ITERATIVE GUIDANCE FOR THE SOLID ROCKET MOTOR LAUNCHER

G. Martens – ELV s.p.a., Italy, O.Lukyanovych – RPI Hartron-Arkos ltd, Ukraine.

1.Abstract

Actual paper is dedicated to the features of the Iterative Guidance Mode (IGM) [1] [2] or so-called Closed Loop Guidance (CLG) implementation in the Solid Rocket Motor (SRM) launcher Guidance, Navigation and Control System (GNC). Development and justification of the computations and of the main parameters of the Guidance algorithm are reported.

Parametrical numerical method of the thrust integrals definition is applied in order to take into account the thrust variability. Proposed guidance method imply the allocation of the SRM nominal performances in terms of thrust and ejected propellant mass versus time and of the initial mass of the SRM stage in the computer memory. Original tables of the thrust integrals are defined once before start and are stored in the computer memory versus the parameter: the accumulated non-gravitational velocity.

Coefficient characterizing the scattering of the Mass Flow Rate (MFR) and of the burning time in a frame of accepted model is introduced in order to cope with in-flight scatterings and to predict remaining motor burning time and time-to-go. The coefficient provides to update the thrust integrals using closed formulas without cyclic re-computation by numerical integration.

Proposed approach simplifies computation algorithm in the cyclic part keeping up the acceptable accuracy.

The studies have been performed and the main results have been achieved in a frame of early phase of the Guidance trade off for the VEGA European small class launcher.

2.Main Statements of the Iterative Guidance

A lot of guidance methods are designed and approved now concerning both atmospheric and exo-atmospheric phases of Launch Vehicle (LV) flight. General problems of the exo-atmospheric guidance were investigated in 60th. In the same time main statements of guidance methods were formulated.

The Closed Loop Guidance is considered as a most acceptable for the launchers with non-controlled thrust that are optimal on payload mass due to following advantages:

- small methodical error,
- computing in flight of the Guidance command versus real cinematic parameters and desired terminal conditions,
- implementation of the trajectory near to the optimal one by energy,
- feasibility of implementation by on-board computer.

Significant experience of the modern LV accumulated up to now confirms this statement, even more no real alternatives on it's use are available.

Main Statements of the Iterative Guidance also were designed and published in 60th [1][2]. There is no necessity to repeat the details, we will note just some points required for the presentation below.

General idea of the iterative guidance consists in simplification of the terminal problem solution basing on following assumptions:

- Thrust vector orientation or LV attitude: pitch and yaw angles are defined as linear tangents law:

(1)
$$\mathrm{tg}\vartheta = a_\vartheta + b_\vartheta \cdot t,$$
$$\mathrm{tg}\psi = a_\psi + b_\psi \cdot t.$$

- Small variation of the disturbances during computation cycle.

These assumptions provide local linearization of the solution and use of the iteration procedure, i.e. the solution obtained at the previous cycle assumed as a reference to precise current computations.

As from [1], in order to resolve terminal problem, i.e. in order to find a_ϑ, b_ϑ, a_ψ, b_ψ coefficients and T_t time-to-go, residual thrust integrals representing all stages performances shall be defined. We recalled the integrals of $\dot{w}(t)$ non-gravitational acceleration and of w(t) non-gravitational velocity of the 3rd stage as an example:

(2) $L(t) = \int_t^{T_3} \dot{w}(\tau)d\tau$;

$S(t) = \int_t^{T_3} w(\tau)d\tau$;

$A(t) = \int_t^{T_3}\int_t^{\tau} w(\tau)d\tau^2$;

$B(t) = \int_t^{T_3}\int_t^{\tau}\int_t^{\tau} w(\tau)d\tau^3$

Hear T_3 is a burning time of the 3rd stage.

Integrals of the product of non-gravitational acceleration by time that are necessary for the guidance terminal problem solution can be expressed by closed formulas:

(3) $I(t) = L(t)\cdot T_3 - S(t)$,

$P(t) = L(t)\cdot T_3^2 - 2\cdot S(t)\cdot T_3 + 2\cdot A(t)$,

$Q(t) = S(t)\cdot T_3 - 2\cdot A(t)$,

$U(t) = S(t)\cdot T_3^2 - 4\cdot A(t)\cdot T_3 + 6\cdot B(t)$.

Computation of these integrals is a base point of iterative schema. Accuracy of the computation impacts on the both prediction of the time-to-go and methodical error of guidance.

It should be noted that most publications of the IGM application assume constant thrust and predefined operating time of all the stages excepting the last one. Rigorously speaking this assumption isn't obligated. There are some cases when it isn't acceptable. The SRM launcher guidance is exactly the case.

The computation of the thrust integrals of SRM stages can be performed just by numerical integration methods. Additional problems linked to the estimation of the scatterings and to their impact on the integrals computational accuracy arise also.

3.Thrust Model of the Solid Rocket Motor

Let us discuss the SRM features from the point of view of IGM. First of all it should be noted that the thrust isn't constant. The nominal thrust is varied in a time and the scatterings are significant. In the same time the variation of the total thrust impulse is very small due to the fixed mass of propellant and to the stable enough specific impulse. Taking above the model of the SRM thrust and of the mass flow rate shall satisfy the series of conditions.

Let us assume a SRM ignition as a reference time $t_{Z9\ ign} = 0$.

The nominal mass-flow-rate $\dot{m}^*$ and the nominal thrust P* shall correspond to the predefined functions usually defined as a lookup table:

(4) $\dot{m}^* = \dot{m}^*(t)$,

$P^* = P^*(t)$

If there is no propellant mass scattering the integral of the mass flow rate for the burning time T_3 shall be constant:

(5) $\int_0^{T_3} \dot{m}(t)dt = \text{const}$

or

(6) $m(T3) = m^*(T3^*)$

what means that the propellant mass burned during the firing time doesn't depend of the burning rate.

Varying the mass-flow-rate and assuming this variation $k_{\dot{m}}$ as constant we derive from (6) that for the disturbed case the burning time is in inverse proportion to the mass flow rate. By another word if we increase the burning rate the burning time is decreased and vice versa. Two plots hereafter present curves of thrust and of mass-flow-rate both nominal and scattered due to ±3σ variation of burning time (or mass flow rate).

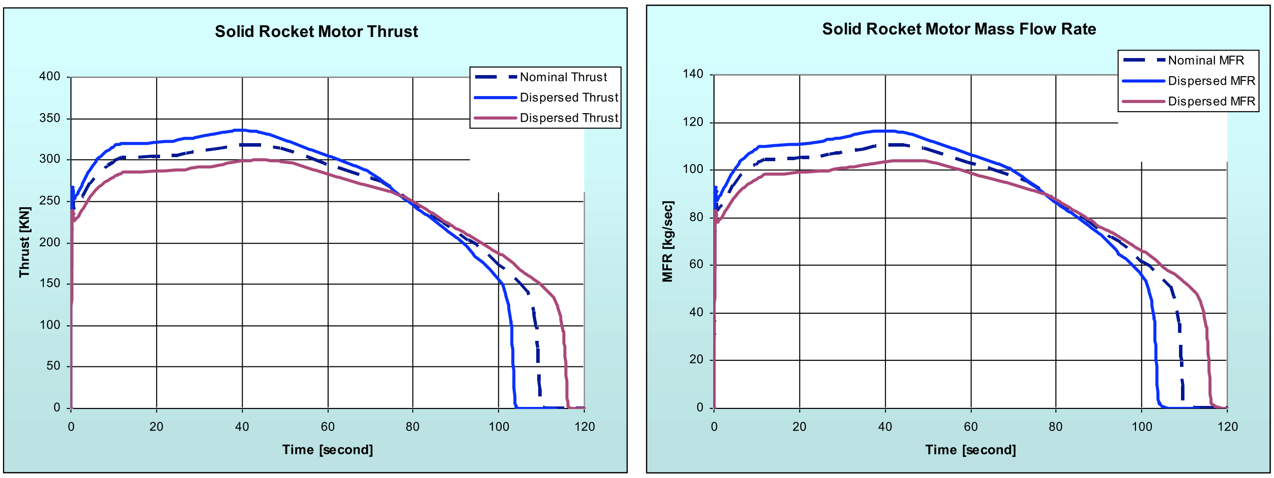

Fig. 31

Following formulas reports above statements:

(7) $$\dot{m}(t) = (1+k_{\dot{m}})\cdot\dot{m}^*[(1+k_{\dot{m}})\cdot t]$$

$$T_3 = \frac{T_3^*}{1+k_{\dot{m}}} = k_{T3}\, T_3^*$$

Here k_{T3} is a burning time scattering coefficient:

(8) $$k_{T3} = \frac{1}{1+k_{\dot{m}}}$$

Nominal figures in the formulas are pointed with asterisk.

Adding two independent scatterings of: δm_{3F} initial mass of propellant and of δIsp specific impulse we obtain the expressions for the disturbed thrust and mass flow rate:

(9) $$P(t) = (1+k_{\dot{m}})\cdot(1+\delta m_{3F})\cdot(1+\delta Isp)\cdot P^*[(1+k_{\dot{m}})\cdot t]$$

$$\dot{m}(t) = (1+k_{\dot{m}})\cdot(1+\delta m_{3F})\cdot\dot{m}^*[(1+k_{\dot{m}})\cdot t]$$

These formulas provide good approximation of the SRM performances and give comfortable tool to be implemented in the LV simulator for the Guidance study.

4.Problem description and general assumption

Guidance at the exo-atmospheric phase is based on the solution of boundary problem: taking the initial phase state known by measurement and the final phase state required by mission the time-to-go and commanded attitude in terms of pitch and yaw angles to be defined. The main difficulty is an account/prediction of the residual motor thrust. Usually it is assumed that the motor performances: thrust, mass flow rate are fixed and all derived parameters as supplied velocity and others thrust integrals (2)(3) can be predicted with required accuracy by closed formulas.

For the SRM launchers such assumption isn't valid as far as the thrust is varied in a time and as pointed in a previous paragraph has significant scatterings on:

- $k_{\dot m}$ MFR or burning time
- δIsp Specific impulse
- δm_{3F} Initial propellant mass.

These scatterings has different impact on the accuracy of the integrals computation and on general guidance accuracy and optimality. Let us deeply discuss this impact.

Initial propellant mass scattering usually is small as far as every stage is weighting at production and unknown scattering is defined just by weighting accuracy.

Impact of two others scatterings is more significant. Both of them have similar impact on the instantaneous thrust and it isn't possible to segregate them by measurement of motion parameters in the flight.

In the same time a relative scattering of the mass-flow-rate and burning time scattering a priory exceed a relative scattering of the specific impulse in for 4–5 time. Therefore it is reasonable to take all the scatterings impacting on the thrust and mass as a scattering of burning time and to estimate exactly this parameter in flight.

So the problem of investigation can be formulated as follow: **to asses in flight the SRM burning time for precise prediction of the iterative guidance parameters.**

In this article two alternatives are proposed on the SRM burning time scattering assessment basing on the measurement of :

- Non-gravitational acceleration
- Accumulated non-gravitational velocity

5.Solution of the problem

More evident at first sight is solution basing on the measurement of acceleration in flight as far as it is proportional to the thrust and mass-flow-rate correspondently.

Let us analyze this approach in details. Nominal value of the acceleration can be expressed

$$\dot w^*_{x1}(t) = \frac{P^*(t)}{m^*(t)} \tag{10}$$

Scattered acceleration for the accepted model (9) is defined as:

$$\dot w_{x1}(t) = \frac{(1+\hat k_{\dot m})\times P^*[(1+\hat k_{\dot m})\times t]}{m^*[(1+\hat k_{\dot m})\times t]} \tag{11}$$

Solving equation (11) we can find assessment of parameter $\hat k_{\dot m}$. The development of P* and m* on $\hat k_{\dot m}$ up to linear members is expressed as:

$$P^*[(1+\hat k_{\dot m})\cdot t] \approx P^*(t) + \hat k_{\dot m}\times t\times \dot P^*(t) \tag{12}$$

$$m^*[(1+\hat k_{\dot m})\cdot t] \approx m^*(t) - \hat k_{\dot m}\times t\times \dot m^*(t) \tag{13}$$

Minus sign in a statement for m* is explained by fact that the $\dot m^*$ value is assumed as positive.

Neglecting by members with $\hat k^2_{\dot m}$ we reduce equation (11) to the format:

$$[P^*(t) + \dot P^*(t)\times t + \dot w_{x1}(t)\cdot \dot m^*(t)\cdot t]\cdot\hat k_{\dot m} = \dot w_{x1}(t)\cdot m^*(t) - P^*(t) \tag{14}$$

Dividing both parts on P*(t) and taking into account (10) we obtain:

(15) $$\left(1 + t \times \left(\frac{\dot{P}^*(t)}{P^*(t)} + \frac{\dot{w}_x(t)}{\dot{w}_x^*(t)} \times \frac{\dot{m}^*(t)}{m^*(t)}\right)\right) \times \hat{k}_{\dot{m}} = \frac{\dot{w}_x(t)}{\dot{w}_x^*(t)} - 1$$

Denoting intermediate variables as:

(16) $$\delta w(t) = \frac{\dot{w}_x(t)}{\dot{w}_x^*(t)} - 1$$

(17) $$\delta P(t) = t \times \left(\frac{\dot{P}^*(t)}{P^*(t)} + [1 + \delta w(t)] \times \frac{\dot{m}^*(t)}{m^*(t)}\right)$$

Final assessment of the $\hat{k}_{\dot{m}}$ mass-flow-rate-scattering will be:

(18) $$\hat{k}_{\dot{m}} = \frac{\delta w(t)}{1 + \delta P(t)}$$

The residual burning time of the third stage can be expressed:

(19) $$T_3(t) = \frac{T_3^*}{1 + \hat{k}_{\dot{m}}(t)}$$

$T_3(t)$ figure is an upper limit for computation of the thrust integrals (2)(3) used in IGM.

Significant disadvantage of the pointed approach is a limitation of use of the instantaneous acceleration measurements. Some averaging or filtering shall be applied in order to increase the reliability and noise protection. Another disadvantage of this approach is that it isn't valid in the final SRM burning phase (more than 70% of a total burning time) because in this phase there is no correlation between acceleration and MFR (see Fig. 31 above).

The second approach used the measurement of accumulated non-gravitational velocity is based on Tsiolkovsky formula.

(20) $$\Delta V = Isp \times \ln\left(\frac{M_0}{M(T)}\right)$$

where

Isp – specific impulse of motor

M_0 – initial mass of rocket

M(T) – final mass of rocket

ΔV – delta velocity supplied by motor

Assuming also that specific impulse is fixed and taking into account (5) we obtain from (20) that accumulated velocity doesn't depend of burning rate:

(21) $$w(T_3) = \int_0^{T_3} \dot{w}(t)dt = \text{const.}$$

Let us accept w accumulated velocity as independent variable. In this case time will be linked to w as

(22) $$t(w) = \int_0^w \frac{dw}{\dot{w}} = \int_0^w \frac{m(w)}{(1 + k_{\dot{m}})P(w)} dw = k_{T3} \int_0^w \frac{m(w)}{P(w)} dw$$

Denoting with t*(w) the nominal predefined dependence

(23) $$t^*(w) = \int_0^w \frac{m(w)}{P(w)} dw$$

we obtain

(24) $$t = k_{T3}\, t^*(w)$$

and

(25) $$k_{T3} = t / t^*(w)$$

At the beginning of burning phase (Δt_{min} = some seconds after ignition) where t*(w) is very small a numerical protection shall be applied in order to avoid division by zero:

(26) $$k_{T3} = \begin{cases} \dfrac{t}{t^*(w)}, & \text{if } t^*(w) > \Delta t_{min}, \\ 1 + \dfrac{t - t^*(w)}{\Delta t_{min}}, & \text{if } t^*(w) \le \Delta t_{min}. \end{cases}$$

And finally assuming k_{T3} as constant estimation of burning time will be expressed:

(27) $$T_3 = k_{T3}\, T_3^*$$

The second approach with respect the first one has no singularity at the end of burning phase even more the estimation accuracy is increased by time. It does not require any averaging as far as accumulated velocity is integral parameter taking into account all the story from the ignition. Therefore the second approach was selected as a baseline and computation procedure implementing this approach and minimizing cyclic integration operation was designed.

6.Computation Algorithm

In order to avoid cyclic repetition of numerical integration let us introduce following function of nominal SRM thrust/acceleration.

(28) $$w^*(t) = \int_0^t \dot{w}^*(t)\,dt,$$

$$S^*(t) = \int_0^t w^*(t)\,dt,$$

$$A^*(t) = \int_0^t S^*(t)\,dt,$$

$$B^*(t) = \int_0^t A^*(t)\,dt,$$

These functions are computed once at mission preparation or during ground operation and stored in a computer memory as a lookup table.

The required integrals (2) can be expressed taking functions above by closed formulas. For the nominal case they are:

(29) $$L(t) = \int_t^{T_3^*} \dot{w}(t)dt = w^*(T_3) - w^*(t);$$

$$S(t) = \int_t^{T_3^*} [w^*(\tau) - w^*(t)]d\tau = \int_t^{T_3^*} w^*(\tau)d\tau - w^*(t)\int_t^{T_3^*} d\tau = S^*(T_3^*) - S^*(t) - w^*(t)\cdot(T_3^* - t);$$

$$A(t) = \int_t^{T_3^*}\int_t^{\tau} [w^*(\tau) - w^*(t)]d\tau^2 = \int_t^{T_3^*} [S^*(\tau) - S^*(t) - w^*(t)(\tau - t)]d\tau =$$

$$= A^*(T_3^*) - A^*(t) - S^*(t)\cdot(T_3^* - t) - w^*(t)\frac{(T_3^* - t)^2}{2};$$

$$B(t) = \int_t^{T_3^*}\int_t^{\tau}\int_t^{\tau} [w^*(\tau) - w^*(t)]d\tau^3 = \int_t^{T_3^*} \left[A^*(\tau) - A^*(t) - S^*(t)(\tau - t) - w^*(t)\frac{(\tau - t)^2}{2}\right]d\tau =$$

$$= B^*(T_3^*) - B^*(t) - A^*(t)(T_3^* - t) - S^*(t)\frac{(T_3^* - t)^2}{2} - w^*(t)\frac{(T_3^* - t)^3}{6}.$$

It could be noted that for the disturbed case with burning duration estimated by formula (27) just first member of equations will be modified. Changing independent variable from time to accumulated velocity as in (22) we can write closed formulas for computation of the thrust integrals

(30) $$L(t) = w^*(T_3^*) - w(t),$$

$$S(t) = k_{T3} S^*(T_3^*) - S^*(w) - w(t)\,\Delta T_3,$$

$$A(t) = k^2_{T3} A^*(T_3^*) - A^*(w) - S^*(w)\,\Delta T_3 - 0.5\, w(t)\,(\Delta T_3)^2,$$

$$B(t) = k^3_{T3} B^*(T_3^*) - B^*(w) - A^*(w)\,\Delta T_3 - 0.5\, S^*(w)\,(\Delta T_3)^2 - w(t)\,(\Delta T_3)^3/6,$$

Where

$\Delta T_3 = T_3 - t$.

Adding equations (3) we obtain full set of closed formulas to define all the integrals required by Iterative Guidance with a solid rocket motor.

So CLG computation procedure taking into account SRM features can be expressed in following steps:

a) Definition of the w non-gravitational velocity accumulated from the motor ignition coming from Inertial Navigation.
b) Computation by linear interpolation versus w of the current value of the functions (23) and (28) pre-stored as a lookup table.
c) Computation of the k_{T3} coefficient by formulas (25)(26).
d) Computation of the thrust integrals by formulas (30)(3).
e) And finally definition of the attitude gains of tangent law and last stage time-to-go by the standard computation procedure [1][2].

7.Simulation Results

Test of the above theoretical statements performed with VEGA LV 3Degree of Freedom simulator including on-board Navigation and Guidance algorithms. Polar circular 700 km, inclination 90 degree Reference mission has been investigated. It shall be noted that the studies were performed at the early phase of VEGA program therefore some parameters can be differ with respect of modern one.

Fig. 71 present the simulation results of one of the scattered case in terms of evolution of the k_{T3} estimation coefficient of the Zefiro 9 Burning time scattering versus flight time.

One can see the impact of the numerical protection at the initial phase of SRM burning, i.e. estimation starts from nominal value 1 and than in some second arrives to the real figure with good accuracy.

Following Fig. 72 presents analysis of the CLG robustness against scattering of Z9 SRM burning duration. Propellant consumption of the Attitude Vernier Upper Module (AVUM) the last stage of VEGA launcher was selected as criterion of the CLG performances. Two curves are plotted: the first blue curve presents classical CLG taking nominal predefined on-ground Z9 burning time while the second magenta curve presents the behavior of CLG algorithm taking into account Z9 burning time corrected due to on-board estimation.

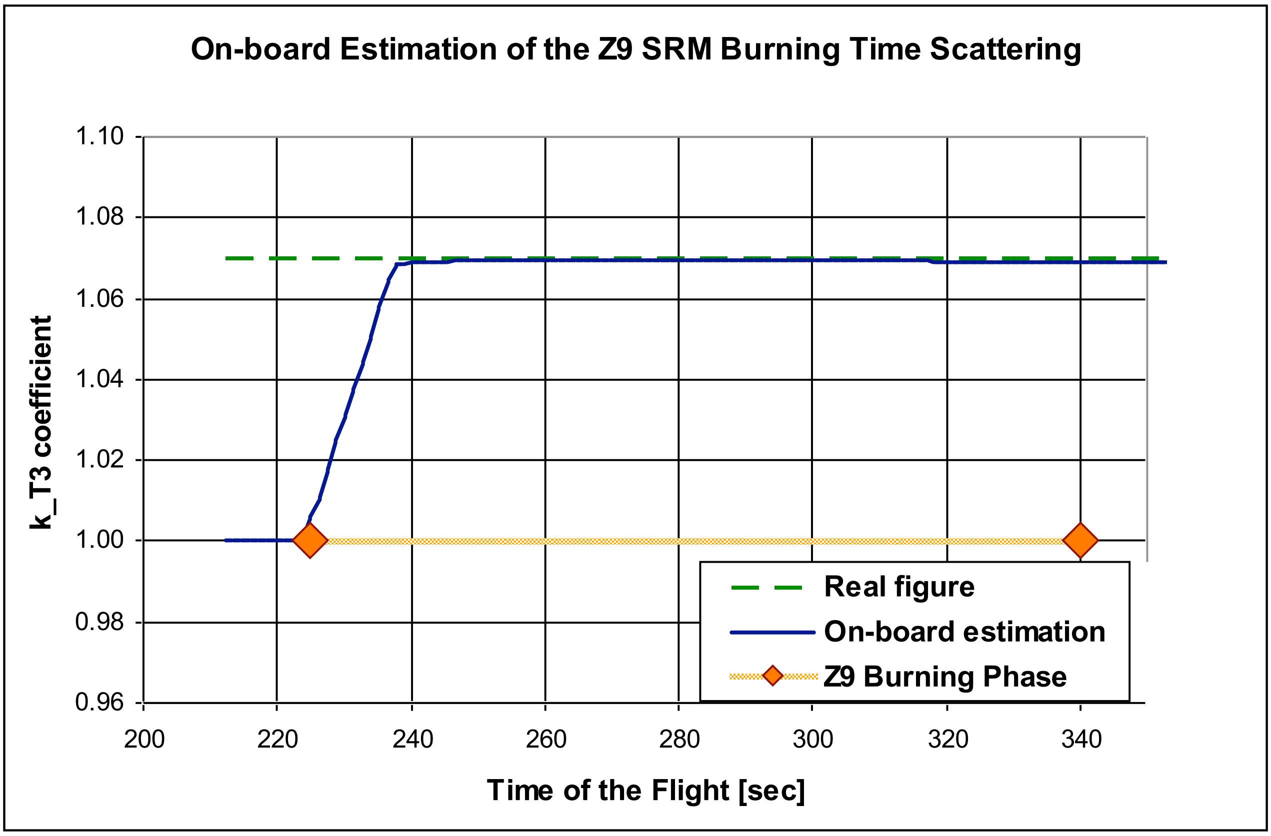

Fig. 71

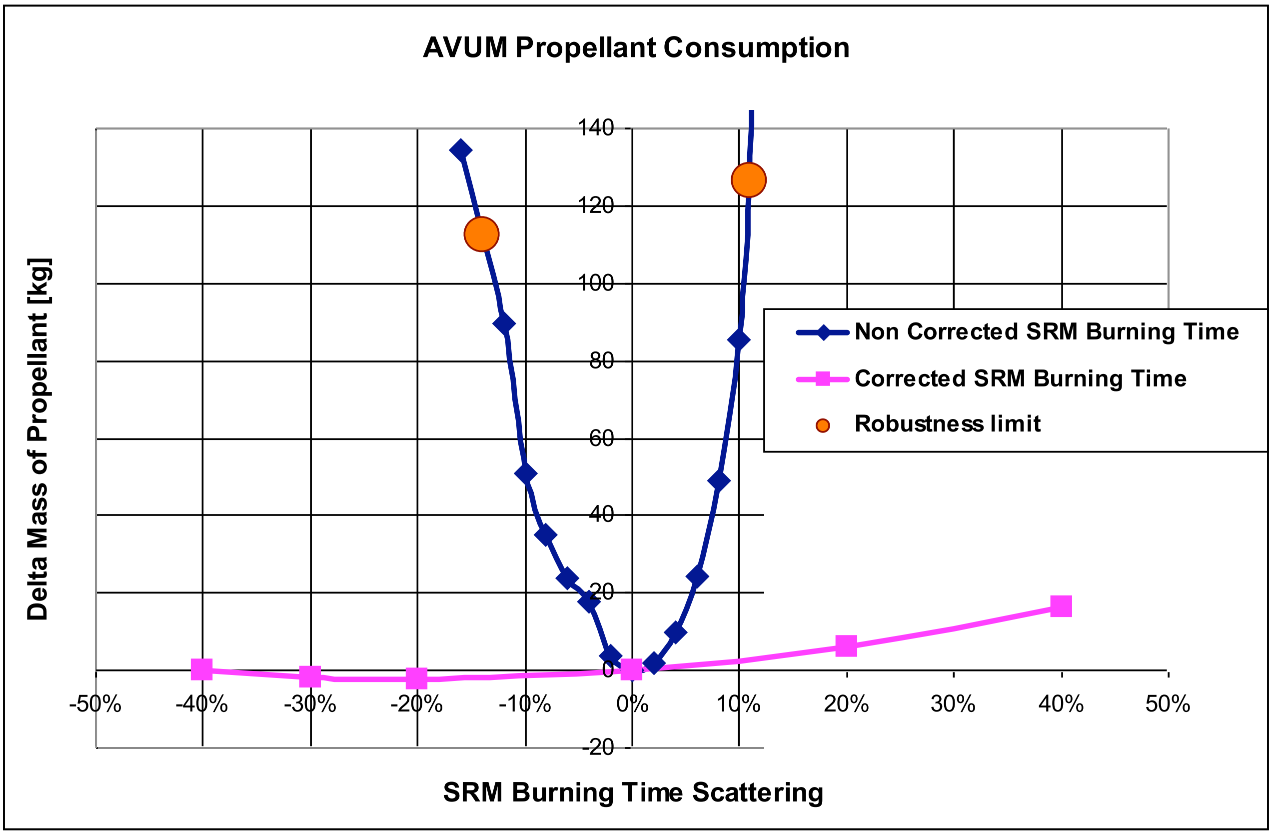

Fig. 72

It is evident that due to on-board estimation of the SRM burning time the CLG become non sensible to this type of scattering.

8.Conclusions

- Proposed model of the Solid Rocket Motor scatterings provide comfortable tool for the Guidance study with good approximation of the SRM performances.
- Two approach of the on-board estimation of the SRM burning time scattering based on the measurement of the non-gravitational acceleration and velocity are presented and discussed. The approach based on the velocity measurement is selected as baseline due to higher stability and accuracy in the final phase that is more important for the CLG performances.
- Computation procedure of the SRM burning time estimation minimizing cyclic computation without significant complication of the CLG calculation schema is reported and implemented in the on-board algorithms.
- The modified Iterative Guidance implementing the SRM burning time estimation procedure becomes non sensible to the SRM burning time scattering that is confirmed by simulation results.
- Presented approach is implemented in the on-board algorithms of the VEGA Closed Loop Guidance.

9.References

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