

# IMPROVED FAST COMPUTATIONAL METHOD INSURING DESIGN OFFICE REQUIREMENTS FOR STRENGTH ANALYSIS OF COMPOSITE STRUCTURES

F. Laurin<sup>1</sup>, N. Carrère<sup>1</sup>, J.-F. Maire<sup>1</sup> et S. Mahdi<sup>2</sup>

1 : ONERA/DMSC

ONERA 29 avenue de la division Leclerc, 92322 Châtillon Cedex France

e-mail : [laurin@onera.fr](mailto:laurin@onera.fr), [carrere@onera.fr](mailto:carrere@onera.fr), [maire@onera.fr](mailto:maire@onera.fr)

2 : AIRBUS/EDASMT

AIRBUS France, 316, route de Bayonne - 31060 Toulouse Cedex 9

e-mail : [stephane.mahdi@airbus.com](mailto:stephane.mahdi@airbus.com)

## ABSTRACT

The use of unidirectional carbon fibre-reinforced composites in the design of primary structures, such as the centre wing box, has spread increasingly over the past few years. However, composite structures can be weakened by the introduction of geometrical singularities, such as holes or notches. The semi-empirical aspect of the current open-hole failure approaches requires the allowable to be systematically fitted against specific test results. This point constitutes a strong limitation for optimal design. A simplified strength analysis method for perforated plates, insuring the requirements of precision and computational time of the design office, is presented. The predictions of the proposed approach are compared successfully with a large experimental data basis, with different configurations of perforations, different stacking sequences and on different Carbon/Epoxy materials.

**Keywords:** Failure, Damage, Multiscale, Open-hole plates.

## INTRODUCTION

Composite materials are being introduced in primary structures in order to answer to the request of aeronautical companies for lighter, safer and less polluting civil aircraft. The design of components, such as the centre wing box, the fuselage or the wings, that ensure the structural integrity of the aircraft, necessitates a high degree of confidence into the current strength analysis methods. However, composite structures can be weakened by the introduction of holes, notches or cut-outs and the strength analysis of high stress gradient parts of the structures, such as in the vicinity of open-hole in perforated plates, still remains a key problem in the design of engineering structures.

From an academic point of view, many advanced strength analysis methods for perforated composite plates can be found in the scientific literature. Some are based on damage modelling to predict the intralaminar failure associated with cohesive zone modelling for delamination [1]. Some use cohesive zone elements to model the most probable cracks [2] (intra and interlaminar damage), or mesh explicitly all the possible cracks [3], and are based on linear fracture mechanics. However, these advanced approaches are too complex to be used in a design office, and the associated computational times remain prohibitive to design aeronautical structures.

From an industrial point of view, some simplified strength analysis methods, specific for the prediction of failure for perforated plates such as the point stress method [4], are widely used. However, the semi-empirical aspect of the current open-hole failure approaches requires the allowables to be systematically fitted against large test campaigns.

The aim of this study is to propose an alternative strength analysis method for perforated plates, based on more physical approaches, but matching the requirements of design office. Therefore, the present approach has to be a fast computational method and easy to carry out (easy to identify, to compute and to analyse the results).

The proposed simplified method for the strength analysis of perforated plates is first presented and decomposed in three main points: (i) the estimation of the membrane loadings within the perforated structures, (ii) the determination of the fracture behaviour of the different plies constituting the laminate, and finally (iii) the prediction of the ultimate rupture of the composite structures. Then, the predictions are compared with uniaxial tension tests performed on perforated laminated plates with different stacking sequences, different ratios width of the plate by the diameter of the hole (noted  $w/d$  ratio), different diameters of perforation with a fixed  $w/d$  ratio, and with off-axis tension tests, and finally with uniaxial compression tests on perforated plates. These comparisons between simulations and test data have been also performed on different composites materials.

## STRENGTH ANALYSIS METHOD FOR OPEN-HOLE PLATES

### Determination of the membrane loadings within the perforated plates

The estimation of the membrane loading within the open-hole plate is performed with an analytical approach [5]. This modelling, initially developed for metallic materials, leads to the exact solution for an infinite perforated plate subjected to multiaxial membrane loadings, in which the materials behaviour is orthotropic linear elastic. Then, an empirical correction factor (eq. 1) is applied to the estimated membrane loading in order to take into account the effect of the finite dimensions, especially the width (noted  $w$ ).

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = C_{w/d} \left( \frac{d}{w} \right) \begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix}_{w=\infty} \quad \text{avec} \quad C_{w/d} \left( \frac{d}{w} \right) = \frac{2 + \left( 1 - \frac{d}{w} \right)^3}{3 \left( 1 - \frac{d}{w} \right)} \quad (\text{eq. 1})$$

The predicted membrane loadings within the open-hole plate have been compared with the ones obtained through finite element (FE) simulations performed with the commercial code Samcef® with different meshes (different sizes or different types of elements).

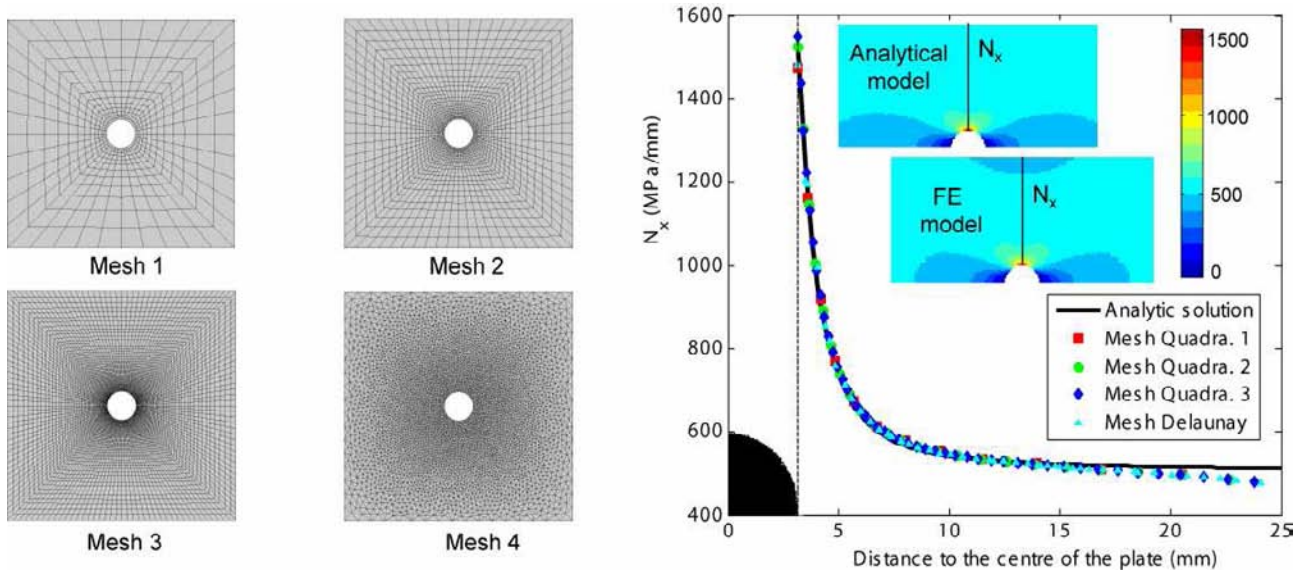


Fig. 1. Comparisons of the membrane loadings within a T700/M21 25/50/25 perforated plates subjected to uniaxial tensile loading estimated with the analytical method or with FE simulations.

The predictions of the analytical method are in very good agreement with the results obtained through finite element simulations and the description of the stress gradient in the vicinity of the perforation is well described, as reported in Fig. 1. However, the predictions of the analytical approach overestimate the membrane loadings near the free edge because the edge effect is not taken into account contrary to the FE simulations.

It has been demonstrated that the validity domain of this analytical approach is limited to perforated plates in which the effect of the edges on the stress gradient is negligible. Therefore, the analytical approach can be used only for perforated plates presenting a  $w/d$  ratio superior to 3.

Otherwise, the estimation of the membrane loading within the perforated plates should be performed through finite elements simulations. The proposed strength analysis method has thus been interfaced with the FE commercial code Samcef®.

### **Multiscale progressive failure approach**

The prediction of the local material behaviour and the determination of the failure of the different plies within the laminate up to the final rupture of the specimen is performed in post-treatment with a multiscale progressive failure approach [6]. The present multiscale failure approach considers the unidirectional (UD) ply as the elementary entity of modelling and is predictive for different stacking sequences. It could be decomposed in four main steps:

In order to predict accurately the failure of a ply in a laminate, it is necessary to estimate in a correct manner the mesoscopic stresses and strains. A non-linear thermo-viscoelastic behaviour, defined at the ply scale, is proposed (eq. 2).

$$\sigma = C^0 : (\varepsilon - \varepsilon^{th} - \varepsilon^{ve} - \varepsilon^{nl}) \quad (\text{eq. 2})$$

where  $\sigma$  is the mesoscopic stress,  $C^0$  the initial elastic rigidity,  $\varepsilon$  the total strain,  $\varepsilon^{th}$  the thermal strain (in order to take into account the thermal residual stresses, which are essential to predict accurately the first ply failure),  $\varepsilon^{ve}$  the viscous strain [7] (in order to predict accurately the final failure of  $[\pm\theta]_s$  laminates or highly disoriented laminates such as the 10/80/10) and  $\varepsilon^{nl}$  the non linear elastic strain describing the non linearity for longitudinal tensile loading [8] (hardening of the behaviour observed on new generations of composite materials such as T700/M21 and T800/M21) or for longitudinal compressive loading [9] (softening behaviour usually observed on Carbon/Epoxy materials).

The prediction of the ply failure within the laminate is performed with a failure criterion, based on Hashin's hypotheses [10], distinguishing the fibre (eq. 3) and interfibre (eq. 4) failure mode and modelling separately the failure mechanisms in tension and in compression for each failure mode.

$$f_1^+ = \frac{\sigma_{11}}{X_t(d_2)} = 1 \text{ if } \sigma_{11} \geq 0 \quad \text{and} \quad f_1^- = \left( \frac{\sigma_{11}}{X_c} \right)^2 + \left( \frac{\tau_{12}}{S_c^f(1 - p\sigma_{22})} \right)^2 = 1 \text{ if } \sigma_{11} < 0 \quad (\text{eq. 3})$$

$$f_2^+ = \left( \frac{\sigma_{22}}{Y_t} \right)^2 + \left( \frac{\tau_{12}}{S_c(1 - p\sigma_{22})} \right)^2 = 1 \text{ if } \sigma_{22} \geq 0 \quad \text{and} \quad f_2^- = \left( \frac{\sigma_{22}}{Y_c} \right)^2 + \left( \frac{\tau_{12}}{S_c(1 - p\sigma_{22})} \right)^2 = 1 \text{ if } \sigma_{22} < 0 \quad (\text{eq. 4})$$

where  $X_t$  and  $X_c$  are respectively the longitudinal tensile and compressive strengths,  $Y_t$  and  $Y_c$  the transverse tensile and compressive strengths,  $S_c$  the in-plane shear strength and  $S_c^f$  the in-plane shear strength for fibre mode. The two main improvements of the fibre failure criterion, as compared to Hashin's criterion, are (i) the introduction of the coupling between the intralaminar damage (noted  $d_2$ ) and the longitudinal tensile strength (allowing to obtain conservative predictions for complex multiaxial loadings) and (ii) to take into account the influence of the in-plane shear on the ply failure in compression, as it has been experimentally observed [11]. The main improvement of the interfibre failure criterion is a better description of the reinforcement of the apparent strength of the material for combined in-plane shear and transverse compressive loadings (thanks to the introduction of the  $p$  parameter).

When the ply is broken, its mechanical properties are progressively degraded using a thermodynamical degradation approach based on damage modellings already developed at Onera [12]. The initial elastic compliance  $S^0$  is increased (eq. 5) by two terms representing respectively the degradation of the ply failed in fibre mode ( $d_1H_1$ ) or in interfibre mode ( $d_2H_2$ ). For each failure mode, the kinetics of degradation (scalar variables  $d_i$ ) are distinguished from the effects of the ply failure (effect tensors  $H_i$ ).

$$S = S^0 + d_1H_1 + d_2H_2 \quad (\text{eq. 5})$$

Finally, for a laminated plain coupon, the final rupture of the laminate is assumed to be due to a ply failure in fibre mode or in transverse compression. For the stacking sequences used in this study, containing only  $0^\circ$ ,  $\pm 45^\circ$  and  $90^\circ$  plies, the final failure is always due to ply failure in fibre mode.

The present multiscale failure approach is identified through the tests results on UD plies and laminates already performed in industrial qualification test campaign at the coupon level of the pyramid of certification. The predictions of the multiscale failure approach on different T700/M21 laminates, with different stacking sequences (noted X/Y/Z with X% of 0° plies, Y% of ±45° plies and Z% of 90° plies) and subjected to uniaxial tensile loading, are compared successfully with available test results (only mean values of the experimental results are reported in Fig. 2).

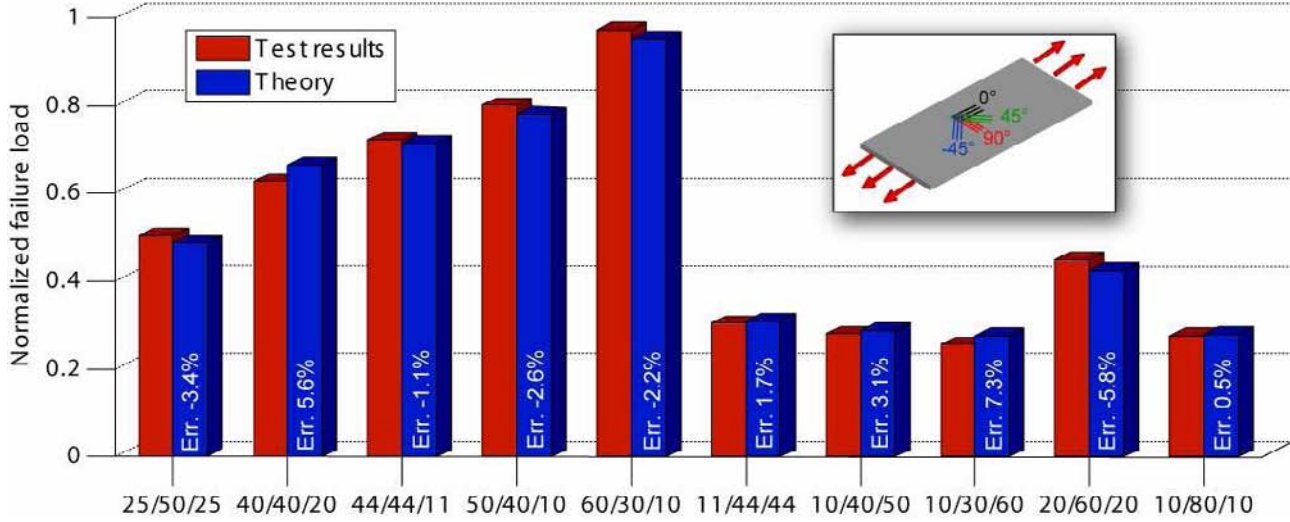


Fig. 2. Comparison between simulations and tests results on different T700/M21 laminated plain plates subjected to uniaxial tensile loading.

### Fracture characteristic volume method

Assuming that the final failure of a perforated plate is due to the first ply failure in fibre mode leads to underestimate drastically the rupture of the specimen (by a factor 2 or 3) because of the overloading in the vicinity of the singularities. A fracture characteristic volume method, based on the approach proposed by Hochard [13] on woven composite, is applied to the laminated perforated plates manufactured with UD plies. It consists in using, in the fibre failure criterion ( $f_1^+$  or  $f_1^-$ ), a stress  $\bar{\sigma}$  averaged on a volume ( $\Omega$ ) defined by an internal length ( $l_0$ ) (see Fig. 3a). A convergence study has been performed to demonstrate that, from 19 integration points, the predicted failure load becomes independent of the choice of the number of integration points (see Fig. 3b). In the following, all the simulations are performed with the minimum number of integration points in order to obtain robust computation with a reduce time of calculation.

The internal length ( $l_0$ ) is independent of the considered stacking sequence but should be defined as a function of the radius ( $r$ ) of perforation as proposed by Whitney [4] (see eq. 6). This internal length should be considered as a material parameter.

$$l_0 = \rho \sqrt{r} \quad (\text{eq. 6})$$

Thus, only one tensile test on perforated plates is required to identify the parameter  $\rho$  (expressed in eq. 6).

The whole strength analysis method dedicated to perforated plates, that includes the analytical method to determine the membrane loading with the plate, and the multiscale failure approach to predict the mesoscopic behaviour and the failure of plies, and the fracture characteristic volume method to predict the final failure of the specimen, has been implemented in Matlab® and the computation time on a laptop is around 2 minutes. Therefore, this simplified strength analysis method presents a low computational time and matches the different requirements of a design office.



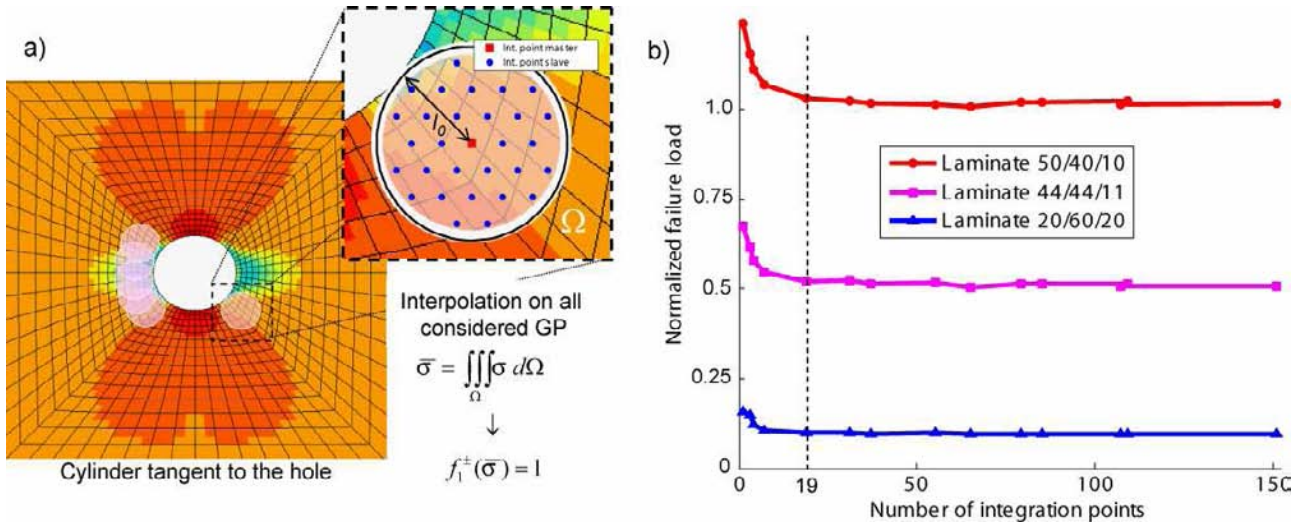


Fig. 3. a) Principle of the fracture characteristic volume method and b) determination of the number of integration points to predict robustly the final failure of perforated plates.

## COMPARISON BETWEEN SIMULATIONS AND EXPERIMENTS ON LAMINATED PERFORATED PLATES

While the design of aeronautical structures necessitates the use of allowables taking into account the material dispersion (A and B values) to insure the required level of confidence, the average values of the stiffnesses and strengths are used in the present article. Indeed, this study deals with the evaluation of the predictive capabilities of the simplified strength analysis method thanks to comparisons with test data extracted from large experimental campaigns. Only the mean values of the experimental data are reported on the different figures presented in this section.

### Tensile tests on open-hole plates with different stacking sequences

The Fig. 4 presents the comparison between the predicted ultimate failure loads on T700/M21 perforated plates with different stacking sequences subjected to uniaxial tensile loading and the test results. For all the perforated tested plates, the diameter of the hole is 6.35mm and the w/d ratio is 5. The internal length has been identified on the 50/40/10 laminated perforated plates. The predictions of the proposed strength analysis method are in very good agreement with experimental data.

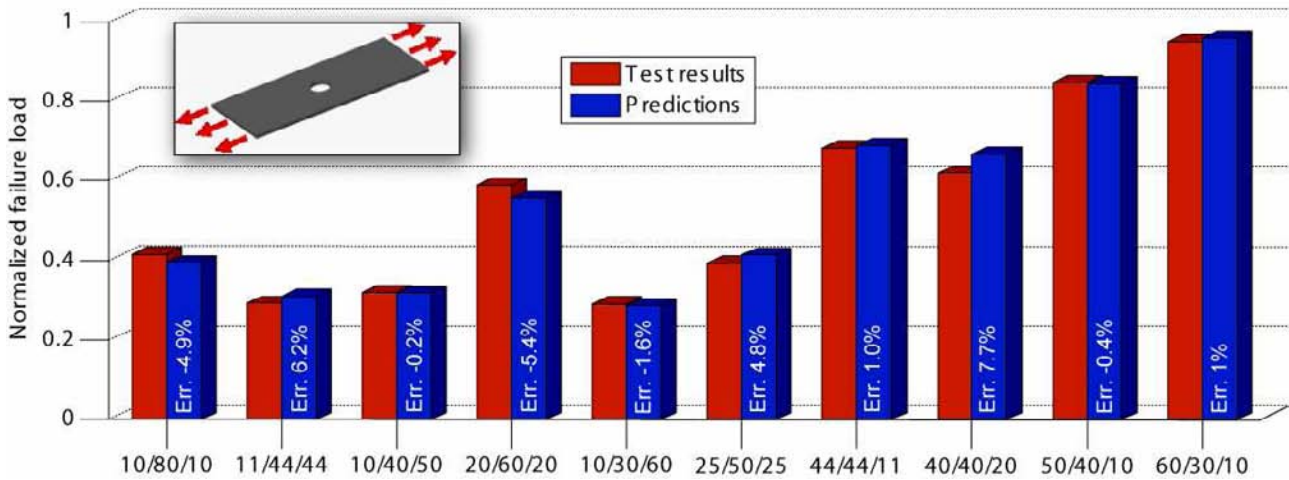


Fig. 4. Comparison between simulations and tests results on T700/M21 laminated perforated plates with different stacking sequences and subjected to uniaxial tensile loading.

These comparisons have also been performed on other Carbon/Epoxy materials, such as the T800/M21 and the IMS/977-2 and are reported on Fig. 5. The internal lengths for these two materials have been

identified on the 40/40/20 perforated plates. Once again, the predictions are in very good agreement with experimental data.

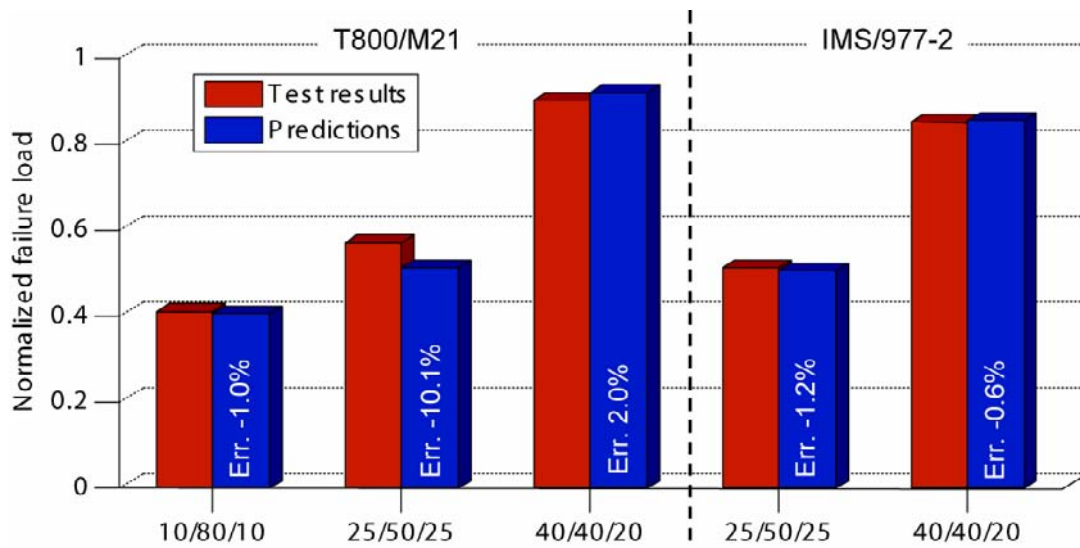


Fig. 5. Comparison between simulations and tests results on T800/M21 and IMS/977-2 laminated perforated plates with different stacking sequences and subjected to uniaxial tensile loading.

#### Tensile tests on open-hole plates with different diameters of perforation

The Fig. 6 presents the comparison between the predicted ultimate failure loads on T700/M21 open-hole plates with different diameters of perforation and subjected to uniaxial tensile loading and the test results. For all the perforated tested plates, the w/d ratio is 5. The variation of the diameters of the perforation induces important variations of the stress gradient in the vicinity of the hole. The increase of the diameter of the hole leads to a decrease of the macroscopic stress at failure (defined by the ultimate load divided by the section), as it has been observed experimentally by Wisnom [14]. The introduction of the dependence of the internal length to the radius of perforation allows obtaining accurate or reasonably conservative predictions, especially for 60/30/10 perforated laminates.

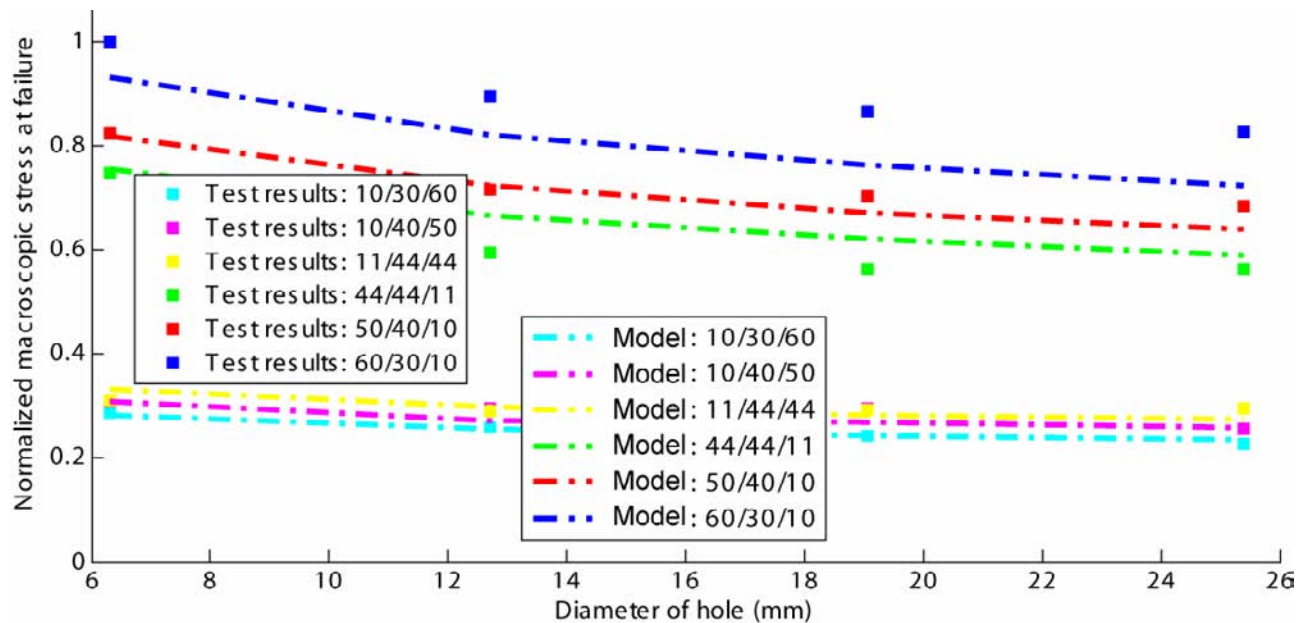


Fig. 6. Comparison between simulations and tests results on T700/M21 laminated perforated plates with different diameters of perforation and subjected to uniaxial tensile loading.

### ***Tensile tests on open-hole plates with different w/d ratios***

The Fig. 7 presents the comparison between the predicted ultimate failure loads on T700/M21 perforated plates with different w/d ratios and subjected to uniaxial tensile loading and the test results. For all the perforated tested plates, the diameter of the perforation is 6.35mm. Because the diameter of the hole remains constant, the stress gradient near the hole remains unchanged. The macroscopic stress at failure for large w/d ratios (superior to 5) remains thus quasi-constant. Nevertheless, for low w/d ratios (inferior to 4), the width of the plate is limited and the influence of the free edges on the stress gradient can not be neglected and induces a decrease of the macroscopic stress at failure. The predicted failure loads are in good agreement with experimental data. Nevertheless, for low w/d ratios, the proposed approach tends to overestimate the final failure because the edge effects are neglected by the analytical method [5] as mentioned previously.

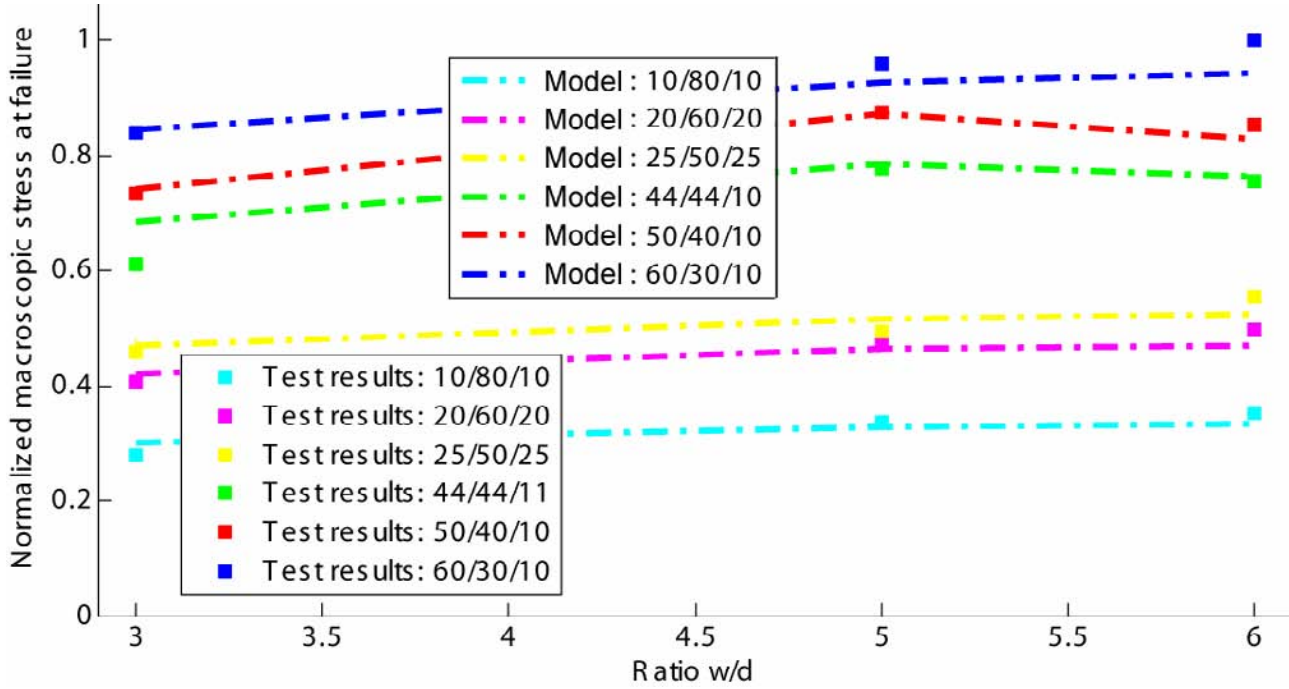


Fig. 7. Comparison between simulations and tests results on T700/M21 laminated perforated plates with different w/d ratios and subjected to uniaxial tensile loading.

### ***Off-axis tensile tests on open-hole plates with different stacking sequences***

The Fig. 8 presents the comparison between the predicted ultimate failure loads on T700/M21 perforated plates with different stacking sequences and subjected to off-axis tensile loading and the test results. For all the perforated tested plates, the diameter of the perforation is 6.35mm and the w/d ratio is 5. This kind of off-axis tensile tests permits to obtain more complex multiaxial stress field in the vicinity of the hole, while the experimental device remains the same and the patterns of rupture evolve as a function of the off-axis angle.

The predicted failure loads have been successfully compared with the experimental data. For a disoriented laminates, the apparent reinforcement of the specimens for off-axis angle around 45° is well reproduced by the proposed modelling.

Nevertheless, the present approach tends to overestimate the final failure of oriented laminates (60/30/10 and 50/40/10) for the off-axis angle evolving between 15° and 35°, where the in-plane shear stresses, in the most loaded plies in the fibre direction, become important.



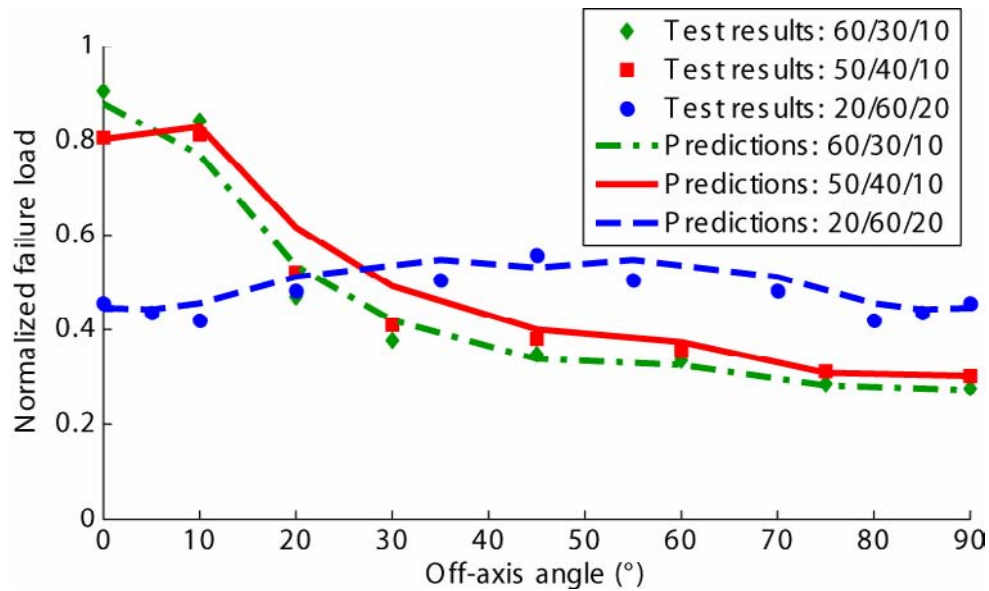


Fig. 8. Comparison between simulations and tests results on T700/M21 laminated perforated plates with different stacking sequences subjected to off-axis tensile loading.

### Compressive tests on open-hole plates with different stacking sequences

The Fig. 9 presents the comparison between the predicted ultimate failure loads on T700/M21 perforated plates with different stacking sequences and subjected to uniaxial compressive loading and the test results. For all the perforated tested plates, the diameter of the hole is 6.35mm and the w/d ratio is 5. The internal length has been identified on the 50/40/10 laminated perforated plates and is different from the one identified in tension. The predicted failure loads are in good agreement with experimental data for oriented laminates (from the quasi-isotropic 25/50/25 laminate to the highly oriented 60/30/10 laminate). Nevertheless, the proposed modelling tends to underestimate the final failure of disoriented laminates (error around -20%). The influence of the delamination on the final failure has been neglected while it should be important in compression and could explain the observed discrepancy.

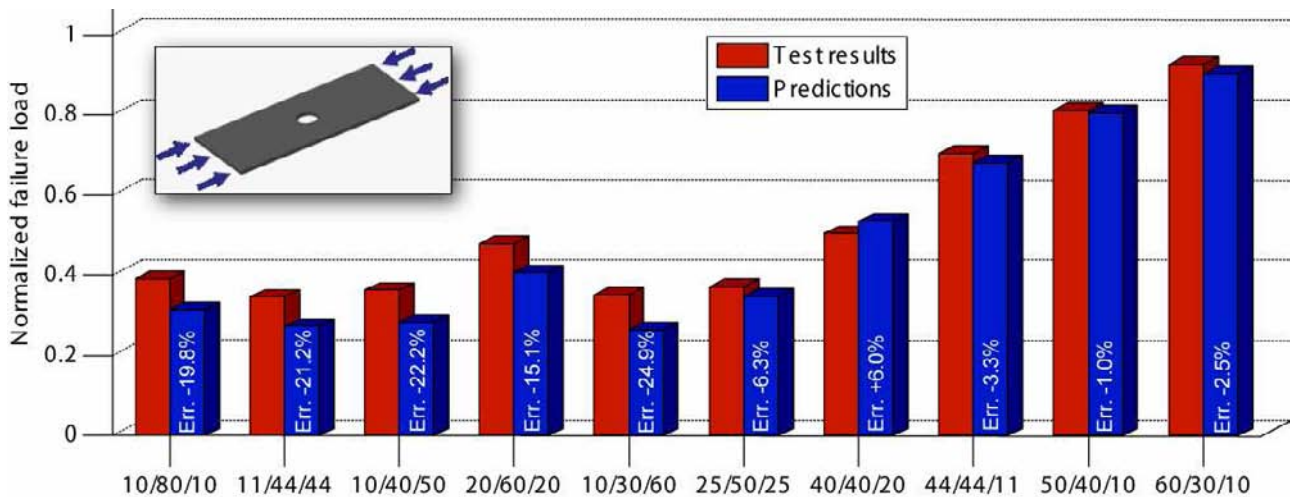


Fig. 9. Comparison between simulations and tests results on T700/M21 laminated perforated plates with different stacking sequences subjected to uniaxial compressive loading.

These comparisons have also been performed on other Carbon/Epoxy materials, such as the T800/M21 and the IMS/977-2 and are reported on Fig. 10. The internal lengths for these two materials have been identified on the 40/40/20 perforated plates. The conclusions remain the same that the ones determined for the T700/M21 composite material. Some additional tests should be performed in order to improve the comprehension of the failure mechanisms occurring in perforated plates subjected to compressive loadings.



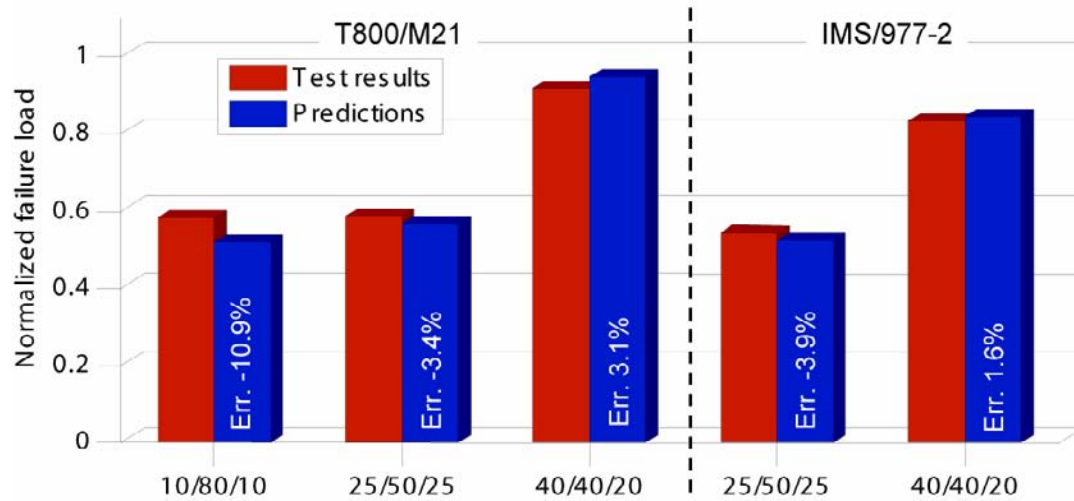


Fig. 10. Comparison between simulations and tests results on T800/M21 and IMS/977-2 laminated perforated plates with different stacking sequences and subjected to uniaxial compressive loading.

## CONCLUSIONS

A simplified strength analysis method dedicated to perforated plates has been proposed and could be decomposed in three main steps: (i) the determination of the membrane loadings within the open-hole plate with an analytical method, which has been validated through comparisons with FE simulations on different test cases, (ii) the estimation of the mesoscopic behaviour and the plies failure in the laminate with a multiscale progressive failure approach, whose predictions of the final failure of laminated plain coupons have been compared successfully with available test data and (iii) the predictions of the final failure of an open-hole plate with a fracture characteristic volume method which consists in using, in the fibre failure criterion, a stress averaged on a volume defined by an internal length. This internal length should be considered as a material parameter independent of the stacking sequence but defined as a function of the diameter of the perforation.

This simplified strength analysis method matches the different requirements of a design office: it is a fast computational method which is moreover easy to carry out (easy to identify, to implement and to analyse the results). The determination of the predictive capabilities of the proposed approach has been performed through comparisons with experimental data on a large test data basis. The predictions of the present strength analysis method have been compared successfully with both uniaxial tensile loading performed on perforated laminated plates with different stacking sequences, different w/d ratios, different diameters of perforation, and off-axis tensile tests. Moreover, these comparisons have also been performed successfully on other Carbon/Epoxy materials.

Finally, some comparisons with compressive tests on laminated open-hole plates have been performed. It is worth mentioning that the internal length in tension is different from the one in compression. The proposed failure method leads to obtain accurate results for oriented laminates but tends to underestimate the final failure of disoriented laminates. The same study has been performed on other Carbon/Epoxy materials and the conclusions remain the same. Taking into account the effect of delamination on the strength of the perforated plates should improve the accuracy of the predictions but increases the computational time.

The predictive capabilities of the proposed simplified strength analysis method still have to be determined for perforated plates with non conventional stacking sequences, or complex configurations of perforations (many holes in one plate) or complex multiaxial loadings.

## REFERENCES

- [1] P.P. Camanho, P. Maimi and C.G. Davila. "Prediction of size effects in notched laminates using continuum damage mechanics". *Composites Science and Technology*, Vol. 67, No. 13, pp. 2715-2727, 2007.
- [2] S.R. Hallett, W.G. Jiang, B. Khan and M.R. Wisnom. "Modelling the interaction between matrix cracks and delamination damage in scaled quasi-isotropic specimens". *Composites Science and Technology*, Vol. 68, No. 1, pp. 90-89, 2008.
- [3] D. Violeau, P. Ladevèze and G. Lubineau. "Micromodel-based simulations for laminated composites". *Composites Science and Technology*, Vol. 69, No. 9, pp. 1364-1371, 2009.
- [4] J.M. Whitney and R.J. Nuismer. "Stress Fracture Criteria for Laminated Composites Containing Stress Concentrations". *Journal of Composite Materials*, Vol. 8, No. 3, pp. 253-265, 1974.
- [5] S. C. Tan, "Stress concentrations in laminated composites". *USA Technomic publishing compagny Inc.*, 1994.
- [6] F. Laurin, N. Carrere and J.-F. Maire. "A multiscale progressive failure approach for composite laminates based on thermodynamical viscoelastic and damage". *Composites Part A*, Vol. 38, No. 1, pp. 198-209, 2007.
- [7] J.-F. Maire. "Etude théorique et expérimentale du comportement de matériaux composites en contraintes planes". *Phd-thesis of the University of Franche-Comté*, 1992.
- [8] T. Ishikawa, M. Matsushima and Y. Hayashi. "Hardening non linear behaviour in longitudinal tension of unidirectional carbon composites". *Journal of Materials Science*, Vol. 20, pp. 4075-4083, 1985.
- [9] T. Yokozeki, T. Ogasawara and T. Ishikawa. "Nonlinear behavior and compressive strength of unidirectional and multidirectional carbon fiber composite laminates". *Composites Part A*, Vol. 37, No. 11, pp. 2069-2079, 2006.
- [10] Z. Hashin. "Failure criteria for unidirectional fiber composites". *Journal of Applied Mechanics*, Vol. 47, pp. 329-334, 1980.
- [11] E.C. Edge. "Does transverse and shear loading affect the compression strength of unidirectional CFC? A reply to Dr Hart-Smith". *Composites*, Vol. 25, No. 2, pp. 159-161, 1994.
- [12] J.-F. Maire and J.-L. Chaboche. "A new formulation of continuum damage mechanics (CDM) for composite materials". *Aerospace Science and Technology*, Vol. 1, No. 4, pp. 247-257, 1997.
- [13] C. Hochard, N. Lahellec and C. Bordreuil. "A ply scale non-local fibre rupture criterion for CFRP woven ply laminated structures". *Composite Structures*, Vol. 80, No. 3, pp. 321-326, 2007.
- [14] M.R. Wisnom and S.R. Hallett. "The role of delamination in strength, failure mechanism and hole size effect in open-hole tensile tests on quasi-isotropic laminates". *Composites Part A*, Vol. 40, No. 4, pp. 335-342, 2009.