EFFECT OF AIR JET VORTEX GENERATORS ON A SHOCK WAVE BOUNDARY LAYER INTERACTION

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The effect of upstream injection by means of continuous Air Jet Vortex Generators (AJVGs) on a shock wave turbulent boundary layer interaction is experimentally investigated. The baseline interaction is of the impinging type, with a flow deflection angle of 9.5° , a Mach Number M_e=2.3 and a momentum thickness based Reynolds number of 5,000. Considered are the effects of the AJVGs on the upstream boundary layer flow topology and on the spatial and dynamical characteristics of the interaction. To this aim, Stereoscopic Particle Image Velocimetry has been employed, in addition to hot-wire anemometry (HWA) for the investigation of the the dynamical characteristics of the reflected shock. It is shown that the AJVGs significantly modify the three dimensionality of the upstream boundary layer. Overall, the AJVGs cause a reduction of the separation bubble length and height. In addition, the energetic frequency range of the reflected shock is increased by approximately 50 percent, which is in qualitative agreement with the smaller separation bubble size.

INTRODUCTION

The effect of a planar shock impinging on a turbulent boundary layer establishes one of the classic interaction phenomena in compressible viscous flow analysis. This particular form of interaction also has a direct technological relevance to the performance of high speed vehicles, affecting notably, for example, the efficiency of supersonic intakes. Furthermore, maximum mean and fluctuating pressure and thermal loads on a structure are most often found in regions of shock wave boundary layer interaction (SWBLI) and are thus important factors in vehicle development. Flow control is seen as an important issue in future vehicle design (see [1]) to negate these adverse effect. In this context the European sixth framework program UFAST "Unsteady effects in shock wave induced separation" was recently initiated.

In the case of a shock wave turbulent boundary layer interaction, provided a sufficiently strong adverse pressure gradient, the boundary layer separation can occur. The resulting bubble of reversed flow has been observed to pulse, leading to low low frequency oscillations of the reflected shock, cause of the detrimental unsteady fluctuations. One of the open issues is the source of the pulsation [2],[3],[4]. Two principal mechanisms have been proposed, ascribing the unsteadiness either to large scale elongated structures in the upstream boundary layer, or to a mechanism based on the entrainment of mass by the shedding of large scale coherent structures. Various systems have been imagined to reduce the separation bubble with the aim of suppressing the unsteadiness. The most sophisticated concern the dynamic control with a feedback loop. A more crude solution consists of static systems based on the generation of vortices near the wall upstream of the interaction. One can think of sub-vortex generators, that consist of mechanical systems fixed at the wall. Another example is based on the injection of fluid by means of continuous Air Jet Vortex Generators in upstream boundary layer. This approach is of particular interest due to the potential of integrating flow control with transpiration cooling. Since this case has been studied principally in the transonic or low supersonic flows, the aim of the current work is to examine this problem in the fully supersonic flow domain where only limited data is available.

In the following, the effect of upstream injection by means of continuous Air Jet Vortex Generators (AJVGs) on

a shock wave turbulent boundary layer interaction is experimentally investigated in the case of a supersonic flow with a Mach number of M_e =2.3. The associated flow deflection angle is 9.5°, leading to a significant mean separation bubble.

EXPERIMENTAL ARRANGEMENT AND FLOW CONDITIONS

The experiments have been performed in the S8 Supersonic wind tunnel at the Institut Universitaire des Systèmes Thermiques Industriels (IUSTI) in Marseille. The tunnel has been equipped with a special measurement section to perform the experiments with air jet vortex generator control. The flow conditions and the set-up of the control experiment are detailed in the following.

Flow facility

The wind tunnel is a closed-loop continuously running with a particularly low free-stream turbulence, the Mach number is 2.3, the unit Reynolds number of 5.5 x10⁶ m⁻¹, the stagnation temperature T₀=295 K, and a total pressure of $p_0=0.5$ atm. At the inlet conditions for the interaction corresponds a Reynolds number based on momentum thickness of approximately Re₀=5,000, a friction coefficient of C_f=2.1 x 10⁻³, and a boundary layer thickness of $\delta_0=10$ mm. An extensive description of the flow facility can be found in [5].

Control experiment set-up

A row of Air Jet Vortex Generators (AJVGs) was be placed upstream of the interaction to study the effect of upstream disturbances on the mean and unsteady flow characteristics for the control of a SWBLI. The vortex generators consist of a row of ten holes, with a spanwise pitch of about one boundary layer thickness. The diameter of the holes is ϕ =0.8mm (ϕ < δ /10). The row is perpendicular to the flow. The axis of the holes is inclined within the spanwise-wall-normal-plane under an angle of ψ =45°. The AJVG array has been located at around 5 δ upstream of the zone of reflected shock oscillations. A settling chamber is installed underneath the complete array of AJVGs to assure a homogeneous and stable air injection. The temperature in the chamber is around the stagnation temperature of the channel flow. It was verified that the pressure spectrum in the chamber filled with the porous medium does not presents any resonant peaks. The stagnation pressure in the chamber is chosen at P_{0jets}=0.4 bar, close to the stagnation pressure of the tunnel (p₀=0.5 atm).

The injected airflow was found to negligible as compared to the mass flow deficit of the boundary layer: for an injection pressure of P_{0jets} =0.4 bar, and considering the row of ten injectors over a span of ΔZ =100 mm, given the boundary displacement thickness of δ^* = 3mm, the ratio of the jets mass flow to the boundary mass flow deficit is:

$$\frac{Jet \ flux}{\rho_e U_e \delta^* \Delta Z} \approx 3\%$$

Flow diagnostics methods

For the hot wire anemometry measurements, the constant temperature system 'Streamline Dantec CTA' was used in balanced bridge mode. The diameter of the hot wire is 5 μ m and overheat ratio was 0.6. The data are sampling with a National Instrument recorder NI6133 using approximately 2x10⁶ samples.

The PIV investigation was made using a Dantec Dynamics system and software. The light sheets are generated by a double pulse ND:YAG laser New wave Solo II, which delivers 30mJ per pulse, with a pulse delay set in the range of 1-2 μ s. The light sheet thickness is 1mm. Incense smoke was used to seed the boundary layer. The particles were injected from the wall, upstream of the wind tunnel nozzle. The time constant of the particles was estimated to be 4.5 μ s, corresponding to a diameter of 0.5 μ m, [6]. The particle images are recorded by Flowsense 10-bit cameras with a CDD size of 1600x1200 pixels, equipped with Nikon Macro Nikkor f=60mm f/2.8 objectives with the diaphragm set to f_#=2.8. The acquisitions were made using Flowmanager 4.71 software via the Dantec Flowmap System Hub. A peculiarity of this system is an internal storage, and therefore long data acquisitions at high rate are possible (12Hz using the two cameras in half frame mode). A maximum of 10,000 image pairs were acquired with two cameras (5000 per camera). The

images were processed with DynamicStudio2.00, statistics and post-processing were done with in-house Matlab routines.

General description of the flow

The flow topology is depicted in figure 1, showing a Schlieren visualisation of the interaction with and without control. As can be observed, the fully turbulent boundary layer which develops on the tunnel floor is subjected to a shock wave produced by a full-span sharp edge plate placed in the external flow. The imposed flow deflection angle is 9.5°, corresponding to a well developed separation. The baseline interaction has been extensively documented in literature [5],[7],[8]. As can be observed, the boundary layer is first perturbed by the AJVG array, which is located at the source of the weak shock-expansion system located upstream of the interaction. More downstream, the incident shock wave impacts on the boundary layer, causing the boundary layer to thicken and to separate. The jets cause a thickening of the reflected shock, indicative of either an increased unsteadiness (shock excursion amplitude) or an increase in three-dimensionality (due to spanwise rippling). As can be observed, the interaction length (distance at the wall between the extrapolated incident and reflected shock) is not significantly affected.



Figure 1: Schlieren visualisation of the interaction; left: baseline interaction without AJVGs; right: modified interaction with AJVGs.

The associated mean streamwise velocity is presented on the figure 2. The flow is from left to right, showing the undisturbed incoming boundary layer on the left hand side of the domain of interest. As can be seen, the boundary layer is perturbed by the jet array at X=212.5mm. The boundary layer thickens, but without a change is free-stream velocity, indicating that the aforementioned shock-expansion system is of weak strength. The reflected shock foot is located at approximately X=270mm, where the flow is lifted away from the wall and a separation bubble appears. The solid black contour line indicates the contour of zero velocity. The dashed contour represents the extent of the zero velocity contour for the undisturbed case. The dashed line indicates the extrapolated incident shock, impacting at X=337mm. As can be observed, the jets significantly decrease the separation bubble size. In the following sections, the effect of the jets will be quantified in more detail.



Figure 2: Mean longitudinal velocity component (m/s) in the streamwise-wall normal plane with the minimum interaction length (Z=-2.5 mm), AJVGs on.

MODIFICATION OF THE UPSTREAM BOUNDARY LAYER BY AIR INJECTION

To visualise the effect of the jets on the incoming boundary layer topology, three-component PIVmeasurements have been made in the horizontal plane at four heights, allowing the reconstruction of the mean three dimensional flow field encompassing the complete domain of interest from the incoming boundary layer up to reattachment. For this data volume, the angular velocity around the local velocity vector has been computed. Figure 3 shows the resulting iso-surfaces for values of $-5x10^3$ rad/s and $5x10^3$ rad/s superimposed on a contour map of the streamwise mean velocity component at Y=1mm.



Figure 3: Iso-surfaces of the angular velocity (blue: α =-5x10³rad/s and cyan: α =5x10³rad/s), superimposed on contour of the mean longitudinal velocity at Y=1 mm.



Figure 4: Iso-surfaces of angular velocity, jets (blue: α =-5x10³rad/s and cyan: α =1x10³rad/s). Black iso-surface represents longitudinal velocity iso-contours of U=350 m/s (low speed fluid). Contours indicate longitudinal velocities at Y=1mm in m/s, as indicated by the colour bar on the right. Red arrows represent the jet location and injection direction.

As a first observation it is noted that the jets induce a spanwise asymmetry, skewing the flow with a small angle of approximately 2.8° with respect to the tunnel axis. Secondly the flow is modulated in the spanwise direction. Pairs of counter-rotating longitudinal vortices, which are induced by each jets, are at the origin of this spanwise modulation. The angular velocity shows (in blue) the main vortices produced by the AJVGs, having negative angular velocity values (turning counter-clock-wise (*ccw*) when looking downstream along the coordinate axis). Also visible are (in cyan) small secondary vortex tubes with a positive angular velocity, which turn clock-wise (*cw*) when looking downstream. These correspond to small secondary vortices generated between the jets and the wall, below the jets (the main vortices are generated between the jets and the outer flow, above the jets).

A zoom of the topology of these vortex is presented in figure 4. As can be observed from this figure, the mean velocity in between the jets is increased from U=350 m/s to U=380 m/s. Since the velocity increase is directly associated to the two vortices, it seems to be an induced effect of the rotation of the longitudinal vortex pairs, which transport fluid from higher up in the boundary layer towards the wall. At the same time, the mean velocity behind each jet is reduced, most likely as a result of the transport of low speed fluid away from the wall by the vortices, in combination with the generation of a wake by the jets themselves.



Figure 5: Schematic representation of the longitudinal vortices generated by the AJVGs, viewed in upstream direction. The green arrow represents the jet.

From the preceding plots, the following vortex structure can be intuited, as illustrated schematically in figure 5 (looking in upstream direction with the negative spanwise coordinate pointing left). An estimate of the angular velocity for the large *ccw* vortex at mid-distance between jets and shock-foot can be obtained as follows:

• diameter:	D = 6mm
 mean out of plane velocity: 	W = 10m/s;
 horizontal velocity component: 	V ₁ = 30m/s (for Z = 1mm); V ₄ = 10m/s (for Z = 4mm)
 distance from jet to interaction: 	L = 50mm
 longitudinal velocity: 	$U_1 = 350$ m/s (for Z = 1 mm); $U_4 = 430$ m/s (for Z = 4 mm)
rotation rate: =	$\alpha = (V4-V1)/(2\pi D) = 40/(2\pi x6x10^{-3})$
	= 1000Hz = 6.7x10 ³ rad/s

This value is in good agreement with the values for the iso-contours in figure 4. Given the travel time from jets to interaction, which is given by:

$$\tau$$
= 2L/(U₄+U₁) = 2x50x10⁻³ /(350+430) = 128µs

the following number of rotations executed by the large *ccw* from its generation until the interaction is obtained: $\alpha \tau$ = 0.13

Performing the same estimation just behind the jet, where the out of plane velocities are stronger, leads to a value of $\alpha \tau = 0.31$. So the total number of rotations may be expected to be around 1/4, certainly less than 1. This means that the mixing induced by the rotation of the longitudinal vortices is limited. The obtained mean

longitudinal velocity profiles at X=260 mm, just upstream of the reflected shock foot, are visualised in figures 6. Shown are the profile for the reference case without jets (L_{ref} , shown in black) and two profiles with jets (shown in blue). In accordance with the spanwise modulation of the flow, the two profiles with jets represent the two extremes of the AJVG effectiveness: L_{min} corresponds to the fullest profile, leading to the smallest local separation length, and L_{max} represents the profile with the largest velocity deficit, inducing the largest local separation length for the case with jets. It is remarked that all profiles are self-similar in the outer part of the boundary layer (y/ δ >0.8). It is noted that the jets may cause a slight increase in boundary layer height, but the increase falls within the measurement error. The increase in velocity observed in figure 4 corresponds well to the increase in fullness of the boundary profile for L_{min} .



Figure 6: longitudinal velocity profiles in the incoming boundary layer at X=260 mm; black: reference profile without jets; blue: profiles with AJVGs control.

Using the rotation rate above and an estimated radius of 3 mm for the *ccw* vortex, it is estimated that the induced vertical displacement caused by the vortex rotation is 2 mm (δ =0.2). Considering the reference boundary layer profile, such a displacement can indeed be held responsible for the change in fullness of the profiles with AJVGs and hence the modulation of the mean longitudinal velocity at Y=1 mm observed in figure 4. This seems to confirm the mechanism proposed in figure 5.

MODIFICATION OF SEPARATION

The effect of injection on the mean flow topology has been investigated. The mean streamwise velocity component in the wall parallel plane at Y=1 mm is shown in figure 7 for the case with or without control jets. The solid black contour lines in this figure indicate the streamwise velocity for 200 m/s (taken as indicative for the extrapolated reflected shock foot location) and for 0 m/s (representing the detachment line and the reattachment line and hence the extent of the separation bubble. The dashed lines indicate for reference the respective contours for the case without jets. It was found that the 200 m/s velocity contour at the reflected shock foot location becomes rippled by the jets, but that its mean spanwise position is only mildly affected, being pushed only slightly downstream as compared to the baseline interaction. This is in accordance with the thickening of the reflected shock observed in figure 1.

Considering the separation bubble, it is clear that the separation line becomes highly corrugated as well in the injection case. This effect is more pronounced than the corrugation of the reflected shock. The reattachment line is displaced upstream with respect to the undisturbed case, but it shows no signs of corrugation. Hence the effect of the jets is to decrease the separation length at each spanwise location.

As a general remark, it is observed that although clear traces of AJVG induced longitudinal vortices exist upstream of the separation bubble, no trace of such vortices is found downstream of the interaction: the reattachment line is uncorrugated, and no sign of the vortex-patterns is visible downstream of the

reattachment. So either the longitudinal vortices are lifted over the interaction by the separation bubble and do not reappear at a height of 1 mm, or they are destroyed by the unsteady processes occurring in the interaction region.



Figure 7: Mean longitudinal velocity component (m/s) with AJVG control, Y=1 mm.

As was shown in the previous section, the AJVGs appear to induce longitudinal vortices that entrain high speed fluid from higher up in the boundary layer. This fluid slightly displaces the reflected shock foot downstream and reduces the separation length. The effect on the separation line is more pronounced than the effect on the reflected shock. To quantify this effect, figure 8 shows the velocity distribution at Y=1 mm for L_{ref} , L_{min} and L_{max} .



Figure 8: Longitudinal mean velocity profiles at Y=1 mm; the vertical blue lines indicate the separation point and the reattachment point for the cases with and without AJVG control.

As can be observed, the reattachment point with AJVGs is moved upstream as compared to the reference case. Furthermore, the separation point is moved downstream for Lmin, while it is identical for Lmax and Lref. Hence, the separation length for L_{min} is significantly smaller than for L_{ref}, while L_{max} is only slightly smaller than L_{ref}. Downstream of the interaction, all cases attain the same mean velocity, and the effect of the jets hence disappears completely. The dip in the velocity in the upstream boundary layer indicates the location of the jets. As can be seen, the velocity for L_{min} increases slightly between the jets and the separation region, while the velocity for L_{max} decreases. This is due to the slight skewing of the flow by the action of the jets, as observed previously, while the velocity distributions have been obtained in planes parallel to the tunnel axis. In addition to reducing the separation length, the AJVGs also reduce the separation bubble height, as has been observed in figure 2, with the largest reduction corresponding to the smallest separation length. The jets cause an overall decrease in separation length and an accompanying decrease in maximum reverse flow velocity. Concerning the modulation of the boundary layer profile, it has been found that a decrease in friction coefficient for the jet 'wake' (L_{max}) is linked to a larger separation length. On the contrary, an increase of both quantities for the fullest profile in between the longitudinal vortices induced by the jets leads to a smaller separation length (L_{min}). The inverse effect has been observed for the shape factor, which is largest for the slim profile and smallest for the full profile. It seems therefore that the most important effect of the AJVGs is to modify the integral boundary layer parameters. There is a direct link between a reduction in separation length and the shape factor and the friction coefficient. This link is not obvious when comparing the reference profile without jets to the two profiles with jets. This might be due to three-dimensional effects. Overall, the action of the jets is to reduce the bubble size.



Figure 9: Shock position and frequency measurements (circles: AJVGs off, asterisks: AJVGs on); left: RMS of the HWA-signal for different positions with and without AJVG control; right: Pre-multiplied HWA spectrum for the reflected shock with and without jets.

MODIFICATION OF THE SHOCK DYNAMICS

The intermittency in the shock position had been detected in the free-stream using hot wire. The RMS values of the HWA-signal induced by the passage of the shock are presented in figure 9 (left side) for different longitudinal positions with and without AJVG control. The maximum value of the RMS can be associated with the median shock location. A downstream shift in this location can be observed for the AJVG control case. This confirm the fact that the interaction length is slightly reduced with AJVG control; However, the shock excursion amplitude L_{ex} (indicated by the width of the peak) is not significantly altered. The observed thickening of the shock in figure 1 can therefore not be attributed to an increased shock excursion length.

The shock frequency range had been detected in the free-stream using a hot wire positioned on the median position of the separation shock. The resulting spectra of the HWA signal for the cases with and without injection is shown on the right side of figure 9. The spectrum is shown in pre-multiplied form (fxE(f) versus log(f), where f is the frequency) to correctly represent the energy concentration. The zone of maximum of spectral energy is not well defined but we can noticed a significant shift in the energy bump of the spectrum to

higher frequencies when the jets are activated. This is in agreement with a quasi constant Strouhal number for the shock frequency:

$$S_t = \frac{fL}{U}$$

A higher frequency does indeed corresponds to a smaller interaction length and a smaller bubble size, at least in the case of a well developed mean separation bubble [3].

RESULTS RECAPITULATION

Effect of the jets on the boundary layer

The horizontal and vertical plane PIV measurements indicate that the Air Jet Vortex Generators induce the formation of large longitudinal vortices. These vortices entrain high velocity fluid from higher up in the boundary layer and hence induce locally higher velocities closer to the wall. The effect of the jets is only noticeable up to $y/\delta=0.8$. Above this height the boundary layer seems not or only slightly affected. In the 'wake' of the jets, the boundary layer profile fullness is significantly lower than the reference boundary layer. At the same time, the boundary layer profile in between the jets is fuller than the reference boundary layer. This coincides with a decrease in friction coefficient and friction velocity for the jet 'wake' and a respective increase of both quantities for the fullest profile in between the jet 'wakes'. The inverse effect is induced on the shape factor, which increases for the slim profile and decreases for the full profile. Moreover the jets also induce a slight skewing of the flow in the upstream boundary layer, deflecting the flow sideways by approximately 2.8° at 1mm height from the wall.

Effect on the separation

The effect of the jets is more pronounced on the separation bubble, the modifications of the incoming boundary layer leads to a global three-dimensionalisation. The overall separation length is reduced since the mean separation line is moved downstream while the reattachment is moved upstream. The corrugation of the separation line is more significant than the corrugation of the reflected shock. The most upstream separation point with AJVGs corresponds to most upstream point for the reference interaction. The reattachment line is not corrugated and no trace of the jets is observed after reattachment. The reattachment location is independent of the separation location.

Beyond these global properties, the following conclusion can be made on the effect of the spanwise modulation induced by the jets. The fullest boundary layer profile corresponds to the smallest separation length and the most downstream reflected shock position; the slimmest profile (the jet wake) corresponds to the largest separation length and the most upstream reflected shock position. The height of the separation bubble is also reduced, with the largest reduction corresponding to the smallest separation length. The most important effect of the AJVGs seems to be due to the integral boundary layer parameters and the mean velocity profile. A reduction in separation length is directly linked to the shape factor and the friction coefficient.

Effect of the jets on the reflected shock

The jets cause an overall corrugation of the reflected shock over its full height. This effect is observed as a thickening of the shock in the Schlieren images, and an undulation of the shock foot in the horizontal plane PIV measurements. The interaction length (distance at the wall of the extrapolated shocks) is only mildly affected, and no increase in shock excursion amplitude has been observed. The HWA results show an increase in reflected shock frequency caused by the AJVGs, in combination with a downstream displacement of the shock. This is in agreement with the properties of the Strouhal number for the shock frequency: a higher frequency corresponds to a smaller interaction length and a smaller bubble size, at least in the case of a well developed mean separation bubble

CONCLUSIONS

The results indicate that Air Jet Vortex Generators (AJVGs), which are inclined under 45% with respect to the wall and blow in transverse direction, generate two longitudinal counter rotative vortices per air jet. The vortices are of unequal strength, with the stronger vortex located above jet and the weaker vortex between the jet and the wall. The rotation rate of these vortices is small, notwithstanding the large injection pressure and a significant modification of the upstream boundary layer structure. Consequently, only a limited amount of mixing should be expected. However, it has been shown that the angular displacement induced by the flow is sufficiently large to entrain high speed flow towards to wall, hence increasing the fullness of the boundary layer profile. At the same time, the opposite effect is observed in the wake of the jets, where the fullness is decreased. No significant effect has been observed of the AJVGs on the shock excursion amplitude and position. However, the AJVGs do reduce the separation bubble size without suppressing it. As a direct consequence of the reduction in bubble size, the shock frequency is increased by 50 percent.

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