GUIDANCE ALGORITHM FOR RENDEZVOUS ON ELLIPTICAL ORBITS

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I ABSTRACT

The current developed guidance algorithms for rendezvous are able to deal with targets placed on circular or quasi-circular orbits. This is typically the case of the far rendezvous automatic on-board guidance of ATV (vehicle developed under ESA responsibility, with Astrium Space Transportation as prime contractor, that successfully performed an AR&D (Automated Rendezvous and Docking) in April 2008). Thanks to recent progress in the field of relative motion modelling, it is now possible to perform an AR&D on an elliptical orbit. The implemented techniques presented hereafter were developed in the frame of in-house studies. They were, afterwards, used in the frame of the HARVD project (Highly Autonomous RendezVous and Docking), under ESA responsibility. A few words about recent progress performed are also added at the end of the article.

II INTRODUCTION

The presented guidance algorithm is based on ATV fundamental ideas. The main difference is in the heart of the algorithm: the relative motion model. For ATV (and others space vehicles whose mission includes a rendezvous), there is a linear relative motion model inside the on-board guidance algorithm. Its purpose is to give an accurate estimation of the boosts effects that the spacecraft has to perform in order to get from a state (position and velocity) to another.

First researches on relative motion model were conducted by Clohessy and Wiltshire [1], in 1960. They developed a method to assess the relative position and velocity of a chaser in a relative coordinate system whose origin is the target and whose directions are given by the local vertical and local horizontal directions. Given the initial position, velocity and true anomaly on a circular orbit, the final position and velocity at a given true anomaly are obtained by solving a linear stationary system of equations. This solution is only accurate when the chaser is close to the target (when the distance target-chaser is small in comparison with the distance target-Earth's centre) and when the eccentricity of the target's orbit is small ($e < 0.01$).

When the target is on an eccentric orbit, the relative motion of the chaser is described by non-linear differential equations with periodic coefficients. The linearized equations are known as the Tschauner-Hempel equations [2]. In that direction, another important research was driven by Carter [3] to change the coordinate system in order to admit non-null eccentricity. The equations of motion are, however, more difficult to solve. Carter proposed a solution that requires integrating some functions and uses the eccentric anomaly. The result is not simple to use in an engineering work and has a singularity for a circular orbit ($e = 0$). Note that other developments exist for relative motion model and associated state transition matrix and/or tensor: linear model with inclusion of J_2 effects [4] and non-linear model [5].

The solution chosen in this document is based on the developments performed by Yamanaka and Ankersen [6], in 2002. It uses the same inertial frame but modifies the integration term expression. Consequently, it can be easily computed by using the variable time instead of the eccentric anomaly. Assessment of position and velocity at a given time/true anomaly, from position and velocity at another given time/true anomaly, is now easier to adapt to a guidance algorithm and faster to compute. An improvement of this guidance algorithm has been recently developed, in the frame of a contract under CNES responsibility, thanks to the solution detailed [4]. A simple comparison is presented at the end of this article.

III NOMENCLATURE

AR&C = Automated Rendezvous and Capture
AR&D = Automated Rendezvous and Docking
ATV = Automated Transfer Vehicle
CoM = Centre of Mass
GC = Guidance and Control
GNC = Guidance Navigation and Control
GTO = Geostationary Transfer Orbit
HARVD = Highly Autonomous RendezVous and Docking
MCI = Mass, Centring and Inertia
WRT = With Respect To

IV LINEAR MOTION MODEL

Hypothesis

The whole relative motion model is based on a single hypothesis: the relative distance between the chaser and the target is small when compared to the distance between the target and the Earth centre. This assumption will help us to simplify the chaser's equations of motion. One recalls that only the gravitational force is taken into account by this model. Neither the J_2 disturbance, nor the atmospheric drag, nor the vehicle thrust, are taken into account.

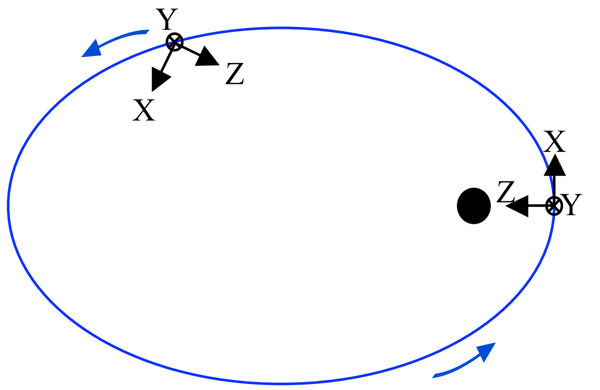

Fig 0 : The LVLH (Local Vertical / Local Horizontal) frame

One notes $\vec{R}$ the absolute position of the target in the equatorial frame and $\vec{r}$ the relative position of the chaser wrt the target, in the LVLH frame based on the target CoM (see Fig. 1). The gravitational force uses the ratio $\frac{\vec{R}+\vec{r}}{\left\|\vec{R}+\vec{r}\right\|^3}$ to be determined. This term is estimated linearly wrt. the relative position of the chaser:

$$\frac{\vec{R}+\vec{r}}{\left\|\left(\vec{R}+\vec{r}\right)\right\|^3} \approx \frac{\vec{R}+\vec{r}}{R^3}\left(1+2\frac{\vec{R}.\vec{r}}{R^2}+\frac{r^2}{R^2}\right)^{-\frac{3}{2}}$$

$$\frac{\vec{R}+\vec{r}}{\left\|\left(\vec{R}+\vec{r}\right)\right\|^3} \approx \frac{1}{R^3}\left(\vec{R}+\vec{r}-3\frac{\vec{R}.\vec{r}}{R^2}\vec{R}\right)$$

This approximation allows us to get simple linear equations for the chaser's relative motion, linking the chaser's relative position, velocity and acceleration as follows:

(I.1)
$$\begin{bmatrix}\ddot{x}\\ \ddot{y}\\ \ddot{z}\end{bmatrix} = \begin{bmatrix} -k\omega^{\frac{3}{2}}x + 2\omega\dot{z} + \dot{\omega}z + \omega^2 x \\ -k\omega^{\frac{3}{2}}y \\ 2k\omega^{\frac{3}{2}}z - 2\omega\dot{x} - \dot{\omega}x + \omega^2 z \end{bmatrix}$$

where $k = \frac{\mu}{C^{\frac{3}{2}}}$, C is the orbital angular momentum of the target, ω is the target orbital rate and $\vec{r}$ is replaced by its components x, y and z.

Equations simplification

These equations cannot be simply solved. Time cannot be used as the reference variable (usually used to place the target on its orbit and to derive the position to get the velocity, and the velocity to get the acceleration). One needs a variable giving the target position on its orbit without ambiguity. By ambiguity, one means leading to an undetermination due to the target's motion periodicity, like the target's radius would do, for instance. The chosen variable is the target's true anomaly _, the angle between the perigee vector and the position vector:

$$\begin{cases} \dot{x} = \omega x' \\ \ddot{x} = \omega^2 x'' + \omega\omega' x' \\ \omega' = -2k^2 e \sin(\theta)\rho \end{cases}, \text{ where } \rho = \left(1 + e\cos(\theta)\right) \text{ and } x' = \frac{dx}{d\theta}$$

Deriving wrt. the true anomaly, the new system is:

(I.2)
$$\begin{cases} \rho x'' - 2e\sin(\theta)x' - e\cos(\theta)x = 2\rho z' - 2e\sin(\theta)z \\ \rho y'' - 2e\sin(\theta)y' = -y \\ \rho z'' - 2e\sin(\theta)z' - (3 + e\cos(\theta))z = -2\rho x' + 2e\sin(\theta)x \end{cases}$$

As it remains difficult to solve, a new variable change is performed: $\begin{bmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{z} \end{bmatrix} = \rho \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

The final solvable system is:

(I.3)
$$\begin{cases} \tilde{x}'' = 2\tilde{z}' \\ \tilde{y}'' = -\tilde{y} \\ \tilde{z}'' = \frac{3\tilde{z}}{\rho} - 2\tilde{x}' \end{cases}$$

Linear problem solution

The system's solution for the out-of-plane motion is simple and corresponds to the y component – its solution is periodical. Given the satellite state at a date t_0, one can get the satellite's state at another date t as follows:

(I.4)
$$\begin{bmatrix} \tilde{y} \\ \tilde{v}_y \end{bmatrix} = \frac{1}{\rho(\theta - \theta_0)} \begin{bmatrix} \cos(\theta - \theta_0) & \sin(\theta - \theta_0) \\ -\sin(\theta - \theta_0) & \cos(\theta - \theta_0) \end{bmatrix} \begin{bmatrix} \tilde{y}_0 \\ \tilde{v}_{y0} \end{bmatrix}$$

The solution of the linear system on the XoZ plane is not trivial. One starts by integrating once $\tilde{x}''$ and one replaces $\tilde{x}'$ in $\tilde{z}''$ equation.

Now, one looks for the homogeneous second-order differential equation. One focuses on the F. Ankersen and K. Yamanaka contribution, [6], to the J. Tschauner and P. Hempel first solution, [2]. Their solution has no singularity:

(I.5)
$$\varphi = C_1 \rho(\theta)\sin(\theta) \int_{\theta_0}^{\theta} \frac{1}{\rho(\tau)^2} d\tau - \rho(\theta)\cos(\theta) + C_2$$

The great interest of such a formulation is on the integral part because it allows a fast computation of its value. Indeed, it can be assessed by using the time separating the two limit true anomalies:

(I.6)
$$J(\theta)=\int_{\theta_0}^{\theta}\frac{1}{\rho(\tau)^2}d\tau=k^2(t-t_0)$$

The final equation form is:

(I.7)
$$\begin{bmatrix}\tilde{x}\\ \tilde{x}'\\ \tilde{z}\\ \tilde{z}'\end{bmatrix}=\begin{bmatrix}1 & -c(1+1/\rho) & s(1+1/\rho) & 3\rho^2 J\\ 0 & s & c & (2-3esJ)\\ 0 & 2s & 2c-e & 3(1-2esJ)\\ 0 & s' & c' & -3e\left(s'J+s/\rho^2\right)\end{bmatrix}_\theta\begin{bmatrix}K_1\\ K_2\\ K_3\\ K_4\end{bmatrix}$$

where $c=\cos(\theta)$, $s=\sin(\theta)$, $c'=-[\sin(\theta)+e\sin(2\theta)]$, $s'=\cos(\theta)+e\cos(2\theta)$, $J=k^2(t-t_0)$ and K_1, K_2, K_3 and K_4 are constants.

Final expression

One transforms the obtained variables in order to get the solution with the initial variables. The chaser's state at a true anomaly _, knowing its state at the true anomaly _0, is:

(I.8)
$$\begin{bmatrix}x\\ \dot{x}\\ z\\ \dot{z}\end{bmatrix}=Mat_{xz}\begin{bmatrix}1 & -c(1+1/\rho) & s(1+1/\rho) & 3\rho^2 J\\ 0 & s & c & (2-3esJ)\\ 0 & 2s & 2c-e & 3(1-2esJ)\\ 0 & s' & c' & -3e\left(s'J+s/\rho^2\right)\end{bmatrix}_\theta\begin{bmatrix}1 & -c(1+1/\rho) & s(1+1/\rho) & 0\\ 0 & s & c & 2\\ 0 & 2s & 2c-e & 3\\ 0 & s' & c' & -3es/\rho^2\end{bmatrix}_{\theta_0}^{-1}Mat_{x_0z_0}^{-1}\begin{bmatrix}x_0\\ \dot{x}_0\\ z_0\\ \dot{z}_0\end{bmatrix}$$

Where: $Mat_{xz}=\begin{bmatrix}1/\rho & 0 & 0 & 0\\ 0 & 1/\rho & 0 & 0\\ \alpha & 0 & \beta & 0\\ 0 & \alpha & 0 & \beta\end{bmatrix}$, $M_{x_0z_0}^{-1}=\begin{bmatrix}\rho_0 & 0 & 0 & 0\\ 0 & \rho_0 & 0 & 0\\ \gamma_0 & 0 & \delta_0 & 0\\ 0 & \gamma_0 & 0 & \delta_0\end{bmatrix}$, $\alpha=\dfrac{\mu^2 e\sin(\theta)}{C^3}$, $\beta=\dfrac{\mu^2\rho}{C^3}$,

$\rho_0=1+e\cos(\theta_0)$, $\gamma_0=-e\sin(\theta_0)$, $\delta_0=\dfrac{1}{\rho_0\mu^2C^3}$

For simplicity sake, one defines:

$$X_1=M(\theta_1,\theta_0)X_0$$

Where M is the product of the different matrices participating in the final state computation, X_t is the chaser state (position and velocity) at a give true anomaly _t.

V GUIDANCE

One assumes that it is possible to drive a satellite from a state to another, in space, by using only two boosts: one performed when leaving a first position, and the other, when arriving at the aimed position. The linear relative motion model is used by the guidance algorithm in order to get boosts directions and durations estimations, able to drive the chaser.

Background

The general on-board guidance problem may be generically described as follows:

i. Find a control u (driving force orientation)
ii. starting from the current estimated state
iii. to reach a targeted state
iv. under a set of constraints, including:
    a. The dynamics of the vehicle
    b. Intermediate path constraints
v. and possibly also allowing minimizing a criterion.

For launchers, the general problem of the on-board guidance algorithm has been solved under the particular case of no criterion considered [7]. The solution uses a segmentation of the trajectory and a time parameterization of the sought control. This scheme is currently part of the Ariane 5 launch vehicle exo-atmospheric guidance.
Following this development, in house advanced studies have also been performed to extend these basic elements through application of collocation and pseudospectral type methods to on-board guidance in the mid-nineties, in [8] and [9]. Indeed, this allows coping with the general problem including both the path constraints and the criterion. However, the parametric problem obtained after transcription of the optimal control setting remains to be solved by a reliable NLP solver, and one knows that for on-board application additional research is needed based on the structure of the problem at hand to get positive convergence results and full mastering of optimizer behaviour.
Considering the state of the art with regard to collocation and pseudo spectral methods, (despite recent progress on the latter approach [10]), a guidance scheme without explicit on board management of intermediate path constraints has been selected, i.e. choice has been made to use the specific dynamics of rendezvous but to rely on off-line mission analysis work to tackle the intermediate constraints (forbidden volume for the trajectory for instance).

Principle

In a guidance algorithm, the interest of the relative motion model may be to predict a future state of the chaser knowing a previous one, or to get the Lambert's problem solution. For the reasons explained in the previous paragraph, one is interested in getting the Lambert's problem solution.

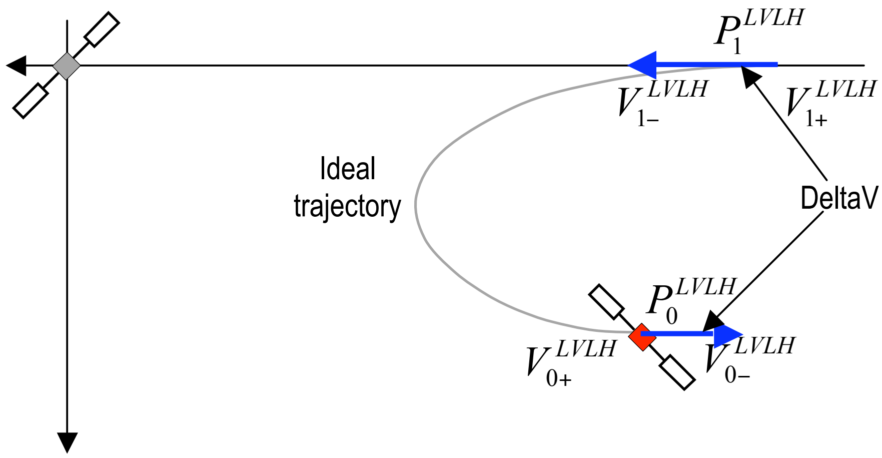

Fig 0 : Two-boosts guidance principle

One recalls that this problem aims at getting the transfer orbit linking two positions P_0^{LVLH} and P_1^{LVLH} in space in a given time duration. Its solution is the first and most important step to deal with. Indeed, the comparison between the velocities on the transfer orbit, V_{0+}^{LVLH} and V_{1-}^{LVLH}, the initial velocity V_{0-}^{LVLH} and the final aimed velocity V_{1+}^{LVLH} gives us the differences of velocity (DeltaV) to perform at the beginning and at the end of the transfer in order to reach the final aimed state. These differences of velocity are recorded as accelerations and durations to be ordered.

However, boosts cannot be performed instantaneously. If the last computed values were commanded, the chaser would not be driven to the correct final state. Therefore, they have to be computed again, taking into account their durations. This is the second step of the guidance algorithm which may now compute the spread boosts effects on the chaser's state. The trajectory is not simulated by the linear motion model but by a gravity model simulator. The dispersions due to the J_2 term of the Earth gravitational field or the atmospheric drag will not be inserted into the simulator. Thus, by iteratively modifying the boosts directions, norms and durations, one will reach a solution theoretically driving the chaser, as exactly as wished, to the final aimed state.

Tests

One performs first series of tests. They show us the guidance algorithm accuracy on elliptical orbits, since this is its design purpose. One considers a manoeuvre on a highly elliptical orbit: a GTO (Geosynchronous Transfer Orbit), whose eccentricity is 0.73. One also considers manoeuvres starting far from the target. Based on ATV manoeuvres, one chooses a transfer lasting half an orbital period and whose initial state is (-25km , 12km) in X_Z LVLH frame and whose finale state is (-25km , 680 m). Since the guidance algorithm is to take into account an ideal gravitational field and nothing more, one tests it by adding the disturbances due to the atmospheric drag and to the J_2 term of the Earth gravitational field.

Open-loop scheme

One first runs tests following an open-loop scheme. By open-loop, one means that the boosts directions, norms and durations are computed once, at the beginning of the transfer, when the chaser is in its initial state. This will show us how far our model is from the reality (see Fig. 3).

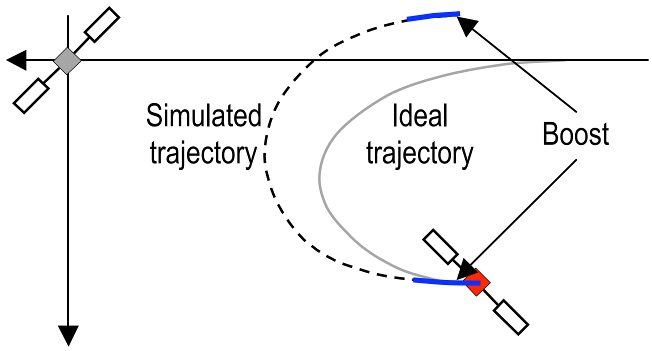

Fig 0 : Open-loop scheme trajectory

One runs 20 tests. For each of them, the initial position and velocity are randomly chosen according to a Gaussian law, around a given position and velocity. One sets the 3-_ dispersion at 3000 m on position and 0.5 m/s on velocity. This shows the initial state influence on the final state dispersion.

One obtains the following results:

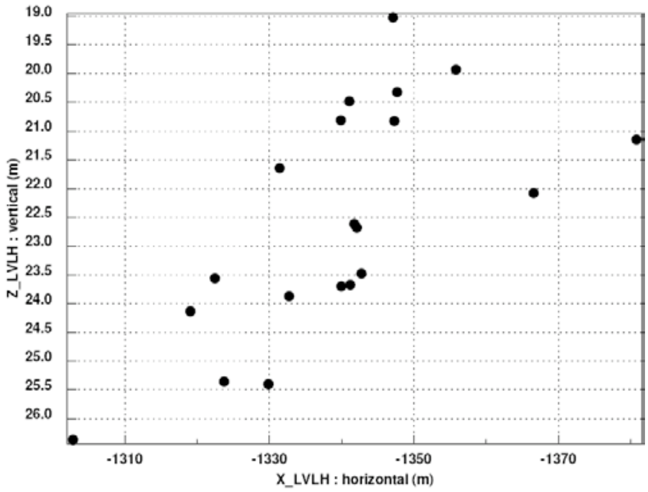

Fig. 1. Position accuracy (Open-loop)

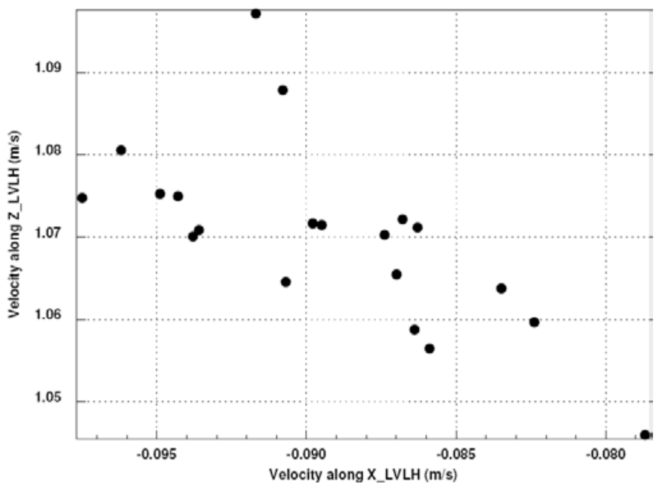

Fig. 2. Velocity accuracy (Open-loop)

On Fig. 4 and Fig. 5 are plot the final position and final velocity errors in the LVLH frame. One observes that the dispersion due to the atmospheric drag and to the J_2 term, may drive the chaser more than 1 km away from the targeted position and more than 1 m/s away from the targeted velocity. Moreover, the obtained final states are not even centred around 0. These results (1 km and 1 m/s) are not acceptable when compared to the ATV GNC requirements (30 m and 0.2 m/s). In spite of not seeking better accuracy than ATV GNC requirements, they are used as a reference for the guidance scheme in order to establish a first conclusion on its possible use on board a chaser.
Concerning the effects of the initial state on the final accuracy, one concludes that it has a small impact on the final velocity, but a strong one on the final position.

Closed-loop scheme

This mode is able to deal with the disturbances occurring during the achievement of the 1st boost and the manoeuvre free-drift phase. During this mode, correction boosts are regularly computed and performed if their magnitude is high enough (see Fig. 6).

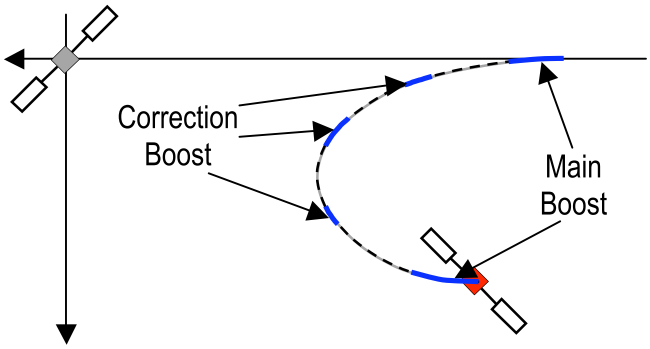

Fig 2 : Closed-loop trajectory

Once again, on Fig. 7 and Fig. 8, the final position and final velocity errors in the LVLH frame are plot. Now one observes that position and velocity accuracy allows concluding the chaser is correctly driven to the final aimed state. The position accuracy is close to 15 m and on velocity is close to 0.08 m/s. Space environment disturbances are correctly treated, as well as dispersions on initial position and velocity.

This kind of guidance scheme can be used on-board as well as during preliminary mission analyses. Indeed, boost timeline and consumption estimations can be assessed and optimized on ground thanks to this same algorithm. It can be let free to perform correction boost whenever it considers it is necessary or it can be forced to respect a given timeline.

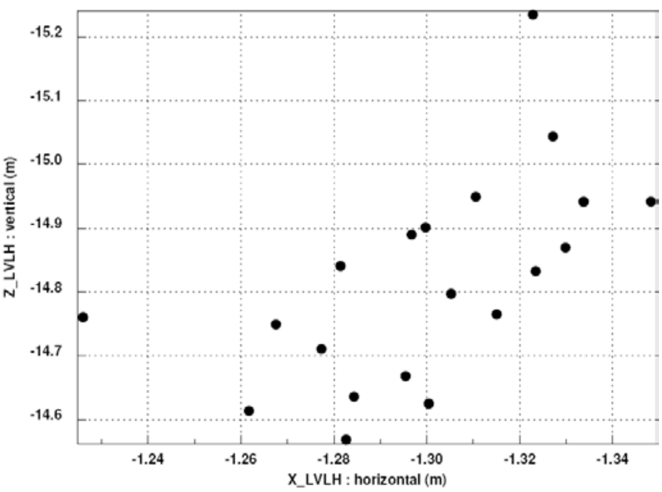

Fig. 3. Position accuracy (Closed-loop)

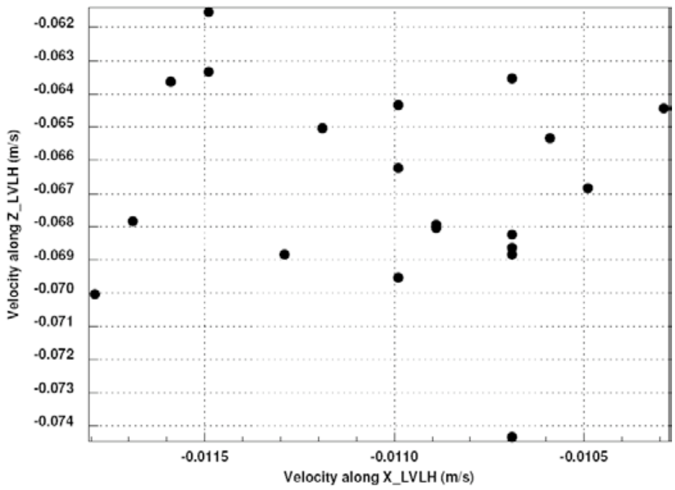

Fig. 4. Velocity accuracy (Closed-loop)

VI IMPROVEMENTS

Recent progress has been achieved, in the frame of a contract under CNES responsibility, thanks to the use of a more precise state transition matrix obtained by Gim and Alfriend [4]. This matrix not only works for elliptical orbits. It also takes into account the J_2 disturbance induced by the Earth oblateness. Therefore, the boosts estimations, computed thanks to this matrix, stick in a better way to the real dynamics of the chaser and generate fewer correction boosts.

The principles of this new guidance algorithm have also been improved: it now studies both 2-boosts and 3-boosts transfers, the latter being two adjacent 2-boosts transfers. Different transfer solutions are assessed by the algorithm which chooses the least consuming one. However, it does not take into account spread boosts but considers they are performed instantaneously.

Consequently, this new algorithm allows reducing the chaser consumption significantly. For example, considering the rendezvous performed in the tests, its cost when performed with the guidance algorithm developed in the article is of 3.82 m/s. With this new algorithm, it only costs 2.53 m/s.

VII CONCLUSION

A guidance algorithm working on elliptical orbits has equally been designed and tested. It is a useful tool to analyze and manage the atmospheric drag and the Earth gravitational field J_2 term disturbances effects, as well as thrusters malfunctions or navigation errors. Moreover, it needs little enough CPU load to be used on board a satellite, making rendezvous on an elliptical orbit possible. The preliminary performances are encouraging as well as the ones of the new algorithm. The latter could, however, be problematic concerning CPU load.

The strong points of these two algorithms (spread boosts, for the first one, and more precise state transition matrix and consumption minimization, for the second one) could, in future developments, be combined to increase final accuracy and overall consumption.

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