MATHEMATICAL ANALYSIS AND IMPROVED DESIGN OF NONLINEAR HELICOPTER FLIGHT CONTROL SYSTEMS

Sébastien Kolb
Flight Dynamics Team
CReA, French Air Force Research Centre
BA 701, 13661 Salon Air, France
sebastien.kolb@inet.air.defense.gouv.fr

Abstract

In a helicopter flight control system, the presence of nonlinear elements such as a nonlinear control stick model or the rate limiting of an actuator can be responsible for pilot-induced oscillations (of category II). In order to assess the PIO proneness and to estimate the first-harmonic properties of the associated limit cycles, the describing function method (DFM) and the open-loop onset-point (OLOP) criterion are employed.

Position or rate limiting of an actuator can also trigger large overshoots and divergence known as the wind-up phenomenon. Specific control laws are designed so as to remedy to the problem by taking into account the level of saturation. The methodology employed lies on the description in the form of linear matrix inequalities (LMI) and on the maximisation of domains of attraction.

1 Introduction

This research paper deals with the analysis and the optimal design of nonlinear rotorcraft command channels. Indeed the presence of a nonlinear element in the flight control system such as a saturation or a dead zone can give rise to abrupt changes concerning the overall system behaviour and must be regarded carefully. The pilot's attitude may then be inadequate to the rotorcraft response and PIO occurrences may appear. It can be noted that in the field of rotorcraft contrary to fixed-wing aircraft, studies dealing with nonlinear PIO (of category II) remain still rare.

Here the content is mainly focused on the methodological aspects and is organised as follows. In the first part, a configuration is analysed which involves a dead zone and a saturation and whose demanded task consists in forcing the system to reach a fixed reference. In the second part, the impact of rate-limited swashplate displacements on the whole rotorcraft behaviour is examined thanks to the describing function method (DFM) and to the open-loop onset-point (OLOP) criterion. Finally, a device is added to the flight control system in order to alleviate the wind-up phenomenon triggered by rate limiting. The mathematical methodology employed is based on the description in the form of linear matrix inequalities (LMI) and on the optimisation of a specific linear criterion whose overall final formulation aims equivalently at the maximisation of (ellipsoidal) domains of attraction.

2 Analysis of PIO occurrences thanks to the describing function method

In this part, after presenting and describing the helicopter command channel under study, some background is furnished about the describing function method and some results are delivered concerning the appearance of possible nonlinear pilot-induced oscillations.

2.1 Presentation of the ADOCS command channel

The studied helicopter is the ADOCS concept whose name is an acronym for Advanced Digital/Optical Control System and which is in fact an equiped UH-60 Black Hawk. The project was carried out by both NASA and its Canadian counterpart aiming at developing an advanced flight control system for a modern combat rotorcraft. The concrete rotorcraft data used for the modelling of the command channel are the ones gathered and identified by M. B. Tischler (and its team) during numerous projects achieved at NASA.[1]

The rotorcraft command channel is adapted for this study and contains a nonlinear control stick model consisting of a dead zone and a saturation. The whole flight control system is exposed in the figure (1) and is implemented with the simulink software. The bare airframe is made of the rigid body and the rotor. As actuators, the command channel contains the upper boost actuator (first-order swashplate model), the ADOCS actuator (second-order model) and a digital-to-analog converter (i.e. a zero order hold block which concretely implies a time delay and is replaced by its Padé approximant for computer processing purpose). The feedback loop concerns the pitch angle and the pitch rate. The feedforward path includes besides the command model, the feedforward model, the stick sampling skew (giving also rise to a time delay) and the stick filter.

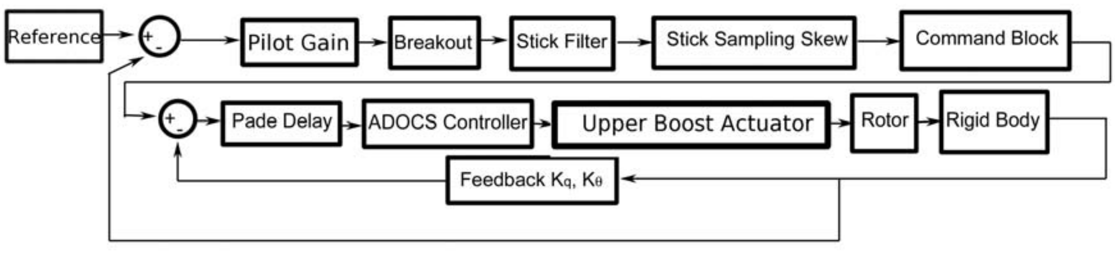

Figure 1: ADOCS command channel with a nonlinear control stick model

2.2 Analysis of the ADOCS command channel

In order to exploit the describing function method,[2] two conditions must hold. The first one states that there must be a clearly identifiable nonlinear element which can be isolated from the linear part whereas the second one stipulates that the linear part must behave like a low-pass filter. For the closed-loop system, the determination of the existence of possible periodical orbits and of their first-harmonic properties requires to solve the harmonic balance equation:

$$1 + L(j\omega) \cdot N(A, \omega) = 0 \tag{1}$$

where $N(A, \omega)$ is the describing function of the nonlinear element (i.e. the nonlinear control stick model) and $L(j\omega)$ is the linear part including the bare airframe, the pilot and the linear actuators. Routines (programmed with matlab) allow to estimate the amplitude, the pulsation and every other variable of the system as the pilot gain. In the figure (2), the amplitude and pulsation of the limit cycles are plotted for different pilot gains i.e. more or less nervous pilots.

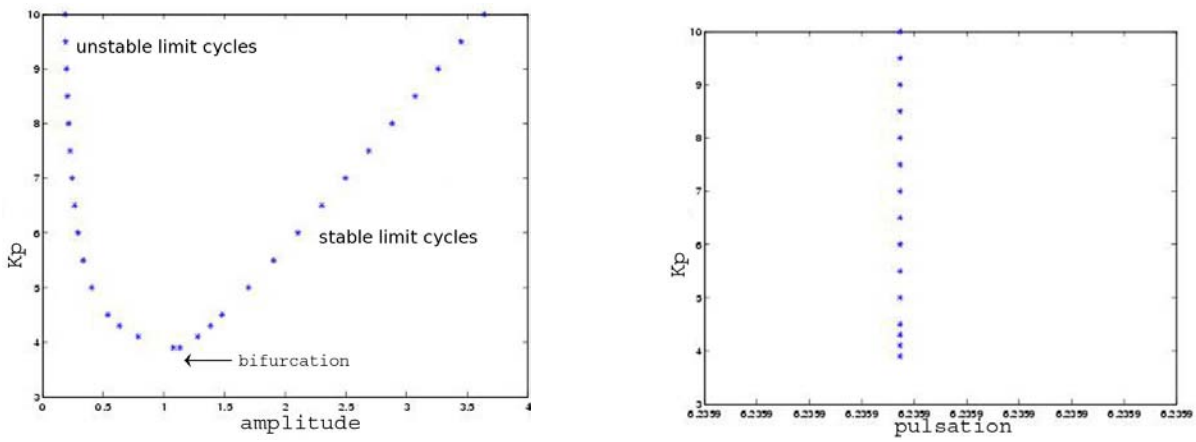

Figure 2: Amplitudes and pulsations of the PIO occurrences for different pilot gains K_p

As a conclusion, for pilot gains lower than $K_p^* = 3.9$, there is actually no limit cycle whereas for pilot gains higher than K_p^*, there exist two limit cycles. The Loeb criterion allows to state that the big one is stable and the little one is unstable. At the critical value K_p^* for which two limit cycles appear, according to dynamical system theory,[3] there is a saddle-node bifurcation of periodical orbits.

In order to verify the predictions of the describing function method, time simulations are performed for pilot gains lower and higher than the critical value K_p^*. Large initial conditions are selected so as to catch the stable periodical orbit if there exists one (the stable periodical orbit has got a larger amplitude than the unstable one).

The figure (3) illustrates the fact that the behavioural prediction is correct. For low pilot gains, the system converges to the zero equilibrium point whereas for high pilot

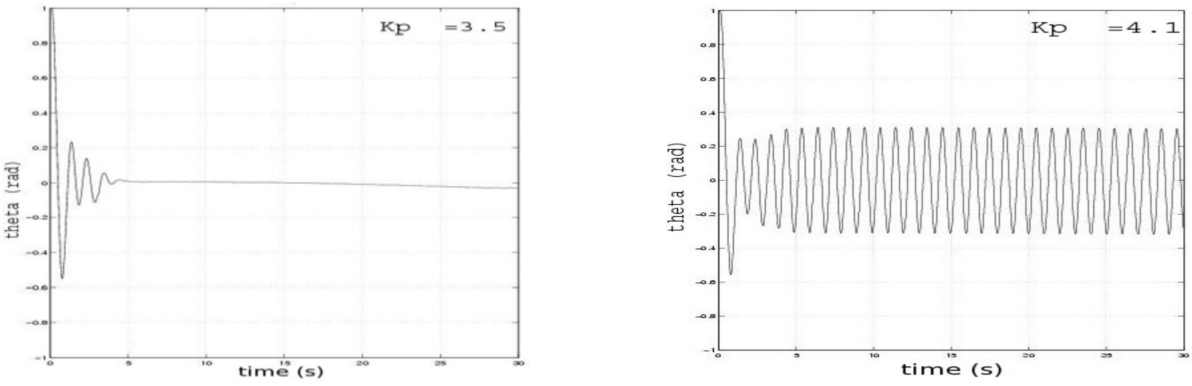

Figure 3: Time simulations of the pitch angle θ for pilot gains $K_p = 3.5$ (left) and $K_p = 4.1$ (right)

gains, the convergence towards a periodical phenomenon is visible (even if a slightly bigger pilot gain than calculated is necessary for a limit cycle to appear).

After having studied a first situation, where some PIO were generated by a very badly designed stick, the interest will be focused on the influence of an actuator rate limiting over the whole helicopter flight control system.

3 Analyses of a rotorcraft command channel performed by the describing function method and the OLOP criterion

Several methodologies[4] were developed in order to examine the impact of rate limiting over a flight control system. Here the describing function method and one of its derivative, the OLOP criterion (elaborated by the German aerospace centre DLR[5]), will be used.

3.1 Describing function method

In this first section, the longitudinal command channel of the ADOCS helicopter[1] presented in the figure (4) and in a modified configuration compared to the one of the previous part is analysed by means of the describing function method. The swashplates are likewise mechanically rate-limited by a maximal variation speed of 10 $inches/s$. The task consists in following a sinusoidal reference of increasing input amplitudes θ_c. The equation (2) requires to be solved so as to estimate the amplitude A and phase delay ϕ of the periodical orbit and involves the describing function $N(A, \omega)$ of the rate limiter. A continuation algorithm is employed in order to compute the solution of an implicit system whose variables are the input amplitude θ_c, the amplitude A of the limit cycles of the rate limiter entry state and the phase delay ϕ between the input reference and the rate limiter entry.

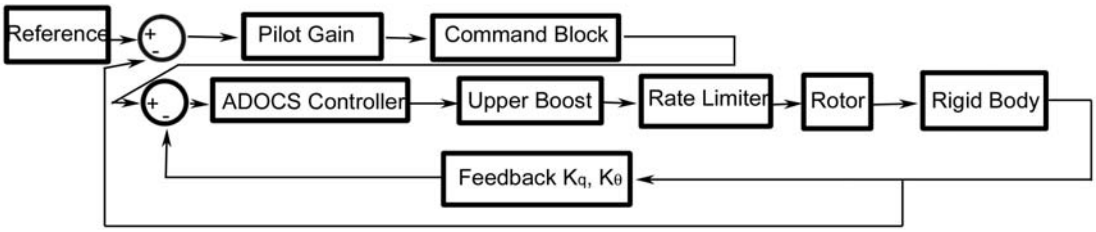

Figure 4: Closed-loop ADOCS command channel with sinusoidal input

$$(1 + Rotor \cdot RigidBody \cdot N(A,\omega) \cdot Actuator \cdot (K_p \cdot CommandBlock + Feedback)) \cdot A\exp(j\phi) = Actuator \cdot CommandBlock \cdot \theta_c \quad (2)$$

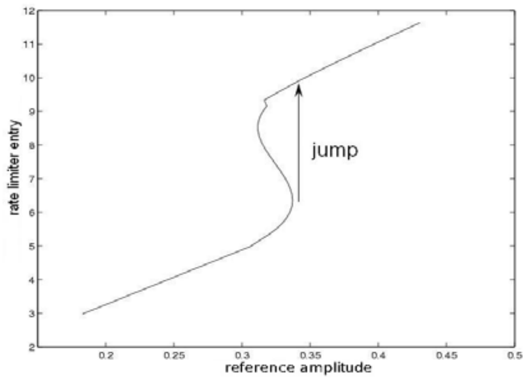

Figure 5: Jump resulting from a little amplitude variation of the input reference

The figure (5) reveals that a little reference amplitude increase from $0.33rad$ to $0.34rad$ makes the rate limiter entry amplitude jumps from 6 to 10. This abrupt change of amplitude of the periodical orbit may surprise the pilot and may prevent him from controlling efficiently the helicopter leading to a possible dangerous situation.

A first configuration containing a rate limiter was studied successfully by means of the describing function method. In what follows, another investigation lying on the OLOP criterion is accomplished.

3.2 OLOP criterion

In this section, two similar rotorcraft command channels are examined thanks to the OLOP criterion. The open-loop onset-point is placed on the Nichols diagram locating the conditions (amplitude and phase of the open-loop system at this specific onset frequency) for which the rate-limiter becomes active. After comparing this location with a pre-defined frontier, the system is considered PIO prone or not according to flight dynamics considerations ("flying qualities cliff") and predictions of dynamical system theory ("jump resonance").[4,5]

A simple pilot model is chosen since the pilot behaves apparently like a simple gain once the PIO is fully developed. It is assumed besides that the pilot remains adapted to the linear aircraft behaviour even after rate limiting onset and he may then react inadequately. A spectrum of pilot gains K_p is constructed depending on the crossover phase angle of the open-loop aircraft-pilot system Φ_{cr} i.e. such that:

$$K_p \cdot \left| F_{\text{open-loop aircraft}}(\omega_{cr}) \right| = 1\,(0\ dB) \tag{3}$$

For the figure (6), on a same graph, there are several points which are linked to different rate limiter bounds varying from $R = 5\ inches/s$ to $R = 25\ inches/s$. Both examined configurations have got the same pilot gain. For the case examined on the left side, the "command model" actuator (a second-order model involving and smoothing the pitch angle reference input) is suppressed of the command channel.

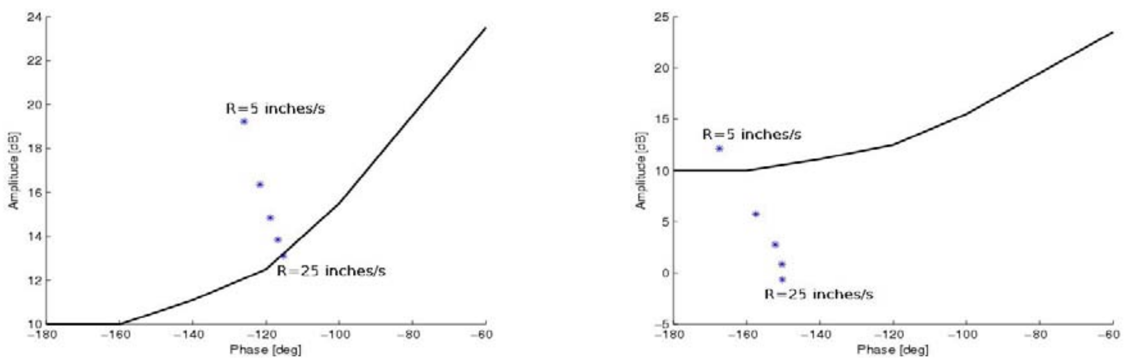

Figure 6: ADOCS criterion applied for different configurations

In the figure (6), the configuration on the left side is likely to meet PIO whereas the other one (on the right side) is not. This observation shows the importance of the command model (a second-order model) and its smoothing effect which contributes to make the PIO phenomenon disappear almost completely.

After having analysed several nonlinear command channels exploiting especially the describing function method and the OLOP criterion, the next part will contribute to improve the flight control system by employing and conceiving an additional device alleviating the nefast effects of the nonlinear element.

4 Design of an anti-windup device for the ADOCS helicopter

Amplitude or rate limiting of an actuator can be responsible for a wind-up phenomenon whose consequences are large overshoots or even a complete system destabilisation. In this particular study of the ADOCS helicopter, the trouble will stem from the rate-limited displacement of the swashplates which doesn't succeed anymore in transmitting an appropriate value to the rest of the command channel because of mechanical limitations.

In order to cope with this problem (or to alleviate it), the level of saturation is taken into account for the design of the flight control laws. The anti-windup device can be conceived such that the system remains stable for high input amplitudes or to improve its $\mathcal{L}_2$-performance as much as possible (i.e. to reduce the quadratic difference between the responses of the linear system and the nonlinear one containing saturations).

4.1 Methodology employed

The flight control law design problem is illustrated in the figure (7). It involves the plant $P(s)$, the controller $K(s)$, the set of dead zones (or saturations) $\phi(v)$ where the vectors correspond to the inputs w, the outputs z, the controls u, the inner states y and the entries of the nonlinear elements v.

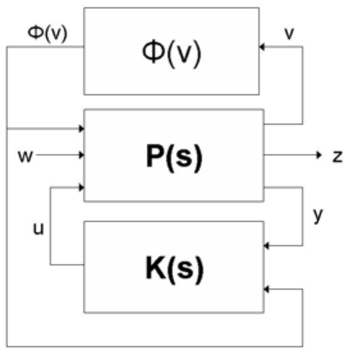

Figure 7: Standard interconnection plant with an anti-windup device

The gains associated with the anti-windup device are here determined via the description in LMI form and the optimisation of basins of attraction according to works on fixed-wing aircraft led by J-M. Biannic from ONERA and S. Tarbouriech from LAAS-CNRS.[6,7] Actually the set of mathematical equations associated with the standard anti-windup synthesis problem and with the figure (7) involves the standard plant $P(s)$ and the controller $K(s)$ containing the anti-windup gain B_{AW} and is defined such that there exists no nested loop and no information fed back from the nonlinear elements to the controller via other ways than the anti-windup signal.

As far as the *stability analysis* is concerned, a *Lyapunov-like theorem* remains the corner stone of this approach. The mathematical developments are exposed in the above mentioned articles.[6,7]

For the specific problem regarded here, the plant $P(s)$ is made of the bare rotorcraft i.e. the rigid body and the rotor whereas the controller is composed respectively of the

ADOCS actuator (second-order model) and upper boost actuator (first-order model), the feedback loop $H(s)$ (for the pitch angle and rate), the feedforward path $(P^{-1}(s) + H(s))$ and the possible supplementary anti-windup device. The unique saturation is actually related to the rate-limited displacement of the swashplates (i.e. "upper boost actuator") which cannot move quicker than $R = 10\ inches/s$. In order to compute easily and successfully optimisation procedures, only low-order models of the rotor and of the rigid body are used.

4.2 Results and improvements

The performance level is a $\mathcal{L}_2$-measure of the difference between the response of the linear model and the response of the system containing saturations. For every input amplitude such that it is possible to find an anti-windup gain stabilising the system, a theoretical optimal achievable performance is determined (conservative bound i.e. stability is guaranteed for lower amplitudes but nothing is predicted for higher amplitudes). The results are summed up in the figure (8).

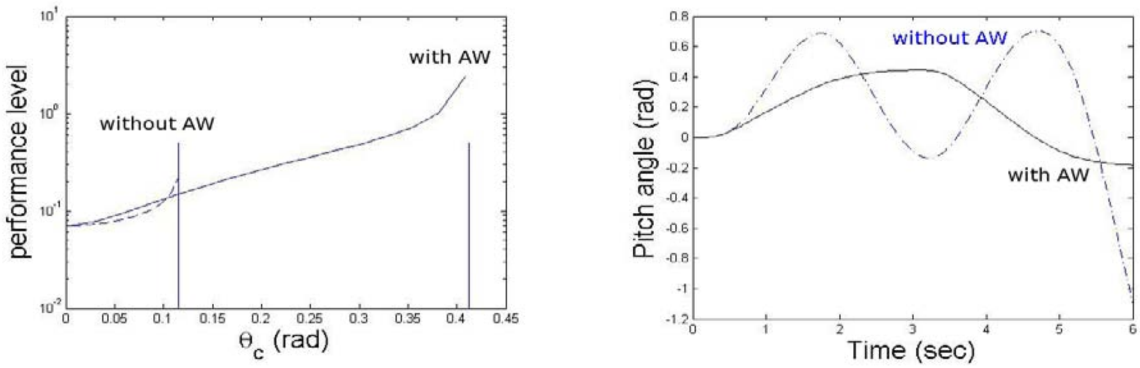

Figure 8: Impact of an anti-windup device : performance level (left) and time simulations of the pitch angle (right)

The time simulation (cf. right figure of (8)) confirms that the anti-windup structure improves the stability of the flight control system. Indeed with an anti-windup compensator, the augmented helicopter succeeds in following pretty well the reference task in pitch angle as the plain curve shows whereas the initial system without anti-device structure diverges as illustrated by the dash-dotted curve.

The predicted pitch amplitude bound beyond which the overall rotorcraft becomes unstable is besides clearly higher with an anti-windup compensator than without. But nevertheless the higher the amplitude is the more the performance level is degraded (cf. left figure of (8)). The anti-windup device works less efficiently when the reference amplitude to follow is too high.

5 Conclusion

The ADOCS helicopter prototype (adapted for this study) was shown to be prone to pilot-induced oscillations. Indeed it was diagnosed that the nonlinear behaviour of certain actuators triggers abrupt flying qualities cliffs. The existence and dangerousness of such events were evaluated thanks to the describing function method (DFM) and to the open-loop onset-point (OLOP) criterion.

A first command channel containing a badly dimensioned stick was analysed to be susceptible to give pilot-induced oscillations and the pilot gains for which there exist limit cycles were determined ; their amplitude and pulsation were estimated as well. In another configuration, the influence of a rate limiter over the whole closed-loop system was considered. The nervousness of the pilot and the rate limit are natural contributors for the appearance of pilot-induced oscillations. The smoothing effect of the command model block in the feedforward path was also observed. The performed calculations allowed concrete observations and estimations of the critical parameters.

At the end, an anti-windup device was added which permits to alleviate the saturation effects. By maximising specific domains of attraction, the amplitude of the admissible inputs and the performance level were successfully improved for the rotorcraft command channel.

Other mathematical methodologies are available in order to analyse pilot-induced oscillations of category II or to design an anti-windup structure which improves the sensitivity of the flight control system to the actuator saturations. They may give rise to further interesting studies in the field of rotorcraft flight dynamics and autopilot.

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