

# NUMERICAL STUDY OF TRANSVERSE JET INJECTIONS INTO SUPERSONIC FLOW\*

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**ABSTRACT:** Numerical investigation of transverse sonic air injection into air flow of Mach 6 has been performed in frame of 2D RANS approach. Comparison of the obtained data with experimental Schlieren photographs and height of the jet penetration into the primary flow demonstrates good agreement. Study of the problem of the air injection into the freestream of Mach 2.8 has been initiated as well as first stage of modeling of the experiments on helium injection into the air flow in a channel with an abrupt expansion performed at ITAM SB RAS. Detailed picture of the flow structure is presented and analyzed.

## INTRODUCTION

The problem of the fluid injection into the supersonic flow has become important due to hypersonic vehicle development and necessity in scramjet engine design. These kinds of flows are met as well in problems of controllable thrust vector of rocket engines, jet-stream control of aircrafts, etc. For the scramjet propulsion device, where chemical reactions and heat release occur in the supersonic stream, fuel and air must be mixed in the near-field of the fuel injection. Proper choice of the injection scheme plays an important role for the efficient design of the combustor. The transverse injector is often used since it gives high fuel jet penetration and good mixing though leads to big pressure losses.

While the problem geometry is simple, the configuration arisen in result of the jet interaction with supersonic freestream is quite sophisticated and includes shock waves, separation zones, areas of significant flow acceleration and stagnation (see Fig. 1). Indeed the jet itself has a complex structure depending on jet/freestream pressure ratio and contains contact discontinuities, barrel shocks, Mach disks and mixing layers. Being imbedded into the supersonic flow it influences the flowfield similar to an obstacle giving rise to the bow shocks and boundary layer separation. All these facts in cooperation with turbulence effects make the numerical problem simulation difficult bearing in mind the need in satisfactory flow resolution.

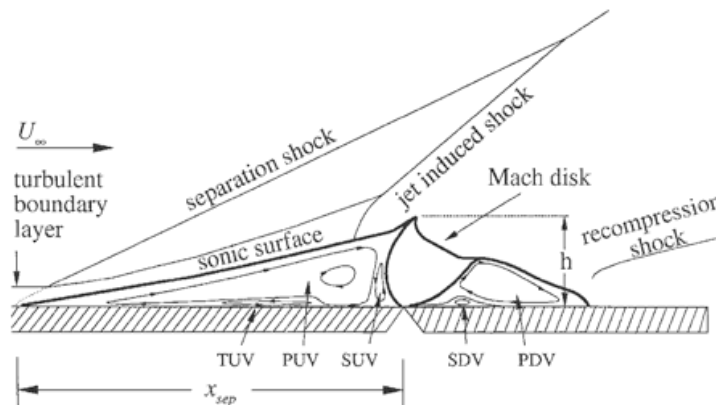


Fig. 1 The schematic representation of the underexpanded jet injection into supersonic flow given in [1]

Information on physical structure of such kinds of flow available at present time is collected mainly from experimental works initiated for several decades ago. A typical two-dimensional flowfield of the underexpanded jet injected normal to the supersonic freestream with sonic conditions given in experimental work [1] is depicted in Fig. 1. As the jet leaves the injector exit, it expands and penetrates into the boundary layer. Upstream of the jet, a jet-induced and separation shocks are generated that causes the boundary layer separation. Inside the separation in front of the jet several vortices may be distinguished such as primary, secondary and third upstream vortices (denoted in the picture as PUV, SUV and TUV

\* The work has been financially supported by Russian Foundation for Basic Research, grant # 08-01-92206-GFEN\_a and by Ministry of Education and Sciences of Russian Federation, AVC Program "Development of Scientific Potentials of Higher School", grant #2.1.1/4674

correspondingly). Behind the jet the downstream separation takes place where primary and secondary vortices (PDV and SDV) are recognized as well and a recompression shock appears. The jet due to its underexpanded nature includes the strong transversal shock called Mach disk.

A brief overview of the contributions to the problem is done below. In paper [2] basic data on interaction processes arising with a normal gas injection into a supersonic flow have been obtained and similarity laws for the most important parameters have been determined on the basis of experimental and theoretical investigation. A low static pressure area has been observed downstream the injection location. An analytical model based on the analogy between flowfield in vicinity of a jet and a blunt body has allowed to obtain a relation for jet penetration depth. In [3], the transverse jet in a supersonic nozzle has been studied experimentally. Wall static pressure distributions and stagnation pressure distributions in cross-sections normal and parallel to the external flow have been obtained along with the separation lengths. Helical structure of the injected flow has been discovered, and 3D flow picture based on the experimental results has been presented. In paper [4], a possibility of an air and helium film cooling of a surface in a supersonic flow is studied. The gas has been injected with a subsonic velocity through a slot parallel to the wall. It has been found that the jet temperature has a weak influence on the flow picture, while the mass flow has a strong effect. Experimental relation of cooling efficiency for different values of the mass flow has been obtained. Experiments on 2D jet interaction with a supersonic stream described in [5] have been aimed at normal force definition. It has been found out that a momentum flux ratio of the primary and secondary flows has sufficient influence on the flow structure. Dependencies of the separation length and transitional area size on this parameter have been obtained experimentally and comparison with a theory and data of other experiments has shown good agreement in a wide range of Mach and Reynolds numbers. The paper [6] is devoted to an investigation of different gases injection through a slot of 0.2 mm width into a supersonic flow of Mach 2.5. The authors have discovered that injected gas concentration inside the separation zone in front of the jet is quite high, and depends on temperature ratio and molecular weights of the jet and external flow gases. It has been shown for a wide range of temperature flow regimes that with pressure ratio rising, the separation length increases. The separation lengths, pressure level within separation and amplification factor are influenced by pressure and density ratios, but almost independent on freestream Reynolds number. Jet temperature and molecular weight has a strong effect on separation length for injected gases possessing low molecular weight or high temperatures. Determination of the physical characteristics of the flow pattern and establishment of the principal relationships governing the geometrical characteristics of the flow had been the goal of the work [7]. The flow picture has been constructed based on experimental data. Semiempirical relation for the jet barrel has been derived and good agreement with the measurements has been achieved. It has been observed that the crossflow Mach number has no effect on relative detachment length with jet/freestream pressure ratio varying.

Mixing parameters of a sonic air jet injected into supersonic flow of Mach number 2.4 have been investigated experimentally in [8]. The ratio of the jet total pressure to the freestream static pressure has been varied. It has been revealed that the flow picture is almost axisymmetric. Velocity and density of the primary and secondary flows are occurred to be the governing parameters. These characteristics have been governing for the mixing process as well. Experimental investigation of mixing and penetration characteristics of three transverse and oblique injector configurations has been carried out in [9]. It has been noted that the transverse jet penetration increases with jet-to-freestream momentum ratio growing. Large scale structures on the interface between the jet and the freestream have been observed for the transverse jets that believed to be main contributors to near field mixing of the injectant. The injector orientation has been found to have a dramatic effect on the flowfield. The subsequent investigations of these authors [10] have been directed toward the large structure convection velocity measurements in transverse injection flowfields. The injector geometry affects mainly the near field behavior, resulting in changing of convection angles. The weaker bow shock takes place for elliptical injection case. Characteristics of a sonic jet mixing in a Mach 1.6 crossflow are studied in [11]. Analysis shows that time-averaged characteristics of the process overestimate the actual level of instantaneous mixing. It has been demonstrated that there are regions where the probability level of the well-mixed fluid is very low in instantaneous frame while mean measurements demonstrate quite high mixing level. It is discovered that the best instantaneous mixing in the near field occurs in the center of the jet wake region. Study [12] has

been devoted to an evaluation of gas jets' penetration into a supersonic flow of various Mach numbers. A new penetration formulation has been suggested taking into account Mach number influence that correlates well with a wide range of experimental data. It has been discovered that with external Mach number growth, the penetration increases. It has been also noticed that the penetration depends primary on the pressure ratio and to a lesser degree on the boundary layer thickness and air Mach number. The effect of Reynolds number is found out to be negligible.

A serious row of theoretical investigations has been performed along with experimental works. Analytical, semiempirical and numerical approaches have been applied to the problem.

Analytical study of the gaseous jets injected into a supersonic flow on a base of three model approaches has been made in [13]. In the paper only the stage of jet penetration is discussed. The assumption is used that the jet acts as a solid body, emerging from the orifice and bent downstream by drag. The jet trajectory has been estimated based on force balance analysis. For underexpanded jets two typical flow pictures have been considered depending on boundary layer thickness in comparison with orifice diameter. Good agreement with a number of subsonic experiments on jet penetration value has been achieved. An extended model proposed in [14] has included simulation of injection at angles others then 90 degrees and turbulent mixing further downstream. Comparison with experimental data on decay of concentration of injectant fluid in the plume has shown good agreement.

Another analytical approach for simulation of 2D supersonic flowfield with a turbulent boundary layer distorted by a sonic jet has been made in [15]. The flowfield has been divided into several areas of very different parameters and each of the area has been described by their own equations and appropriate boundary conditions. After that the parts of the model have been combined into one common solution. The comparison between predicted and experimental results has shown good agreement.

Semiempirical model of equilibrium 2D flow resulting from normal injection of sonic hydrogen into supersonic air flow has been designed in [16]. The authors made an attempt to present a method for two-dimensional flow calculation in vicinity of the jet injection into the freestream. As an assumption bow and separation shocks have been combined in one composite shock. Control volume has been chosen and conservation laws for mass, fuel mass concentration, momentum and energy have been written in integral form and then integrated within the control volume with unit width in transverse direction. Missing information has been taken from experiment. Derived system of the equations has been solved by iteration method and density and temperature profiles at the computation cross-section have been constructed. Experimental and calculated total pressure profiles and wall pressure distributions have been compared and have shown satisfactory agreement.

To obtain additional insights into 3D unsteady supersonic jet, numerical simulations have been performed in a frame of LES and DES approaches [17, 18] that were shown to be able to resolve large-scale structures responsible for mixing processes.

The goals of the current paper is to get detailed picture of the flow which is difficult to obtain in experiments and to predict optimal configuration of the jet locations in order to attain the maximal mixing levels in future. Here the transverse sonic jet interaction is modeled in 2D approach by means of URANS code. Two flow configurations are considered: the first initial data set has been taken in [19] where experiments on air slot injection into freestream flow of Mach 6 at different pressure ratios have been conducted. The second case parameters have been chosen similar to experiments performed at ITAM [20, 21] where a helium injection into air flow of Mach 2.8 is realized in a channel with an abrupt expansion. As the first stage of the investigation in the computations the air jet has been considered. The main parameters of the both cases are presented in the Table.

Table

Case number	Freestream			Jet				
	Mach	Density, kg/m <sup>3</sup>	Jet/freestream total pressure ratio	Mach	Density, kg/m <sup>3</sup>	Static temperature, K	Static pressure, Pa	Size, mm
<b>1</b>	6	0.092	0.11	1	1.9	249.48	1523.13	1.37
<b>2</b>	2.8	0.676	1.177	2.93	27.29	219.46	1718122.9	2

## MATHEMATICAL MODEL AND NUMERICAL ALGORITHM

The calculations have been made in frame of full Favre-averaged Navier-Stokes equations closed by  $k-\omega$  Wilcox turbulence model [22]. The numerical algorithm has been designed and validated previously at ITAM SB RAS, Novosibirsk [23]. It includes third-order TVD-scheme based on van Leer flux vector splitting for the convective terms, central difference scheme for the viscous terms and two variants of the time approximation schemes: implicit and explicit with the first order of accuracy. The Case 1 has been calculated with MPI parallelized code based on 1D domain decomposition and ITAM parallel cluster T-Edge-32 (16 nodes, 2xXeon 5420 2.5 GHz 16 GB RAM) has been used for the computations.

### Boundary conditions

In Fig. 2, a the computational domain and boundary conditions for the Case 1 are presented. The inflow conditions have been taken from the paper [19] according to the Table data, at the top “simple wave” conditions are put. At the bottom non-slip boundary conditions for the velocity and adiabatic temperature conditions are implemented for the wall. Jet density, velocity and temperature are kept constant through the orifice area. At the outlet zero-gradient conditions for all parameters are applied.

In the Case 2 (Fig. 2, b) an abrupt expansion is appeared behind the jet location, and both the grid generation and domain geometry becomes slightly more complex. At the step surfaces and on the bottom part of the domain non-slip velocity conditions are used. Inlet and jet parameters have been taken from [20]. All the rest boundary conditions are the same as for the Case 1.

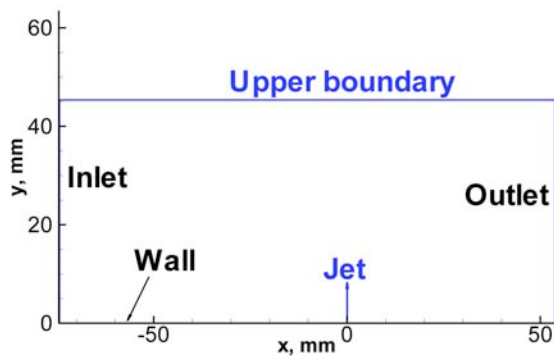


Fig. 2, a Boundary conditions, case 1

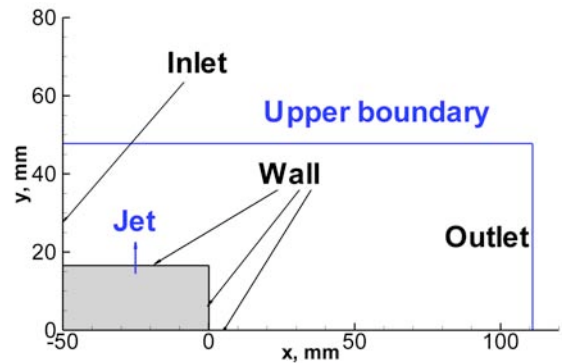


Fig. 2, b Boundary conditions, case 2

### Grid

All the constructed grids have been regular and rectangular, with toward-the-wall and toward-the-jet refinement both in  $x$  and  $y$  directions. For the Case 1, 14 grid cells are placed inside the jet hole, so that the step size is equal to 0.098 mm. Further from the jet, the cell size increases as shown in Fig. 3, a, where the part of the grid is presented. As the flow is evolved in time the grid nodes have been added to the computational domain and finally the whole domain in  $x$  direction has spread from -130 to 54 mm so that to include the separation bubble and to keep supersonic outlet conditions. The minimal size of the grid step in  $y$  direction is equal  $2 \cdot 10^{-4}$  mm. It assures the law of the wall variable  $y^+$  to be lower then 1.

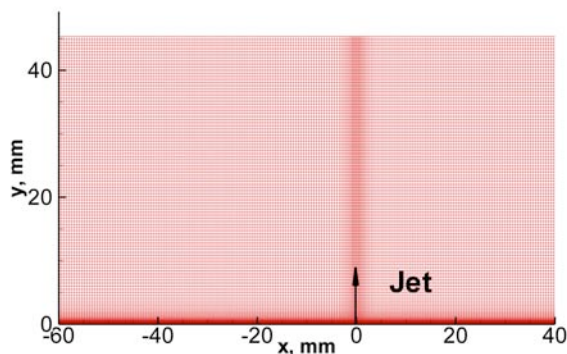


Fig. 3, a Grid for the Case 1

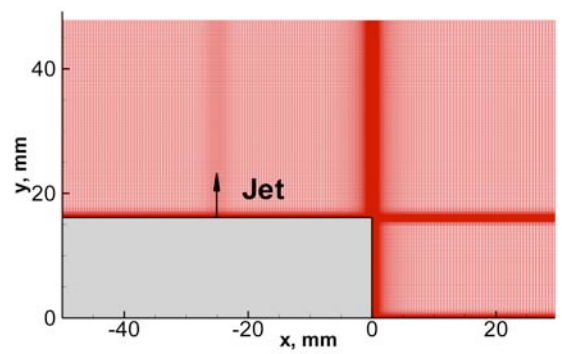


Fig. 3, b Grid for the Case 2

For the Case 2, 10 cells are put inside the jet and the x-step equal 0.2 mm. We should note that the length scale in this case is 2 mm while in case 1 it is 1 mm. The computational field has been of size 160 mm in x direction and 47.8 mm in y direction. The node number is 350x500 in y and x directions correspondingly.

## NUMERICAL RESULTS

### Case 1

First we consider the air jet spreading in the transverse to Mach 6 freestream flow direction. The Mach number distribution is shown in Fig. 4, a. Here the jet issuing from the slot forms a zone of supersonic flow (5) typical for underexpanded jets [1, 24]. The flow bounded by the jet contact surfaces accelerates and compression waves reflecting from this curved surface generate inner shocks. Further a strong shock (6) arises also called Mach disk and flow behind it becomes subsonic. As a result transverse structure appears that interferes with the primary supersonic flow. It leads to the turbulent boundary layer (1) separation; the recirculation area (3) gives rise to the jet induced shock (4) and the separation shock (2). Finally the recompression shock (7) appears behind the interaction zone. One can note that the obtained flowfield reproduces the experimental one depicted in Fig. 1 very close. As can be seen in Fig. 4, b where the pressure field is presented the maximal flow stagnation is attained in vicinity of the front jet side where the pressure has the local maximum. The maximal temperature is obtained near the leading point of the separation, where the separation shock meets the boundary layer (Fig. 4, c) while inside the jet low temperatures are realized.

The flow details can be seen in Fig. 5, a where an enlarged fragment of density gradient picture in vicinity of the interaction area is drawn. The contact boundaries of the jet (2) and (2') form the barrel-wise shape of the jet that leads to the internal shock waves (3) and (3') appearing and final strong shock (4) origin. In vicinity of the collision of the shocks (3) and (4) the high pressure zone emerges (see Fig. 4, b) due to the meeting of the two streams of different directions. In result of the interaction an essential flow deceleration takes place and a secondary jet confined by the main shock and the jet boundary is generated. It heads toward the wall and opposite to the external flow direction. Further downstream of the secondary jet the recompression shock (1) appears due to expansion and acceleration processes similar to those in the main jet.

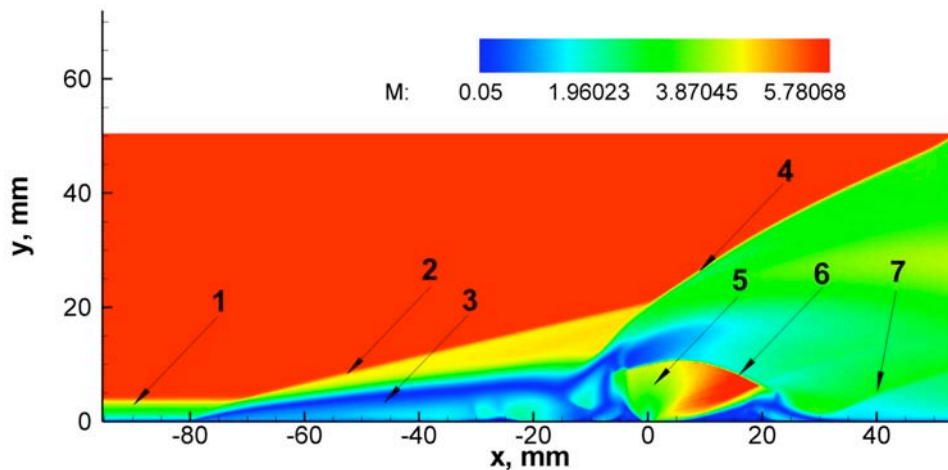


Fig. 4, a Mach number field for the Case 1 computations

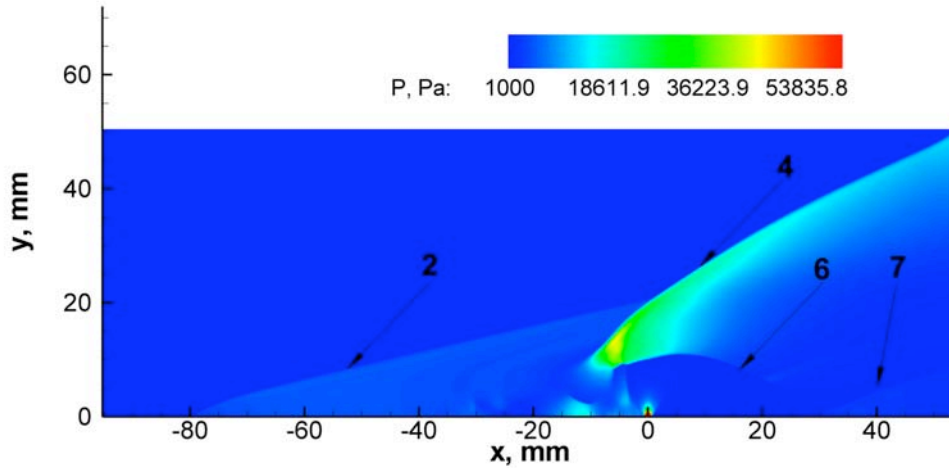


Fig. 4, b Pressure field for the Case 1 computations

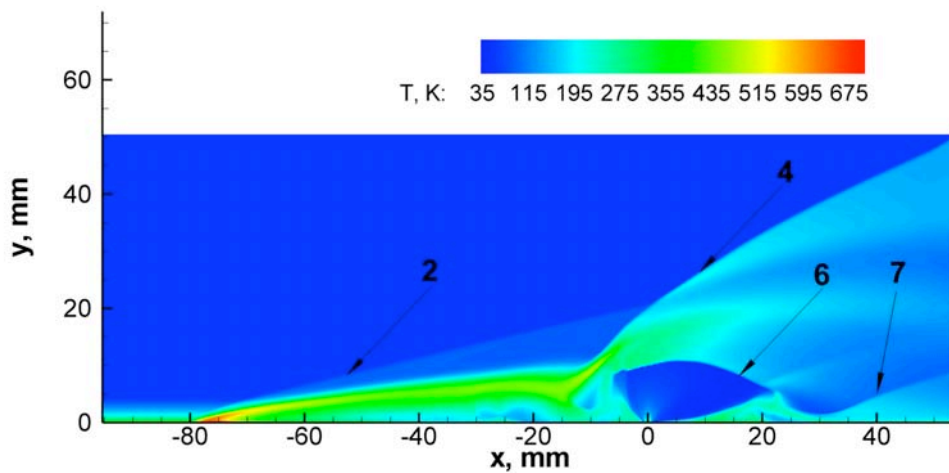


Fig. 4, c Temperature field for the Case 1 computations

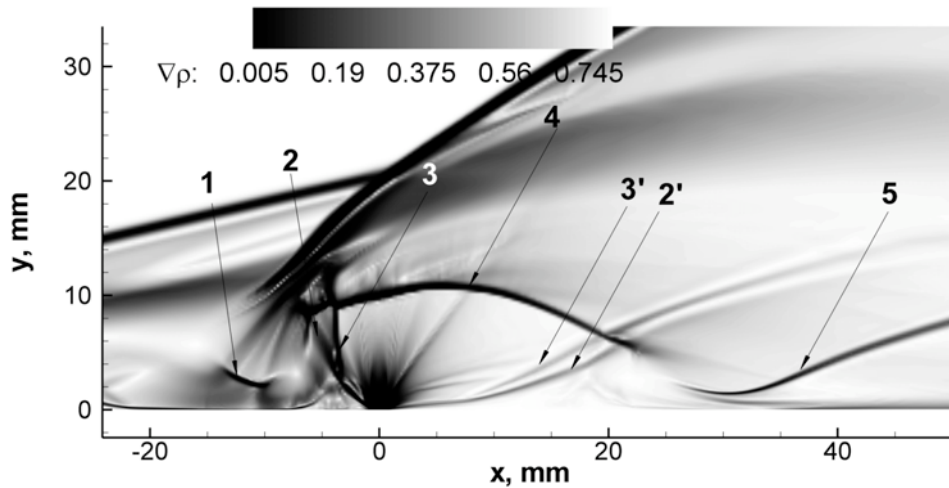


Fig. 5 Numerical Schlieren picture of the flow for Case 1, enlarged

The basic flow features are shown in Fig. 6 where the separation shock (6) can be seen along with the contact line (7) dividing the two opposite directed flows. As it has been mentioned above, behind the interaction area due to the downstream flow separation and reattachment the recompression shock wave (5) occurs. Here a qualitative comparison of the numerical Schlieren picture with the experimental one is performed. The experimental drawing is made in [16] under the initial conditions similar to those in the computations but with higher jet/freestream pressure ratio. As can be seen all the basic structures of the

flow are accurately reproduced by the computations. The shock behind the interaction is not seen in the experiment since there is no plate behind the jet nozzle. The jet penetration height has been estimated in the experiments and its value lies in the range from 14.5 to 18 mm. The computed magnitude is about 10.5 mm which agrees well with the experimental number. Some disagreement may be conditioned by the fact of the plate absence behind the injection in the experiment.

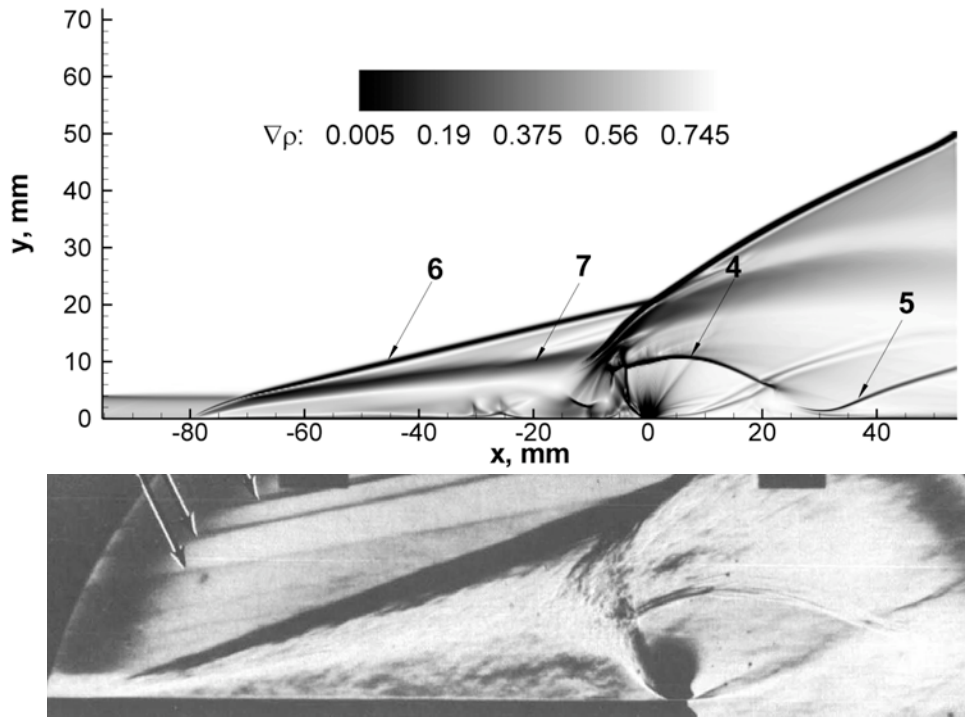


Fig. 6 Numerical Schlieren picture of the flow for Case 1 (top) and experimental Schlieren photograph [19] (bottom)

All the separation bubbles and their inner flow directions can be clearly seen in Fig. 7 where the streamtraces in the jet vicinity are represented in front of the vertical velocity field background. The big recirculation zone arisen due to the primary and secondary flow interaction and fed by the jet mass flow is spread approximately from  $x=-80$  to  $0$  mm (only zoom view of the field is shown in Figure). Inside the region  $x=-10$  to  $0$  mm several additional vortexes are observed with alternative counter and clockwise directions of rotation. Behind the jet, a low pressure zone arisen at  $x=10$  to  $30$  mm. The downstream recirculation zone with low pressure and the recompression shock (5) are observed that agree with basis scheme in Fig. 1.

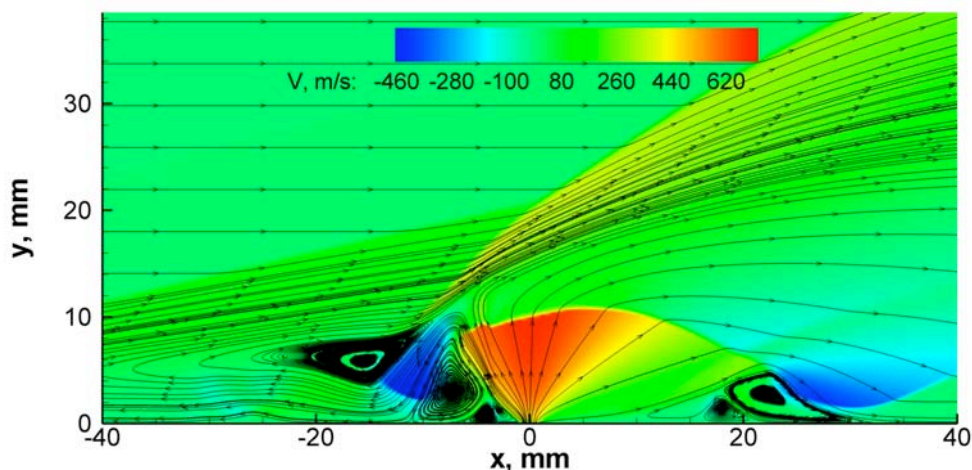


Fig. 7 Zoom view of vertical velocity field and streamtraces of the flow for the Case 1

## Case 2

The Case 2 problem corresponds to the experimental conditions realized in the hot-shot wind tunnel IT-302 ITAM SB RAS. The parameters of the external flow are as follows: total pressure  $P_0=30$  bar, total temperature  $T_0=1400$  K, Mach number  $M_\infty=2.8$ . The experimental model is a channel consisting of constant cross-section and diverging parts. In the constant area part, the backward-facing step of a height 16 mm is located. At the tests, pressure distributions along the channel wall and flow visualization have been obtained. Numerical study [20, 21] has been performed for a flow without mass supply. The comparison of the wall pressure distribution computed and measured experimentally is presented in Fig. 8. Good correspondence of the data can be noticed. Experimental Schlieren photograph and numerical density field drawn in Fig. 9 show identical location of the rarefaction fan (EF), the tail shock (TS) and the recirculation zone (RZ) size. The influence of temperature factor on a flow structure in the separation zone and temperature distribution has been investigated numerically at different Mach numbers. It is shown that the wall temperature essentially influences on quantity and sizes of vortexes in recirculation zone and also on temperature fields in the separation and reattachment zones.

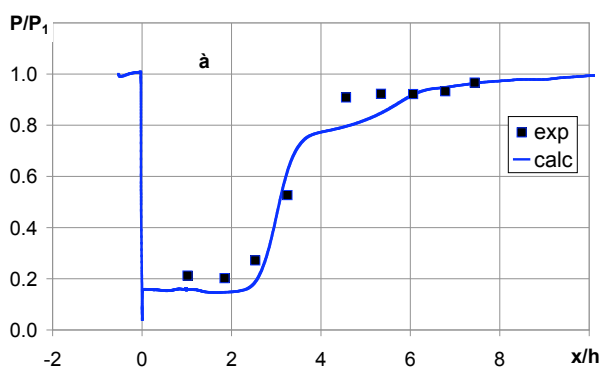


Fig. 8 Wall pressure distributions obtained experimentally and numerically

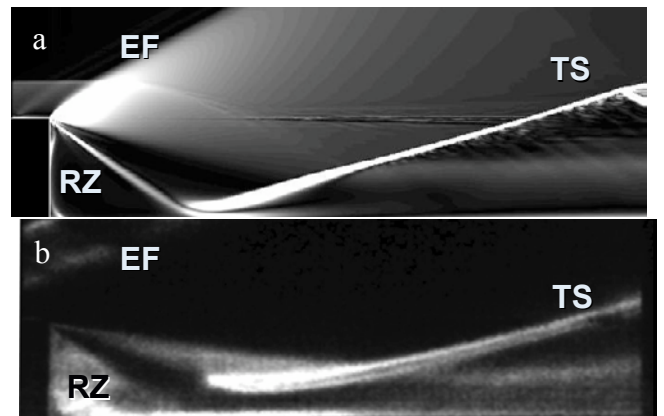


Fig. 9 Computed density field (a) and experimental Schlieren visualization (b) of the flow behind the back-step

The goal of the present paper is to simulate the injection taking into account a mass supply. In the experiments, the helium has been injected normally to the main stream. In the computations, we started with the air jet which parameters have been calculated so as to be equal to the experimental static pressure and temperature values at the jet exit. Then density has been computed from the equation of state for ideal gas with air specific heat ratio. The obtained jet velocity is 872.7 m/s that correspond to the jet Mach number of 2.934. The rest parameters are shown in the Table.

First the part of the field with the injection will be analyzed. The Mach field of the problem is presented in Fig. 10, a. The jet centre is located at  $x=-25$  mm. Similar to the previous case the bow shock (1) appears due to the transverse injection and subsonic area (2) behind the shock is clearly seen. Recirculation zone is generated near the wall in front of the jet and flow separation occurs. The subsonic flow (2) between the shock (1) and the jet surface is accelerated similar to a nozzle flow and becomes supersonic in region (3).

The jet structure differs from the Case 1 since the jet to freestream static pressure ratio is about 10 which is much lower than that in the Case 1. The contact surfaces (4) and (4') of the jet restrict supersonic area and a strong inner shock (5) is formed. The barrel structure is not accurately distinguished though the shock (6) can be seen both in Mach and temperature (Fig. 10, b) fields. Inside the jet structure the rarefaction waves (7) arising due to the flow expansion are observed. After they meet the reflected rarefaction waves (8) appear, though owing to the strong displacement in the direction of the main flow the left reflected wave is almost not visible. The temperature distributions in Fig. 10, b demonstrate the maximum in the area behind the separation shock (1) where the jet and the primary flow collide. Behind the jet in the near-wall area the low pressure level exists and weak separation takes place.

The whole jet remains supersonic with Mach numbers close to that in the external flow. Due to the freestream influence, the jet bent downstream of the orifice. Unfortunately no experimental Schlieren pictures are available that would allow a qualitative comparison to be performed.

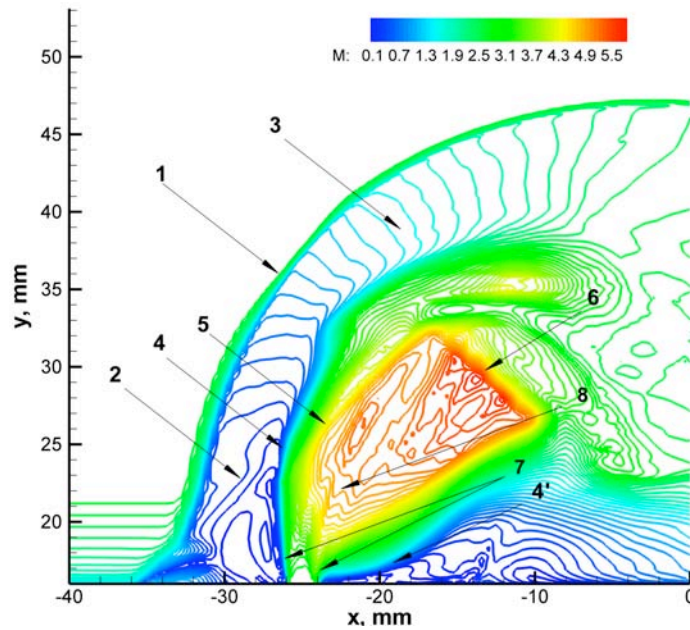


Fig. 10, a Mach number field for the Case 2

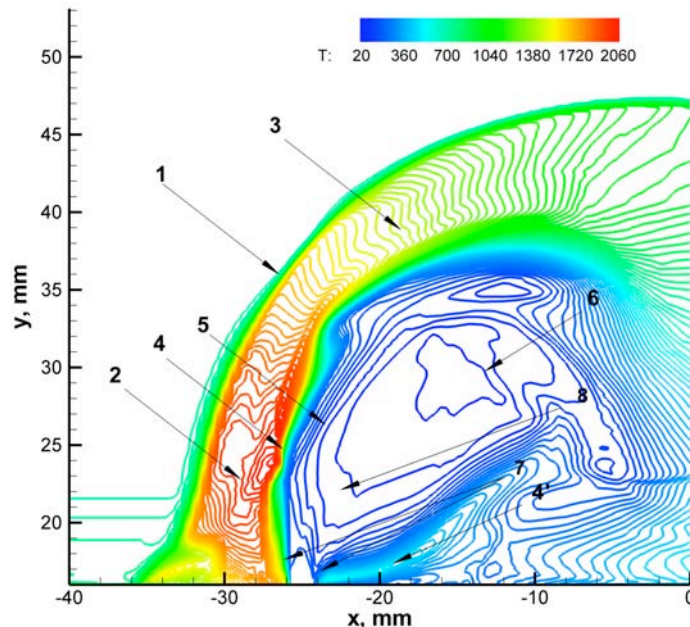


Fig. 10, b Temperature field for the Case 2

## CONCLUSIONS AND FUTURE WORK

Numerical simulation of sonic and supersonic injections into supersonic flow has been performed in frame of RANS approach. All the typical structures of the underexpanded jets have been reproduced by the computation algorithm as well as the main features of the ambient flow. Comparison with experimental Schlieren picture showed good qualitative agreement and some details of the flowfield were revealed. The future investigations will be directed toward the computations of the flow configurations of different geometry and jet locations and comparison with experiments will be performed.

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