides and nodes are diverged under perturbations, control (17) provides trajectory convergence to the final orbit. In case of absence perturbations in the argument of perigee and RAAN control (17) is coincided with optimal control (up to errors due to averaging and interpolation), otherwise it is differed from optimal control, less for weak perturbations and more for strong ones. Let name control (17) as a suboptimal feedback control.

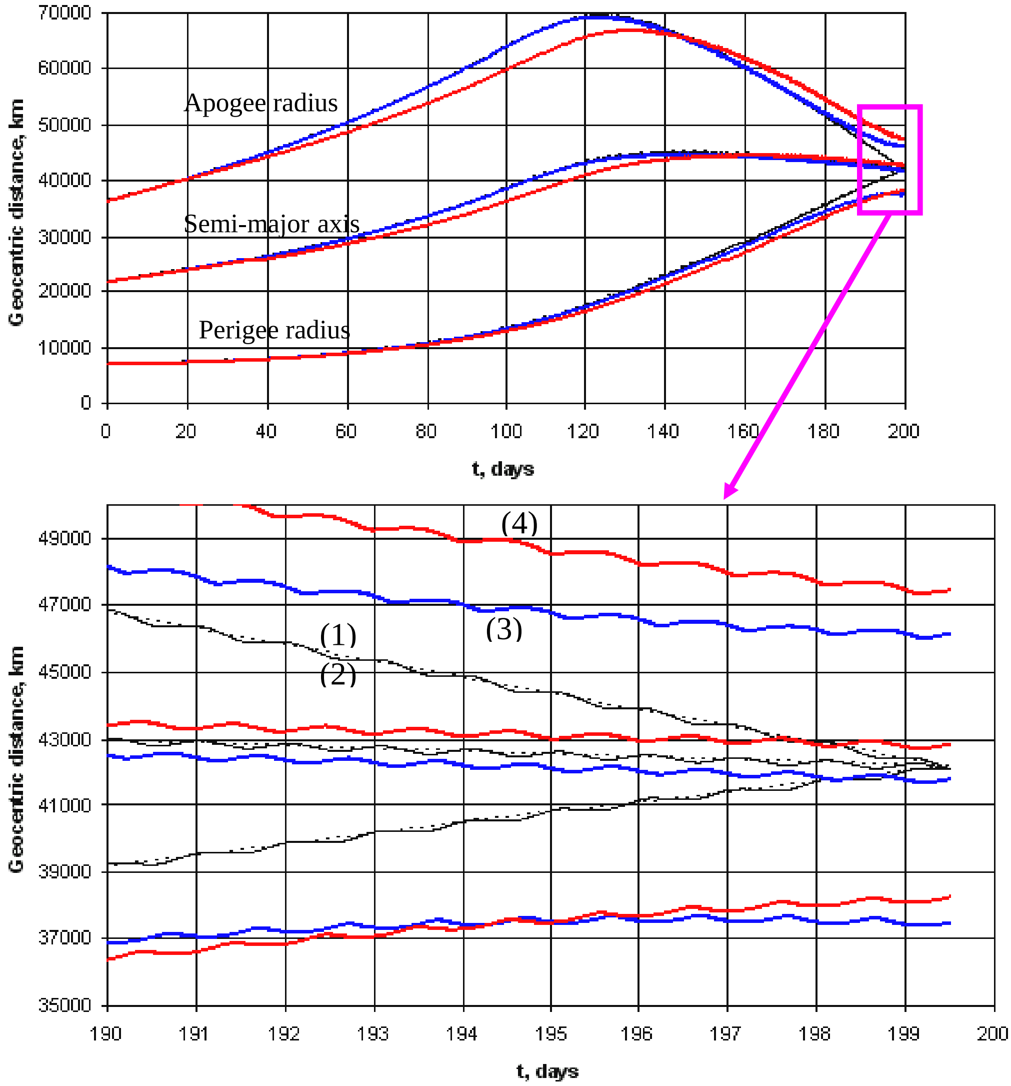
Figure 2 shows time history of semi-major axis, perigee and apogee radii in the considered above problem of transfer to GEO for final time range in case of unperturbed averaged trajectory and two trajectories using the suboptimal feedback control (17). In one case there are taken into account perturbations due to geopotential (up to 10^{th} degree and 10^{th} order), lunar and solar gravity and in another case these coasting during eclipses supplements the same perturbations.

Figure 2 demonstrates the solution convergence to the final orbit and closeness of trajectory using feedback control to optimal trajectory.

Control (17) was found stable with respect to large deviations of initial conditions too. Figure 3 shows time history of perigee and apogee radii and semi-major axis in case of transfer from high-elliptical orbit of “Molniya”-type to GEO. It was considered initial orbit having perigee altitude 1000 km, apogee altitude 40000 km, inclination 63.4 degrees, and argument of perigee 250 degrees. Optimal trajectory is denoted by blue line, and trajectory using suboptimal feedback control is denoted by white line. Despite non-zero value of initial argument of perigee, feedback control provides convergence to the final orbit due to transformation (17) if line of apsides has large deviation from line of nodes.

Suboptimal feedback control (17) was successfully used for computation of 3-dimensional transfers between circular earth and lunar orbits within frame of restricted problem of four bodies Sun-Earth-Moon-spacecraft taking into account earth oblateness. To carry out these computations, trajectory was divided on two parts: transfer from a earth orbit to the lunar libration point L1 and transfer from the L1 to a lunar orbit. It is easy to show L1 osculating orbit is elliptical both in geocentric and selenocentric reference frames. This fact allows to apply considered here technique if geocentric part of trajectory is integrated using inverse direction of time and thrust vector having opposite direction with respect to optimal one (6), (7).

Figure 4 presents an example of suboptimal trajectory from the circular earth orbit having radius 42164 km and inclination 51.6 degrees to the polar circular lunar orbit having altitude 100 km. Spacecraft mass at L1 is 100 tons, thrust equals to 10 N, and specific impulse equals to 6000 s.



Fugure 1 – Dependency of semi-major axis, perigee and apogee radii versus time. (1) Dashed line – averaged optimal trajectory in inverse square gravity field; (2) black solid line – integration of unaveraged equation of optimality motion using initial co-states from averaged problem; inverse square gravity field; (3) blue line – the same as (2), but taking into account geopotential 10×10 and lunisolar perturbations; (4) red line – the same as (3), but taking into account coasting during eclipses.

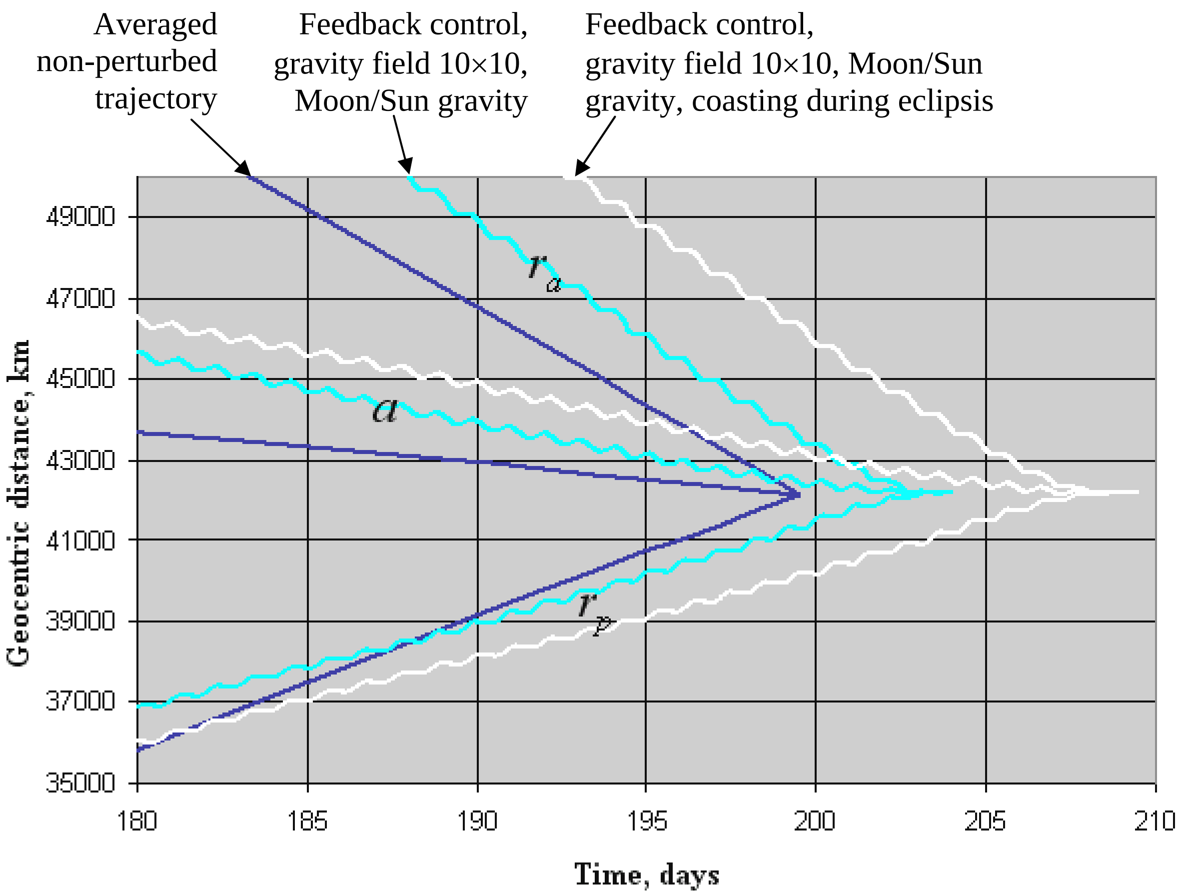


Figure 2 – Perturbations impact on trajectory using suboptimal feedback control

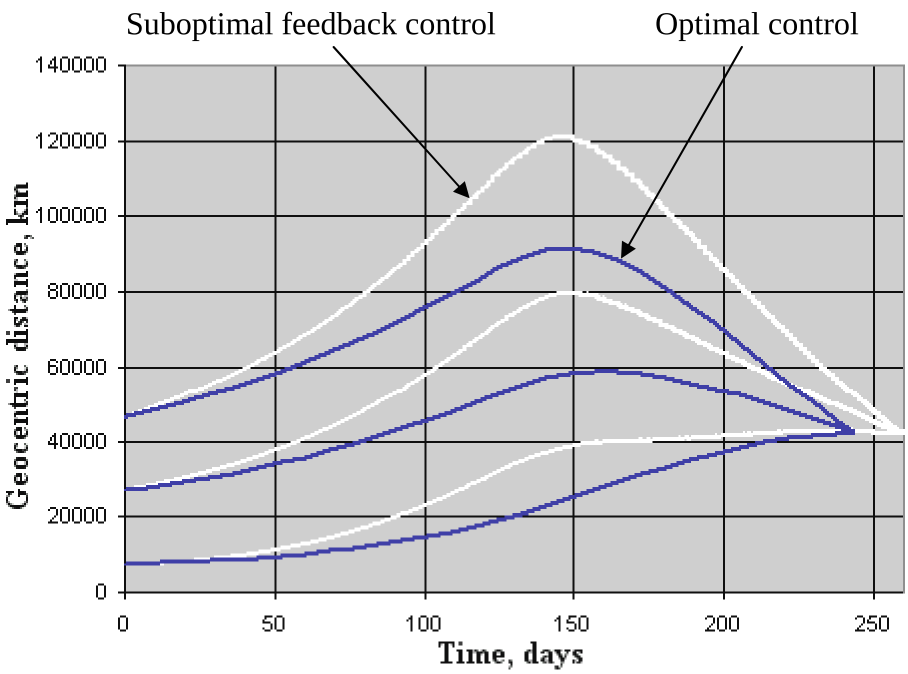


Figure 3 – Transfer from “Molniya”-type orbit to GEO

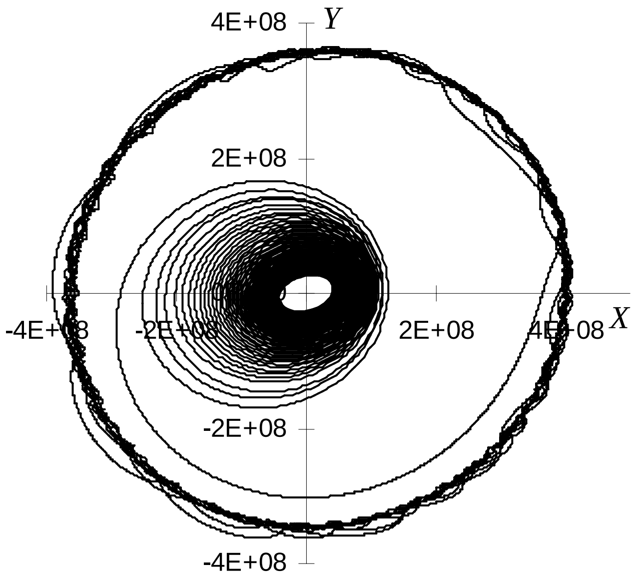


Figure 4 – Projection of trajectory between earth and lunar obits onto equatorial plane

6. CONCLUSION

It is derived robust suboptimal feedback control (17) for low-thrust transfer between non-coplanar elliptical and circular orbits. This control can be used for providing of spacecraft autonomy or for mission design purpose. On-board application of this control leads to necessity to have on-board estimation of current phase vector which should be periodically refined using either spacecraft autonomous navigations or ground stations. On-board realization of considered control algorithm is simple enough. It does not require large consumptions of processor time and computer memory. Proposed algorithm is robust in wide range of spacecraft orbital parameters with respect to typical orbit perturbations. High stability of considered control with respect to perturbing accelerations allows it using for high-perturbed trajectory design, for example to design of low-thrust transfer between earth and lunar orbits.

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