STRUCTURAL TOPOLOGY OPTIMIZATION WITH STRESS CONSTRAINTS: GENERAL METHODOLOGY AND APPLICATIONS IN AEROSPACE SCIENCE

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ABSTRACT

Structural optimization is a well known discipline in aerospace applications since most of the optimization problems were stated in order to solve aeronautic structures. Since first works about structural optimization were published, different formulations had been proposed to obtain the most adequate structure. Topology optimization of structures is the most recent branch of structural optimization. In this discipline, the aim is to distribute a predefined amount of material in a predefined domain. The most usual formulation tries to maximize the stiffness of the structure by using a predefined amount of material. In this paper, we propose a minimum weight with stress constraints formulation that avoids most of the drawbacks associated to maximum stiffness approaches as it can be observed in the applications examples that we have analyzed.

1. INTRODUCTION

Topology optimization of structures is a relatively recent discipline in the field of structural optimization. Since the first model was introduced a lot of effort has been dedicated to deal with this problem. However, the most of the works about this topic has been driven to maximum stiffness formulations due to computational reasons, among other considerations. More recently, different approaches with stress constraints have been proposed due to the important advantages that they offer (avoids checkerboard solutions, guarantees the feasibility of the solution, ...). However, the computational resources required are more restrictive because the underlying optimization problem is much more complicated.

In this paper we present and compare three different approaches for topology optimization of structures that incorporate stress constraints. Finally, two application examples are presented.

2. TOPOLOGY OPTIMIZATION PROBLEM

The minimum weight with stress constraints topology optimization problem can be written, according to [1], in the following terms:

$$
\begin{array}{lll}
Find & \boldsymbol{\rho} = \{\rho_e\}, & e = 1, \ldots, N_e \\
that\ minimizes & F(\boldsymbol{\rho}) & \\
verifying & g_j(\boldsymbol{\rho}) \leq 0, & j = 1, \ldots, m \\
 & 0 < \rho_{min} \leq \rho_e \leq 1, & e = 1, \ldots, N_e
\end{array} \tag{1}
$$

where the design variable ρ_e is the relative density of element e (assumed uniform within the element) and N_e is the total number of elements in the mesh (the lower limit of the relative density, ρ_{min}, is slightly higher than zero to avoid the stiffness matrix to become singular). The model of microstructure used is the SIMP model without any penalization of the intermediate densities. The penalization of the intermediate densities is included in the objective function as

$$
F(\boldsymbol{\rho}) = \sum_{e=1}^{N_e} (\rho_e)^{\frac{1}{p}} \int_{\Omega_e} \gamma_{mat}\ d\Omega, \tag{2}
$$

where Ω_e is the element number e, γ_{mat} is the density of the material, and $p \geq 1$ is the penalization parameter of the intermediate densities used to favor a mainly compact distribution of material [1].

3. STRESS CONSTRAINTS

In order to consider stress constraints we analyze three different formulations. First, we propose to impose one stress constraint in the central point of each element, which is usually known as the local approach of the stress constraints [1, 2, 3, 4, 5]. This local stress constraint can be introduced as

$$
g_e(\boldsymbol{\rho}) = \left[\widehat{\sigma}\left(\boldsymbol{\sigma}_e^h(\boldsymbol{\rho})\right) - \widehat{\sigma}_{max}\ \varphi_e \right] (\rho_e)^q \leq 0 \tag{3}
$$

being

$$
\varphi_e = 1 - \varepsilon + \frac{\varepsilon}{\rho_e}, \tag{4}
$$

where g_e is the stress constraint of the element e and $\widehat{\sigma}$ is the reference stress used (usually the Von Mises criterion) obtained through the calculated stress tensor $\boldsymbol{\sigma}_e^h$ in the central point of the element. In order to avoid singularity phenomena when the relative density tends to zero this constraint has been relaxed by using the function φ_e [2, 6]. The “relaxation parameter” ε usually takes values between 0.001 and 0.1. In addition, the exponent q allows to deal with real stress (when $q = 0$) or effective stress (when $q = 1$). According to [1, 3] the use of effective stress reports important advantages because it reduces the non-linearity of the stress constraints when the relative density tends to zero.

The local approach of stress constraints usually requires to impose a high number of constraints due to the number of elements (and design variables). Consequently, this approach requires, nowadays, a high computing effort when thousands of design variables are used.

Due to this fact, several alternative formulations have been derived in order to reduce the computing effort required: thus we propose to use a global function that aggregates the effect of all the local constraints from a global point of view [3]. This global function was first proposed by Kreisselmeier-Steinhauser (and later used in [7], for example) and it is defined as

$$G_{KS}(\boldsymbol{\rho}) = \left[\frac{1}{\mu}\ln\left(\sum_{e=1}^{N_e} e^{\,\mu(\widehat{\sigma}_e^* - 1)}\right) - \frac{1}{\mu}\ln(N_e)\right] \leq 0 \tag{5}$$

being

$$\widehat{\sigma}_e^* = \frac{\widehat{\sigma}\big(\boldsymbol{\sigma}_e^h(\boldsymbol{\rho})\big)}{\widehat{\sigma}_{max}\ \varphi_e}, \tag{6}$$

where μ is the aggregation parameter and it usually takes values between 15 to 40 [3, 5]. Values smaller than 15 allow an excessive violation of the local constraints and values higher than 40 produce a highly non-linear function.

The use of this global function reduces enormously the computing effort required but it also leads to a loss of information in the sensitivity analysis due to the aggregation. In addition, the non-linearity of the stress constraints is increased.

Due to this fact, we have also proposed a different strategy that forms groups of elements that we call blocks (figure 1). Each block contains approximately an equal number of elements.

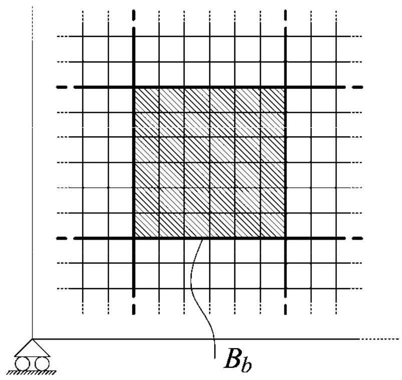

Figure 1. Example of block definition

Thus, the main idea is to impose over the elements of each block one global stress constraint like the proposed in the global approach. The global function to impose over each block of elements is defined as

$$G^b_{KS}(\boldsymbol{\rho}) = \left[\frac{1}{\mu} \ln \left(\sum_{e \in B_b} e^{\,\mu(\widehat{\sigma}^*_e - 1)} \right) - \frac{1}{\mu} \ln(N^b_e) \right] \leq 0 \tag{7}$$

being

$$\widehat{\sigma}^*_e = \frac{\widehat{\sigma}\big(\boldsymbol{\sigma}^h_e(\boldsymbol{\rho})\big)}{\widehat{\sigma}_{max}\ \varphi_e}, \tag{8}$$

where N^b_e is the number of elements aggregated in block b and B_b is the set of elements in block b.

This approach allows to define the number of blocks to use and consequently the number of stress constraints to impose. Thus, this formulation is more general than the local or the global ones and includes them as a particular case [5, 8].

The number of blocks used and the way of defining the geometry of the blocks are the most important features of this formulation. However, we have observed that the geometry of the blocks does not influence considerably the final solution. The number of blocks and the parameter of aggregation are much more critical.

In the application examples presented in this paper, the definition of the blocks of elements is developed according to the numbering used in the FEM formulation. This algorithm usually produces deformed long blocks for the most usual FEM meshes used in topology optimization problems. As it was mentioned before this fact does not influence significantly the final solution obtained. However, a more compact definition of blocks (like the proposed in figure 1) could lead to a more efficient problem. The global constraints and the sensitivity analysis would produce more appropriate information to the optimization algorithm. Thus, further research is necessary to develop more specific techniques to define the blocks in order to obtain a better performance.

4. OPTIMIZATION ALGORITHM

According to the approaches introduced in the previous section, the topology optimization of structures with stress constraints leads to mathematical programming problems type (1) with a large number of highly non-linear constraints type (3), (5) or (7) and a non linear objective function.

An improved SLP algorithm with quadratic line-search seems to be a right choice to solve this kind of problems [1, 4, 9] . Thus, the linear approximation to problem (1) is stated (with additional side constraints) and solved at each iteration by means of the Simplex method [10]. This algorithm has demonstrated to work properly even if the global approach is used (only one constraint) [11]. The inactive constraints are disregarded, with the aim of saving computational resources.

The required sensitivity analysis can be computed analytically. Full first order derivatives of the stress constraints are obtained via the adjunct variable method in order to reduce the computational effort. However, the second order directional derivatives are computed analytically via a direct differentiation technique. With this procedure, directional derivatives of all the stress constraints can be obtained although full first order derivative have not been calculated.

5. APPLICATION EXAMPLES

In this section, we present two structural problems frequently analysed in topology optimization. These examples are 2D structures in plane stress but we show three dimensional figures to better understand the solutions obtained.

The first example corresponds to a classic MBB-type beam with sliding supports [12]. Only half of the structure is analysed because of symmetry. Figure 2 shows the dimensions of the domain and the position of the external load. Self-weight is considered. The domain of the structure is discretized in $N_e = 120 \times 40 = 4800$ eight-node quadrilateral elements. The material being used is steel with density $\gamma_{mat} = 7650\ \mathrm{kg/m^3}$, Young's modulus $E = 2.1\ 10^5\ \mathrm{MPa}$, Poisson's ratio $\nu = 0.3$ and elastic limit $\widehat{\sigma}_{max} = 230\ \mathrm{MPa}$. The thickness of the structure is $1\ \mathrm{m}$

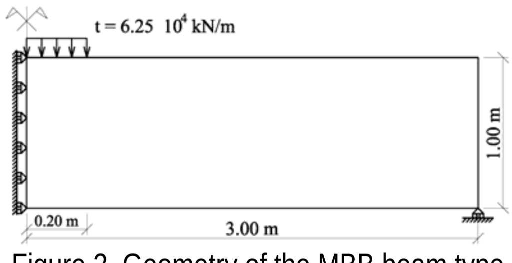

Figure 2. Geometry of the MBB beam type

This example is solved with the three formulations of stress constraints proposed in section 3 in order to compare the solutions obtained with them. Figures 3, 4 and 5 show the solutions obtained with the local, the global and the block-aggregation approaches of the stress constraints.

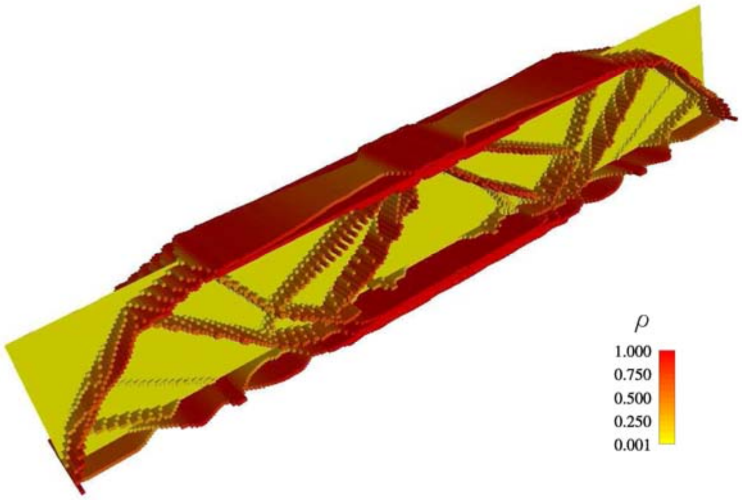

Figure 3. MBB solution with the local approach of the stress constraints

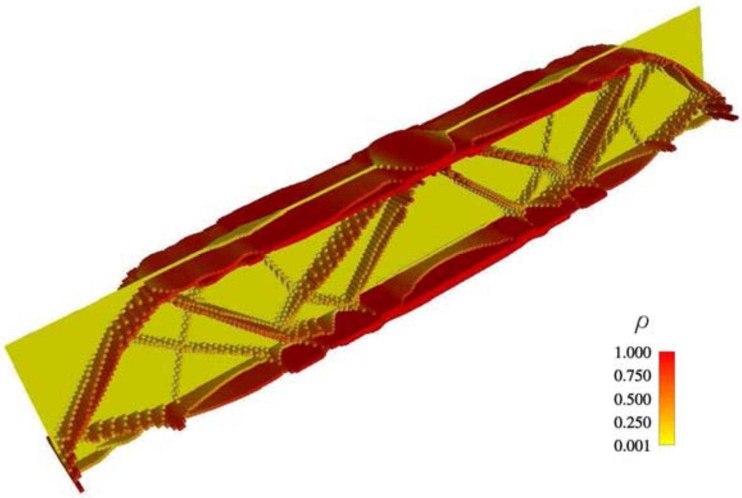

Figure 4. MBB solution with the global approach of the stress constraints

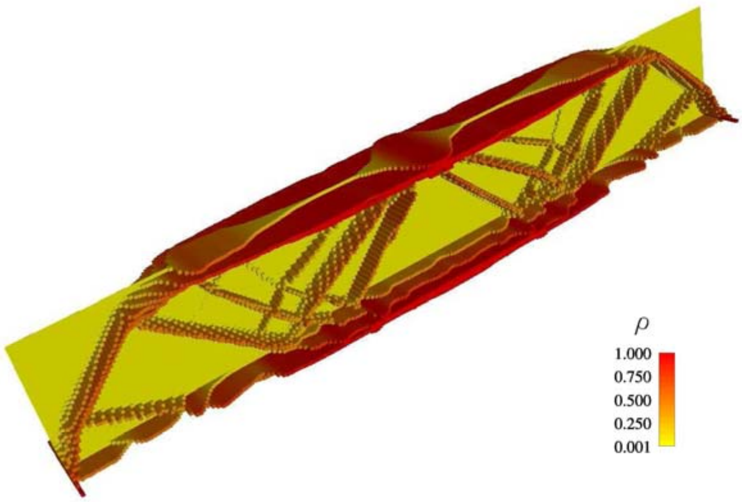

Figure 5. MBB solution with the block-aggregation approach of the stress constraints $[N_e^b = 6C]$

Table 1 shows the most important parameters of this problem in order to better understand them and in order to verify the weight of the final solutions obtained with the approaches of the stress constraints proposed.

Table 1. Summary of the MBB-Beam solutions

	q	ε	p	μ	Final Weight
Local approach (Figure 3)	1	0.01	4	-	15.41 %
Global approach (Figure 4)	-	0.01	4	40	13.62 %
Block aggr. Approach (Figure 5)	-	0.01	4	40	14.24 %

The second example is the optimal design of a cantilever beam with null displacements in the left border and with a vertical force applied in the middle of the right border. Figure 6 shows the dimensions of the domain and the position of the vertical forces applied. In this example, self-weight of the structure has also been included as a structural load. The domain of the structure has been discretized by using a homogeneous mesh with $120 \times 60 = 720$ 8-node quadrilateral elements. The thickness of the structure is $0.2\ \mathrm{m}$. The external force applied $(2\ 10^3\ \mathrm{kN})$ has been distributed on four contiguous elements in order to avoid stress accumulation phenomena. The material being used in this problem is steel with density $\gamma_{mat} = 7650\ \mathrm{kg/m^3}$, Young Modulus $E = 2.1\ 10^5\ \mathrm{MPa}$ Poisson ratio $\nu = 0.3$ and elastic limit $\widehat{\sigma}_{max} = 230\ \mathrm{MPa}$.

Figure 6. Scheme of the cantilever beam problem (units in m)

Figure 7 shows the optimal solution for the cantilever beam problem obtained by using the local approach of stress constraints.

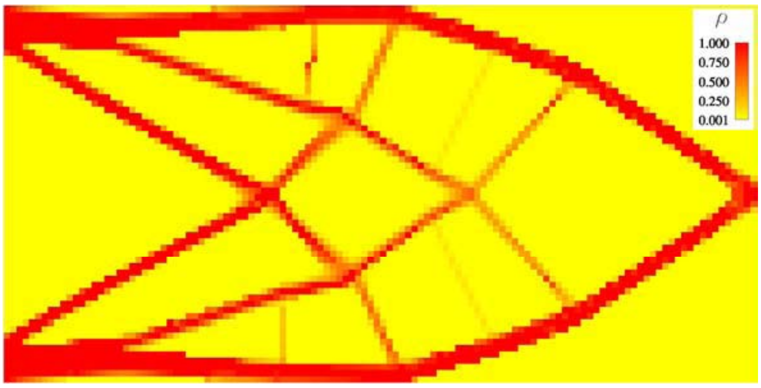

Figure 7. Optimal solution of the cantilever beam problem by using the local approach of stress constraints.

Figure 8 shows the optimal solution for the cantilever beam problem obtained by using the global approach of stress constraints.

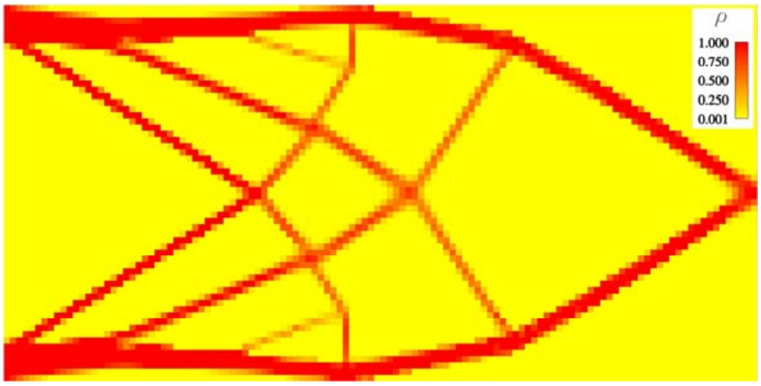

Figure 8. Optimal solution of the cantilever beam problem by using the global approach of stress constraints.

Figure 9 shows the optimal solution for the cantilever beam problem obtained by using the block aggregated approach of stress constraints. The number of blocks used is 120.

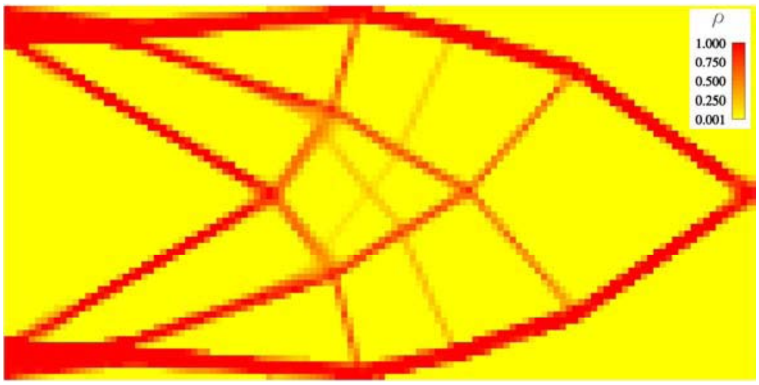

Figure 9. Optimal solution of the cantilever beam problem by using the global approach of stress constraints.

The solution obtained must be symmetric since the material being used presents an equal behaviour in tension and compression. However, we have analyzed the entire domain and do not consider this fact in order to verify the methodology proposed. Note that the optimal material distribution obtained is symmetric although this issue has not been forced. This symmetric distribution remarks the validity of the techniques proposed. In addition, the relaxation parameter also introduces a small effect on the stress constraints for intermediate values of the relative densities. This small decrease of the stress constraints due to the relaxation allows a larger weight reduction than the expected one without any relaxation.

Table 2 shows the most important parameters of this problem in order to fix their value and better understand them.

Table 2. Summary of the Cantilever Beam solutions

	q	ε	p	μ	Final Weight
Local approach (Fig. 7)	1	0.01	4	-	18.27 %
Global approach (Fig. 8)	-	0.01	4	40	16.22 %
Block aggr. Approach (Fig. 9)	-	0.01	4	40	16.61 %

6. CONCLUSIONS

Structural Topology optimization with stress constraints is an unusual branch in topology optimization problems. However, this kind of formulations offers important advantages versus maximum stiffness approaches because it avoids checkerboard layouts without using stabilization. In addition, it imposes a more realistic objective function from an engineering point of view and guarantees the feasibility of the solution because stress constraints are considered.

In this paper we propose three different formulations to impose stress constraints. The most usual and reliable procedure is the local approach of stress constraints because one stress constraint per element is introduced. However, this methodology imposes a large number of constraints when fine FEM meshes are used. Thus, the optimization algorithms require much more effort to obtain the optimum solution and the computing time increases considerably.

Due to this fact, two additional procedures are analyzed in order to reduce the computational effort: the global approach and the block aggregation approach. The global approach imposes only one global constraint that tries to aggregate the effect of all the local constraints. However, this formulation does not strictly guarantee the feasibility of the local constraints. On the other hand, the block aggregation of elements allows to define a more general methodology that takes the most important advantages of the local and the global approaches. The stress constraints are much more conveniently satisfied and the computing time is only slightly increased. Thus, if a large number of design variables is used, the block aggregation of elements is the most appropriate technique due to computational considerations. However, if the computing time is not too much restrictive the local approach is the most reliable formulation.

Finally, it is important to remark that minimum weight with stress constraints formulations produce fully satisfactory results versus maximum stiffness approaches. The use of maximum stiffness formulations should be replaced by minimum weight formulations because they offer very important advantages and the computational effort required is completely affordable nowadays.

ACKNOWLEDGEMENTS

This work has been partially supported by Grant Numbers DPI-2006--15275 and DPI-2007-61214 of the *“Ministerio de Educación y Ciencia”* (Spanish Government), by Grant Numbers PGIDIT03--PXIC118001PN and PGIDIT03--PXIC118002PN of the *“Dirección Xeral de I+D”* of the *“Consellería de Innovación, Industria e Comercio”* (*“Xunta de Galicia”*), and by research fellowships of the *“Universidade da Coruña”* and the *“Fundación de la Ingeniería Civil de Galicia”*.

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