MATHEMATICAL MODEL FOR ELECTRICAL POWERPLANT OF ADVANCED AIRCRAFT

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ABSTRACT

Mathematical models convenient for analytical investigation for all the components of electrical powerplant were formed on the basis of experimental results and physical principles of their functioning. Basing on the relationships proposed a set of useful results was obtained: angular characteristics of solar battery, efficiency of motor with friction as function of motor characteristics and engine behavior. The model of the powerplant as a whole with the control through PWM (pulse width modulation) was formed and investigated. Particularly, with the help of this model optimal design parameters for the propeller and electrical drive can be obtained for the required airplane and flight conditions. On the basis of the formulas obtained, the expressions for the efficiency of the whole powerplant as function of the thrust and velocity were derived. The comparisons of real results with the mathematical model proposed were made.

1. INTRODUCTION

Now there exists a set of aircraft types with electrical powerplant. One of them is High Altitude Long Endurance (HALE) aircraft such as “Helios”. Another type is Micro Air Vehicles (MAVs). At last, now there have appeared small piloted airplanes with an electrical drive and propeller.

An ordinary electrical powerplant consists of electrical drive, propeller, accumulator or some other energy storage, electrical devices for motor control (the so called speed controllers) and, in some cases, gear box between the motor and propeller. In case of multi-day mission solar cells are also onboard.

For the current technologies the energy density (ED) of accumulators is about 100 times lower than the energy density of “traditional” fuels (kerosene, petrol). Even though ED of fuel cells is higher than for accumulators, it is not comparable with petrol. On the other hand, the efficiencies of piston engines and jet engines are lower than for the electrical drive. Besides, electrical drive does not pollute the environment. But for electrical powerplant to be comparable with traditional powerplants it is necessary to maximize the efficiency of all the electrical powerplant components and powerplant as a whole.

For the cruise conditions in a set of cases it is rather simple to choose the parameters of the powerplant for the maximal efficiency. For most cases the efficiencies of the drive and propeller are chosen so that they are near to their maximums on this regime. But it is also important to define the powerplant characteristics for all the flight conditions. This requires analytical formulas or great number of experimental investigations.

The first way seems to be more preferable. Here, it is required to analyze the functioning of all the components and powerplant as a whole.

2. PROPELLER MODEL

It is well known that for the region of the so-called self-similarity ($Re>10^6$) it is very convenient to use dimensionless propeller characteristics: advanced ratio λ, thrust coefficient α and power coefficient β defined as [1]

$$\lambda = \frac{V}{nD}, \ \alpha = \frac{F}{\rho n^2 D^4}, \ \beta = \frac{W_P}{\rho n^3 D^5}, \ \eta_P = \frac{\alpha\lambda}{\beta} \quad (1)$$

where V is the velocity of the stream far from the propeller, n – rotational frequency, D – propeller diameter, F – propeller thrust, ρ – air density, W_P – power consumed by propeller, η_P – propeller efficiency. For the region of self-similarity coefficients α and β depend only on the value of the advanced ratio.

Typical graphs [1] for the thrust coefficient, the power coefficient and the efficiency for the various angles φ of propeller blade inclination are given in Figures 1-2.

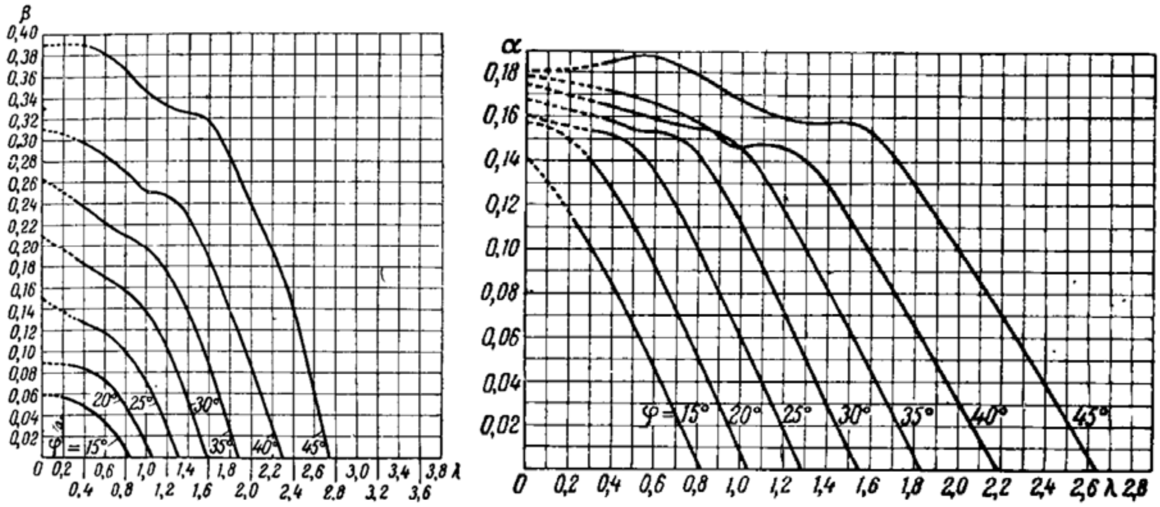

Figure 1. Thrust coefficient α and power coefficient β vs. advanced ratio for various angles of propeller blade inclination [1].

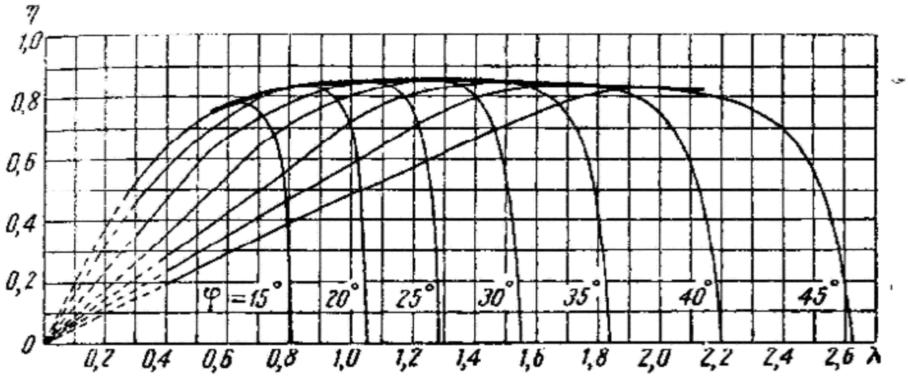

Figure 2. Efficiency vs. advanced ratio for various angles of propeller blade inclination [1].

From this graphs one can see that for the values of λ near the maximal efficiency the thrust coefficient is practically a straight line. Moreover, it remains to be a straight line within the whole range from $\lambda=0$ to $\alpha=0$ for the small values of φ ($\varphi < 20^0$).

Efficiency near its maximum can be assumed as constant, and for small values of λ it looks like a straight line beginning from the zero point.

It should be mentioned that the graphs are also the same for $Re<10^6$ for the propellers with high value of maximal efficiency (see Fig.3 [2]).

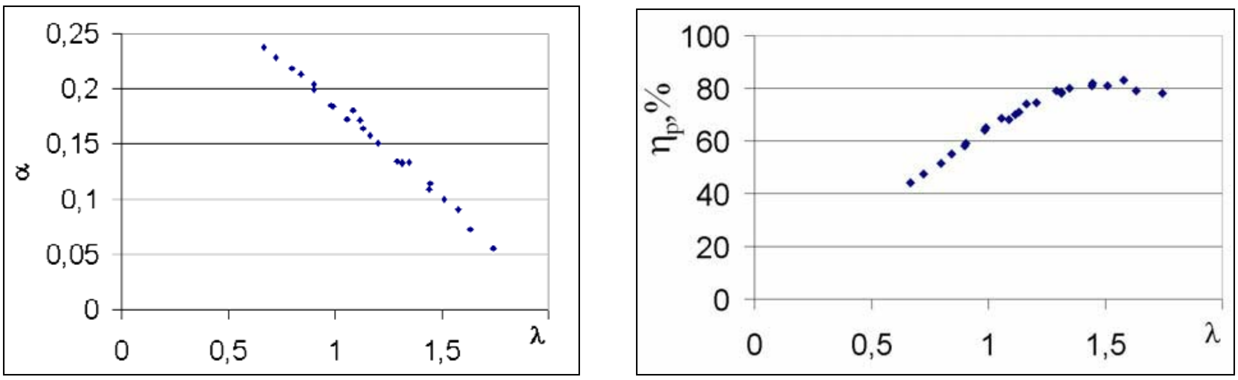

Figure 3. Thrust coefficient α and efficiency η vs. advanced ratio for "Black Widow" propeller [2].

Coefficient β can be obtained from α, η_P, λ. But for $\lambda=0$ the value of β must be defined in another way. From fig. 2, it is possible to assume that near $\lambda=0$ the coefficient β is practically constant.

So, proposed is the following model of the propeller (see fig.4):

1st region: $\lambda<\lambda(\eta_{max})$, $\eta_P = k_0\lambda$, $k_0 = k_0(\varphi)$;

2nd region: $\lambda\approx\lambda(\eta_{max})$, $\eta_P = \eta_{PM} = \text{const}$;

3rd region: $\lambda(\eta_{max}) <\lambda< \lambda(\eta=0)$, $\eta_P=\delta-\gamma\lambda$, $\delta=\delta(\varphi)$, $\gamma=\gamma(\varphi)$.

For all regions

$$\alpha=\alpha_0-\sigma\lambda, \tag{2}$$

where $\alpha_0 = \alpha_0(\varphi)$, σ = const.

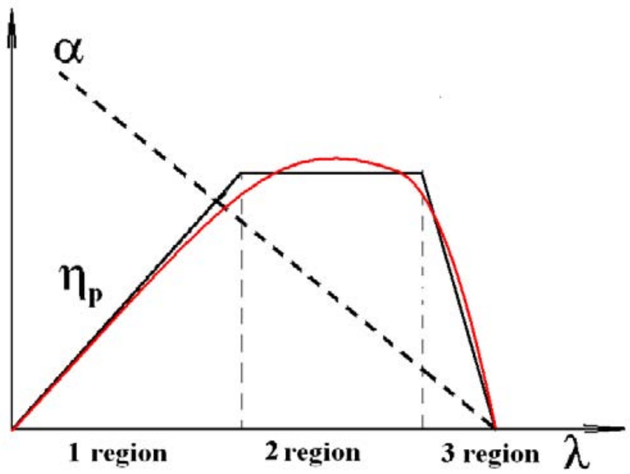

Figure 4. Mathematical model of propeller. Red curve – real efficiency, black polygonal line – model of efficiency, dash line – thrust coefficient.

As we will see later, the analytical solution can be found only for the cases when the efficiency can be expressed by the polynomial not higher than the first order with respect to λ. So, the more complicated models of the efficiency are not proposed.

Also, it should be mentioned that the margins between the regions are taken rather arbitrary. The main requirement is that the lines must be rather close to the real curve.

The thrust required for level flight must be equal to the drag force of airplane. Assuming that viscous drag coefficient C_{D0} is constant one can obtain drag D as the function of the flight velocity V and aircraft weight G as

$$D = C_{D0}\rho\frac{V^2}{2}S + \frac{2G^2}{\rho V^2 L^2},$$

where S is wing area, L – wingspan.

Traditionally, the aircraft is characterized by its cruise velocity and other cruise conditions (for example, altitude and corresponding air density). The propeller is usually designed to work at these conditions with maximal efficiency or efficiency near the maximum with the corresponding advanced ratio λ^*. According to the above expression for drag and formulas (1), the higher velocity corresponds to the higher λ for level flight, the lower velocity corresponds to the lower λ. Also, the aircraft acceleration requires lower λ, deceleration requires higher λ.

According to (1) and (2), one can obtain

$$F = \frac{\rho\sigma}{2\pi}D^3\omega\left(\frac{\alpha_0 D}{2\pi\sigma}\omega - V\right),$$

or

$$F = c\omega(f\omega - V), \tag{3}$$

where

$$c = \frac{\rho\sigma}{2\pi}D^3,\ f = \frac{\alpha_0 D}{2\pi\sigma},\ \omega = 2\pi n.$$

It should be mentioned that c depends only on the blade shape, air density and diameter and is independent of the angle φ of the blade inclination and f depends on φ and is practically independent of the blade shape (only on the diameter).

3. ELECTRICAL DRIVE MODEL

Consider the electrical motor [3]. According to Ohm law

$$E=\Phi\omega + IR, \tag{4}$$

where E – electromotive force of energy source, I – current through the drive, R – motor resistance, $\Phi=BNS$ – flow of magnetic field through the motor windings, B – magnetic field induction, N – number of coils in winding, S – cross-section area of windings.

Torque on the motor shaft is defined as [3]

$$M = \Phi_1 I.$$

Value of Φ_1 is not equal to Φ but rather close to it, so assume $\Phi = \Phi_1$.

Shaft power W_S which must be equal to propeller power in stationary case is defined as

$$W_S = M\omega = BSN\omega I = \Phi\omega I.$$

Using (4) one can obtain

$$W_S = \frac{\Phi^2}{R}\omega\left(\omega_0 - \omega\right), \tag{5}$$

where $\omega_0=E/\Phi$. Total power W_{SUM} is defined as

$$W_{SUM} = EI = I(\Phi\omega+IR) = W_S+I^2R.$$

Efficiency of electrical drive is defined as

$$\eta_D = \frac{W_S}{W_{SUM}} = \frac{\Phi\omega I}{EI} = \frac{\omega}{\omega_0}. \tag{6}$$

In case of energy losses in drive due to friction the shaft power will be smaller by a certain value ΔW. Then, the useful power W_D that can be consumed by propeller is

$$W_D=W_S - \Delta W.$$

Torque due to friction M_F has a constant component (due to dry friction) and a component dependent on the rotational frequency. Experiments show that the second component is small enough comparing to the first one. So, one can assume that in the first approximation the torque due to friction is constant. For this case

$$\Delta W = M_F\omega.$$

It is rather easy to measure the no-load current I_0 of the drive. Then, the last formula can be expressed as $\Delta W = \Phi I_0\omega$, so

$$W_D = \Phi\omega\left(I - I_0\right) = \frac{\Phi^2}{R}\omega\left(\omega_0 - \omega - \frac{I_0 R}{\Phi}\right). \tag{7}$$

The efficiency in this case is

$$\eta_D = \frac{\Phi\omega\left(I - I_0\right)}{EI} = \frac{\Phi\omega(E - \Phi\omega - I_0 R)}{E(E-\Phi\omega)} = \frac{\omega(\omega_0 - \omega - I_0 R/\Phi)}{\omega_0(\omega_0 - \omega)} \tag{8}$$

It should be noted that by the change of variables

$$\omega_1=(\omega_0 - \Delta\omega),$$

where $\Delta\omega=I_0 R/\Phi$, formula (7) will correspond to (5).

In case of no friction in drive the maximal value of the efficiency is equal to 1. In the presence of friction the maximal efficiency is

$$\eta_D = \left(1 - \sqrt{\frac{I_0 R_D}{E}}\right)^2 \tag{9}$$

at

$$\omega = \omega_0\left(1 - \sqrt{\frac{I_0 R_D}{\Phi\omega_0}}\right). \tag{10}$$

Let's evaluate some values. Imagine that the **maximal** efficiency of an electrical drive for a value of ω_0 is 70%. Then the value of $\Delta\omega/\omega_0$ is about 3% and ω/ω_0 corresponding to this maximum is about 83% (for the case of no friction the value ω/ω_0=0.83 corresponds to the drive efficiency of 83%). For ω/ω_0=0.7 and the same $\Delta\omega/\omega_0$ the efficiency is 63%, for ω/ω_0=0.5 the efficiency is 47%, The increase of $\Delta\omega/\omega_0$ gives higher values of the efficiency with respect to the efficiency in no-friction case for the same $\Delta\omega/\omega_0$.

So, the formulas for non-friction case can be applicable at high values of shaft power when W_D is much higher than power losses due to friction. For small values of shaft power one must use the formulas for case with friction.

4. GEARBOX

Sometimes the gearbox is situated between the electrical drive and propeller. The main reason of the gearbox presence is the following: the drives with higher rotational frequencies for the fixed power are lighter. But their working frequencies are higher than those required for the propellers. So, the main reason is to decrease the weight of the powerplant. On the other hand, the gearbox has its own weight and own efficiency. So, the total advantage of the gearbox application must be analyzed in each case.

The main characteristics of gearbox are transmission ratio N (frequency of drive/frequency of propeller) and efficiency.

5. ENERGY SOURCES

Now there exists a set of sources of the electrical energy suitable for airplanes. The most usable are accumulators, fuel cells and solar cells (SC). Accumulators and fuel cells use the “stored” energy, solar cells use energy from the external source.

It is well known that the tension of the accumulator and fuel cell decreases linearly with the current increase, so one can use the concept of internal resistance of the energy source R_0. Then, during the analysis of the powerplant one can assume the internal resistance as zero and the resistance of the drive as $R+R_0$ if the energy source consists only of accumulators.

The performance of the solar cell depends on the intensity of the radiation and cell orientation with respect to the source of radiation.

Preliminary experiments and analysis show that for the solar radiation (nonpolarized radiation) the reflection coefficient remains practically constant in a wide range of light angles. From this, if we assume that the accepted power for SC for transversal incidence is P_0, then for the angle of incidence of θ the accepted power $P(\theta)$ will be

$$P(\theta)=P_0\cos(\theta).$$

The voltage-current characteristic of solar cell is [4]

$$U = B \cdot \ln\left(\frac{I_M - I}{s}\right), \tag{11}$$

where U – voltage of cell, I – current of cell, $B = kT/e$, k – Boltzmann’s constant, T – absolute temperature, e – electron’s current, s is some constant dependent on the concrete cell characteristics, I_m is characteristic dependent on the $P(\theta)$ and the concrete cell characteristics.

The power W_{PV} of solar cell can be expressed as

$$W_{PV} = IB \cdot \ln\left(\frac{I_M - I}{s}\right)$$

The calculations show that near the maximum of W_{PV} one can imagine SC as power source with constant EMF and constant internal resistance (as in case of galvanic cell or accumulator).

As the maximal voltage of SC is of order of 1 Volt, the cells must be connected in series to provide the necessary voltage and in parallel to provide the necessary current. The investigations show that for the best performance of the battery as a whole every cell must work near its maximal power. Then, the number of cells in series and the number of series in parallel connection can be defined through the current and voltage of SC at maximal power and voltage and current required.

It should be mentioned that the intensity of solar radiation changes during the day time. So, the connection of cells is not optimal for every moment of the day. But the analysis of (11) shows that the decrease of $P(\theta)$ changes the voltage corresponding to the maximum of W_{PV} very slightly. The current corresponding to the maximum of W_{PV} decreases nearly proportional to $P(\theta)$.

6. POWERPLANT CONTROL

The powerplant can be controlled by various methods. Some of them are listed below:

1. by changing the propeller’s blade angle;
2. by changing the characteristics of gearbox;

3. by changing the voltage of power source;
4. by changing the resistance;
5. by PWM.

Nowadays, the fifth method is the most popular. Also, it should be mentioned that the effect of PWM from physical point is the same as changing the voltage. So, below we will consider the third method with keeping in mind that we use PWM.

The special device (named as "Speed Controller") is used to make PWM signal for electrical drive. For the further investigations assume that the efficiency of the speed controller in the working range of powerplant remains constant.

7. MODEL OF POWERPLANT AS A WHOLE

Now gearboxes between the propeller and electrical drive are practically unused. So, at the stationary regimes the total power consumed by the propeller is equal to the shaft power of the electrical drive. In other words,

$$W_P = W_D,$$

or

$$FV = W_D \eta_P .$$

Using the relationships obtained above one can find the characteristics of the powerplant for the given value of EMF. In general case

$$c\omega(f\omega - V)V = \eta_P \frac{\Phi^2}{R}\omega(\omega_0 - \omega).$$

As it can be seen one can solve this equation with respect to ω only if the propeller efficiency can be expressed by the polynomial not higher than the first order of λ ($\lambda \sim V/\omega$). So, the analytical solution can be found for 6 cases:

1. No friction in drive. Propeller efficiency is constant (2nd region of propeller efficiency);
2. No friction in drive. Propeller efficiency is proportional to advanced ratio (1st region of propeller efficiency);
3. No friction in drive. Propeller efficiency is linear decreasing function of advanced ratio (3rd region of propeller efficiency);
4. Drive with friction. Propeller efficiency is constant (2nd region of propeller efficiency);
5. Drive with friction. Propeller efficiency is proportional to advanced ratio(1st region of propeller efficiency);
6. Drive with friction. Propeller efficiency is linear decreasing function of advanced ratio (3rd region of propeller efficiency).

So let's analyze these cases.

7.1. No friction in drive. Propeller efficiency is constant

For this case

$$c\omega(f\omega - V)V = \frac{\Phi^2 \eta_{PM}}{R}\omega(\omega_0 - \omega).$$

From this,

$$\omega = \frac{cV^2 + \omega_0 \dfrac{\Phi^2 \eta_{PM}}{R}}{Vcf + \dfrac{\Phi^2 \eta_{PM}}{R}}, \tag{12}$$

and

$$F = c\frac{\Phi^2 \eta_{PM}}{R} \frac{\left(cV^2 + \omega_0 \dfrac{\Phi^2 \eta_{PM}}{R}\right)(f\omega_0 - V)}{\left(Vcf + \dfrac{\Phi^2 \eta_{PM}}{R}\right)^2}. \tag{13}$$

Formula (11) gives the thrust as the function of the stream velocity and EMF.

The efficiency of the powerplant can be expressed as

$$\eta=\eta_P\frac{\omega}{\omega_0}=\eta_{PM}\frac{cV^2+\omega_0\dfrac{\Phi^2\eta_{PM}}{R}}{\left(Vcf+\dfrac{\Phi^2\eta_P}{R}\right)\omega_0}=\eta_{PM}\frac{cV^2}{\left(Vcf+\dfrac{\Phi^2\eta_{PM}}{R}\right)\omega_0}+\eta_P\frac{\dfrac{\Phi^2\eta_{PM}}{R}}{\left(Vcf+\dfrac{\Phi^2\eta_{PM}}{R}\right)}. \tag{14}$$

As the combination of (12) - (14) one can obtain very important relationship between the thrust and the efficiency:

$$F(\eta)=\frac{\Phi^2\eta_{PM}}{R}c^2V^3\frac{\eta(\eta_{PM}-\eta)}{\left(\eta\left(fcV+\dfrac{\Phi^2\eta_{PM}}{R}\right)-\eta_{PM}\dfrac{\Phi^2\eta_{PM}}{R}\right)^2}. \tag{15}$$

Analysis of (12)-(15) shows that the efficiency decreases with the thrust increase for this region.

7.2. No friction in drive. Propeller efficiency is proportional to advanced ratio

As in previous case, $FV=W\eta_P$. Taking into account that

$$\eta_P=k_0\lambda=kV/\omega$$

one can write

$$c\omega(f\omega-V)V=k\frac{V}{\omega}\frac{\Phi^2}{R}\omega(\omega_0-\omega).$$

The solutions of this equation are

$$\omega=\frac{cV-k\dfrac{\Phi^2}{R}\pm\sqrt{\left(cV-k\dfrac{\Phi^2}{R}\right)^2+4cfk\dfrac{\Phi^2}{R}\omega_0}}{2cf}.$$

As ω must be higher than zero, then

$$\omega=\frac{cV-k\dfrac{\Phi^2}{R}+\sqrt{\left(cV-k\dfrac{\Phi^2}{R}\right)^2+4cfk\dfrac{\Phi^2}{R}\omega_0}}{2cf}, \tag{16}$$

and

$$F=\frac{1}{2cf}k\frac{\Phi^2}{R}\left[k\frac{\Phi^2}{R}+c(2f\omega_0-V)-\sqrt{\left(cV-k\frac{\Phi^2}{R}\right)^2+4cfk\frac{\Phi^2}{R}\omega_0}\right]. \tag{17}$$

One can see that F is monotonically increasing with ω_0 increase for fixed V.

The total efficiency can be expressed as

$$\eta=\eta_B\frac{\omega}{\omega_0}=k\frac{V}{\omega}\frac{\omega}{\omega_0}=k\frac{V}{\omega_0}. \tag{18}$$

The total efficiency monotonically decreases with ω_0 decrease for the fixed V. So, η monotonically decreases with F increase.

7.3. No friction in drive. Propeller efficiency is linear decreasing function of advanced ratio

First of all, it should be noted that the thrust and the efficiency are equal to zero at the same point. So, one can express the efficiency in the 3rd region as

$$\eta=\psi\left(f-\frac{V}{\omega}\right),$$

where ψ is a certain constant. The rotational frequency for this case can be expressed as

$$\omega=\frac{\psi\Phi^2}{cRV+\psi\Phi^2}\omega_0. \tag{19}$$

So, ω is proportional to PWM. As the thrust (for positive thrust) increases monotonically with ω increase, it monotonically increases with PWM.

Thrust can be expressed as

$$F = c\omega_0\left(\frac{\psi\Phi^2}{cRV+\psi\Phi^2}\right)\left(f\frac{\psi\Phi^2\omega_0}{cRV+\psi\Phi^2}-V\right). \tag{20}$$

The total efficiency of the powerplant can be expressed as

$$\eta = \frac{\omega}{\omega_0}\psi\left(f-\frac{V}{\omega}\right)=\psi\frac{f\omega-V}{\omega_0}=\psi\left(f\frac{\psi\Phi^2}{cR+\psi\Phi^2}-\frac{V}{\omega_0}\right). \tag{21}$$

So, the efficiency increases with the thrust increase. As it was shown above, for 1st and 2nd regions the efficiency decreases with the thrust increase. So, the margin between 2nd and 3rd regions is the point of the maximal total efficiency of the powerplant.

But it is only for the case of the absence of friction in drive. To obtain a more precise result we must take the friction into account. It is rather evident that the maximum cannot shift to 3rd region, but it can shift to 2nd or, maybe, to the 1st region.

7.4. Drive with friction. Propeller efficiency is constant

As in previous cases,

$$FV=W\eta_{PM}$$

gives

$$\omega = \frac{cV^2+(\omega_0-\Delta\omega)\dfrac{\Phi^2\eta_{PM}}{R}}{Vcf+\dfrac{\Phi^2\eta_{PM}}{R}}, \tag{22}$$

$$F = c\frac{\Phi^2\eta_{PM}}{R}\frac{\left(cV^2+(\omega_0-\Delta\omega)\dfrac{\Phi^2\eta_{PM}}{R}\right)\left(f(\omega_0-\Delta\omega)-V\right)}{\left(Vcf+\dfrac{\Phi^2\eta_{PM}}{R}\right)^2}, \tag{23}$$

and

$$\eta = \eta_{PM}\frac{\left(\dfrac{\Phi^2\eta_{PM}}{R}(\omega_0-\Delta\omega)+cV^2\right)\left((\omega_0-\Delta\omega)cVf-cV^2\right)}{\omega_0\left(cVf\omega_0+\dfrac{\Phi^2\eta_{PM}}{R}\Delta\omega-cV^2\right)\left(cVf+\dfrac{\Phi^2\eta_{PM}}{R}\right)}. \tag{24}$$

It is rather interesting to find if the function $\eta(F)$ has the maximum for this task. So

$$\frac{d\eta}{dF}=\frac{d\eta}{d\omega_0}:\frac{dF}{d\omega_0}=0.$$

The analysis of (22) and (23) shows that the derivative of $\eta(F)$ is equal to zero only if the derivative of $\eta(\omega_0)$ is equal to zero. The corresponding ω_0 can be expressed analytically but is rather complicated.

One can expect that this value corresponds to (10) and the maximal efficiency corresponds to (9) with taking into account (22). The analysis shows that the values of ω_0 for these cases are rather close to each other but do not coincide in the general case. It can be explained by the fact that in the general case the value of ω corresponding to ω_0 in accordance with (22) does not coincide with ω obtained by (10). But as it is mentioned above, the values of ω_0 are close to each other and one can use any of them to find the maximum in the first approximation.

Also it should be mentioned that ω corresponding to the maximum of the powerplant efficiency can be outside of the 2nd region.

7.5. Drive with friction. Propeller efficiency is proportional to advanced ratio

For this case

$$\omega = \frac{cV - k\dfrac{\Phi^2}{R} + \sqrt{\left(cV - k\dfrac{\Phi^2}{R}\right)^2 + 4cfk\dfrac{\Phi^2}{R}\omega_1}}{2cf}\ , \qquad (25)$$

$$F = \frac{1}{2cf}k\frac{\Phi^2}{R}\left[k\frac{\Phi^2}{R} + c\left(2f\omega_1 - V\right) - \sqrt{\left(cV - k\frac{\Phi^2}{R}\right)^2 + 4cfk\frac{\Phi^2}{R}\omega_1}\right], \qquad (26)$$

$$\eta = \eta_B\frac{\omega\left(\omega_1 - \omega\right)}{\omega_0\left(\omega_0 - \omega\right)} = k\frac{V}{\omega}\frac{\omega\left(\omega_1 - \omega\right)}{\omega_0\left(\omega_0 - \omega\right)} = k\frac{V}{\omega_0}\frac{\left(\omega_1 - \omega\right)}{\left(\omega_0 - \omega\right)}\,. \qquad (27)$$

For small values of $\Delta\omega/\omega_0$ there will be no maximums of the function $\eta(F)$.

7.6. Drive with friction. Propeller efficiency is linear decreasing function of advanced ratio

As in chapter 7.3

$$\eta = \psi\left(f - \frac{V}{\omega}\right).$$

The rotational frequency for this case can be expressed as

$$\omega = \frac{\psi\Phi^2}{cRV + \psi\Phi^2}\left(\omega_0 - \Delta\omega\right) = \frac{\psi\Phi^2}{cRV + \psi\Phi^2}\omega_1\,. \qquad (28)$$

The thrust can be expressed as

$$F = c\left(\omega_0 - \Delta\omega\right)\left(\frac{\psi\Phi^2}{cRV + \psi\Phi^2}\right)\left(f\frac{\psi\Phi^2\left(\omega_0 - \Delta\omega\right)}{cRV + \psi\Phi^2} - V\right). \qquad (29)$$

The total efficiency of powerplant can be expressed as

$$\eta = \frac{f\psi^2\Phi^2 cRV\left(\omega_0 - \Delta\omega\right)}{\omega_0\left(cRV + \psi\Phi^2\right)\left(\omega_0 cRV - \Delta\omega\psi\Phi^2\right)}\,. \qquad (30)$$

The efficiency for this case decreases with the trust decrease.

8. HOVER PHASE

Now for MAV the ability not only to fly but also to hover for some time is investigated. As it was shown above, the coefficient β for $\lambda\approx 0$ is practically constant. So,

$$\frac{\beta\rho\omega^3 D^5}{\left(2\pi\right)^3} = \frac{\Phi^2}{R}\omega\left(\omega_0 - \omega\right).$$

So,

$$\omega = \frac{\left(2\pi\right)^3\Phi^2}{R\beta\rho D^5}\left(-1 + \sqrt{1 + 4\frac{R\beta\rho D^5}{\left(2\pi\right)^3\Phi^2}\omega_0}\right). \qquad (31)$$

The thrust and the power can be expressed from formulas (1).

9. RESULTS ANALYSIS

Formulas (12)–(31) allow to obtain the main characteristics for the powerplant for any set of devices characteristics (c, f, R, Φ, I_0), flight velocity and control ω_0. But they are the most useful for the analysis of thrust and efficiency changes with the flight conditions and powerplant characteristics change. Also they give the possibility to analyze the powerplant with the propeller of the changeable angle of blades inclination.

Preliminary calculations show that for the velocity that corresponds to the global maximum of total efficiency all the models proposed are rather close to the real efficiency within their regions. No-friction models are not good enough only near the margin between 2nd and 3rd regions. Also, the model with friction for 2nd region works rather good also in 3rd region. So, for the first steps of powerplant investigation one can use more simple models or smaller number of models.

As it was seen above, for the first region of the propeller efficiency the total efficiency of the powerplant decreases with the thrust increase, for the third region the total efficiency increases with the thrust increase for the constant flight velocity. So, the maximum of the efficiency with respect to the thrust can be found in the second region.

It is evident that the greatest errors in this model correspond to the greatest differences between the models and the real dependencies. More complex model of the propeller efficiency consisting of more number of lines can be proposed, but it will be more complex for the analysis.

Also it should be mentioned one more time that we assume that the efficiency of the speed controller is constant. So, the real data will differ slightly from the data from the model proposed.

10. CONCLUSIONS

1. Models of all the powerplant elements have been formed and analyzed.
2. Characteristic features of elements have been discovered with the help of these models.
3. On the basis of the models of the powerplant elements the models of the powerplant as whole have been formed.
4. Methods of powerplant control have been discussed. Pulse Width Modulation method have been chosen for the investigation.
5. Models of whole powerplant were analyzed. Efficiency models have been compared with real efficiency.

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