

THE INFLUENCE OF A STATIONARY VIBRATIONAL NONEQUILIBRIUM ON THE STABILITY AND THE STRUCTURE OF SHOCK WAVES

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The influence of a vibrational nonequilibrium sustained by an external heat source (in particular, electric pumping in the discharge) on the shock wave structure is investigated. Nonequilibrium leads to the strong modification of the weak shock waves. Weak shock waves are unstable. They disintegrate into the sequence of the self-sustaining waves. Two types of the self-sustained waves (the pulse and the wave with non-zero asymptote) are obtained.

1. INTRODUCTION

Klimov et al. [1-3]; Basargin and Mishin [4-6]; Bystrov, Ivanov, and Shugaev [7]; Gridin, Klimov, and Molevich [8]; Ganguly, Bletzinger, and Garscadden [9] have investigated the weak shock wave propagation in the weakly ionized nonequilibrium plasma. The different types of electric discharges (direct current discharges, radio frequency discharges and pulsed discharges) are used in these experiments. It was observed both the shock wave amplification (in molecular gases, such as air, nitrogen, CO₂) and the shock wave decay. It was also observed the acceleration of shock waves, the precursor generation before the shock wave, the shock wave structure modification, and the shock front splitting. The modification of SW structure was also observed in the chemically active gas [10].

Bailey and Hilbun [11]; Macheret et al. [12] have tried to explain the shock wave decay, the acceleration of shock waves, and the shock front splitting by the thermal mechanism. The thermal mechanism takes into account non-uniform gas heating in a plasma region and the shock wave front curvature. However, Klimov et al. [13], Mishin, Klimov, and Gridin [14]; Gridin and Klimov [15] have observed the shock wave splitting (Fig.1) and plasma precursor generation in pulsed transverse discharges, where the high temperature homogeneity and the absence of the shock wave curvature were controlled. Thus, the thermal mechanism can't be responsible for the new shock wave structure in these conditions. It can't explain the shock wave amplification at all.

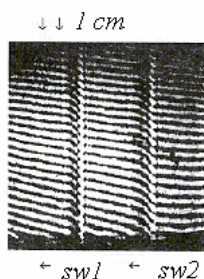


Fig 1. Splitting of the shock wave in transverse discharge.

The amplification and the modification of weak shock waves can be caused by the new viscosity-dispersion properties of nonequilibrium media. In [16-19], we have discussed in detail the principle difference between acoustics of equilibrium media and acoustics of such nonequilibrium media as a vibrationally excited gas, a nonisothermal plasma, chemically active mixtures, media with nonequilibrium phases etc. In such media, the second (bulk) viscosity coefficient ξ and the sound dispersion can be negative: $\xi < 0$ and $c_0 > c_\infty$. Here, c_0, c_∞ are the equilibrium (low-frequency) and frozen (high frequency) sound velocities, respectively. Media possessing the negative viscosity are acoustically active. Moreover, the low-frequency coefficient of the gas dynamic nonlinearity Ψ_0 is a complicated function on the nonequilibrium degree. Only the frozen coefficient of gas dynamic nonlinearity has the usual form $\Psi_\infty = (\gamma_\infty + 1) / 2$. There are ranges of the nonequilibrium degree, where $\Psi_0 < 0$.

In the present work, we investigate theoretically the possible modification of the shock wave structure in stationary nonequilibrium media, which is caused by their new acoustical properties.

2. SHOCK ADIABATS IN NONEQUILIBRIUM MEDIUM

The initial system of gas dynamics equations has the form

$$\begin{aligned} P &= \frac{\rho T}{M}, \quad \frac{d\rho}{dt} + \rho \frac{\partial v}{\partial x} = 0, \quad \rho \frac{dv}{dt} = -\frac{\partial P}{\partial x}, \\ C_{V\infty} \frac{dT}{dt} + \frac{dE_v}{dt} - \frac{T}{\rho} \cdot \frac{d\rho}{dt} &= Q - I \\ \frac{dE_v}{dt} &= \frac{E_e - E_v}{\tau_v(T, \rho)} + Q \end{aligned} \quad (1)$$

In Eq. (1), E_v is the energy of the vibrational degrees of freedom of the molecules, E_e is its equilibrium value, τ_v is the vibrational relaxation time, and Q is the power of an external heat source (in particular, electric pumping in the discharge, chemical or optical pumping), that is sustaining the nonequilibrium degree $S = (E_{v0} - E_{e0})/T_0 = Q\tau_v/T_0$, v, T, ρ, P are, respectively, the velocity, temperature, density, and pressure, $I=Q$ is the heat removal and $d/dt = \partial/\partial t + v\partial/\partial x$.

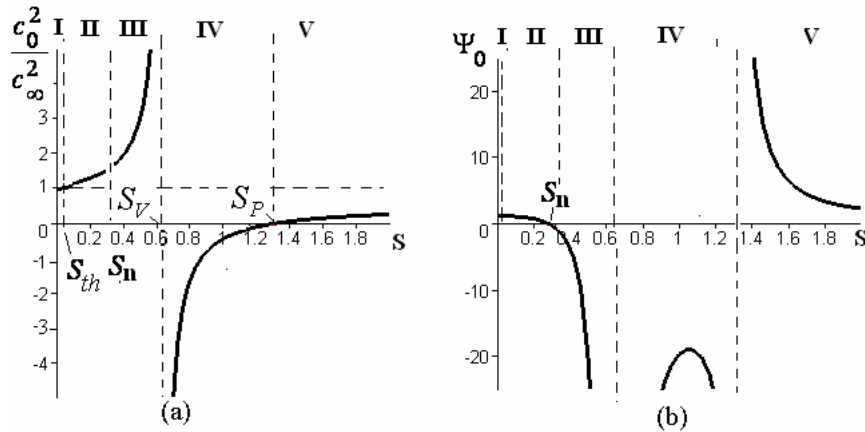


Fig.2. Dependences of the sound speed relation (a) and the low-frequency nonlinearity coefficient (b) on nonequilibrium degree in the CO_2 -containing mixture ($CO_2:N_2:He=1:2:3$, standart conditions)[19]. Fields of nonequilibrium I–V are characterized by appreciably different acoustical properties.

The gas with the stationary nonequilibrium and Landau- Teller dependence of the relaxation time has the five fields of the nonequilibrium degree S with qualitatively different properties [17-19] (Fig.2).

Field I: $S < S_{th} = C_v / (C_{V\infty} - \tau_T)$, where $C_v = dE_{e0} / dT_0$, $\tau_T = \partial \ln \tau_{v0} / \partial \ln T_0$. Here, we have the positive viscosity $\xi > 0$, the positive dispersion $c_0 < c_\infty$, and the positive nonlinearity coefficient $\Psi_0 \approx (\gamma_0 + 1) / 2$ similar to equilibrium media.

Field II: $S_{th} < S < S_n$. The dispersion and the second viscosity are negative ($\xi < 0; c_0 > c_\infty$). The low frequency nonlinear coefficient $\Psi_0 > 0$. Here, S_n is defined from the equation $\Psi_0(S_n) = 0$, where

$$\Psi_0 = \left[\frac{S_0 \tau_T (1 + S_0)}{C_{P0} C_{V0}} + \frac{1 + 2C_{V0}}{2C_{V0}} - \frac{S_0 (1 + S_0)^2}{2C_{P0} C_{V0}^2} \tau_{TT} \right], \quad \tau_{TT} = \frac{T_0^2}{\tau_{v0}} \frac{\partial^2 \tau_{v0}}{\partial T_0^2},$$

$C_{V0} = C_{V\infty} + C_v + S_0\tau_T$, $C_{P0} = C_{P\infty} + C_v + S_0(\tau_T + 1)$, are the low-frequency heat capacities at constant volume and constant pressure in the vibrationally excited gas.

Field III: $S_n < S < S_V = \frac{C_{V\infty} + C_v}{-\tau_T}$. Here, $\xi < 0$, $c_0 > c_\infty$, $\tilde{\Psi}_0 = \gamma_0\Psi_0 < 0$.

Field IV: $S_V < S < S_P$; $S_P = \frac{C_{P\infty} + C_v}{-(\tau_T + 1)}$. Here, $\xi < 0$, $\gamma_0 < 0$, $C_{V0} < 0$, $C_{P0} > 0$.

Field V: $S > S_P$. Here, $\xi < 0$, $c_0 < c_\infty$, $\tilde{\Psi}_0 > 0$, $C_{V0} < 0$, $C_{P0} < 0$.

In relaxation gas dynamics, two shock adiabats drawn through a given initial point (P_0, V_0) are

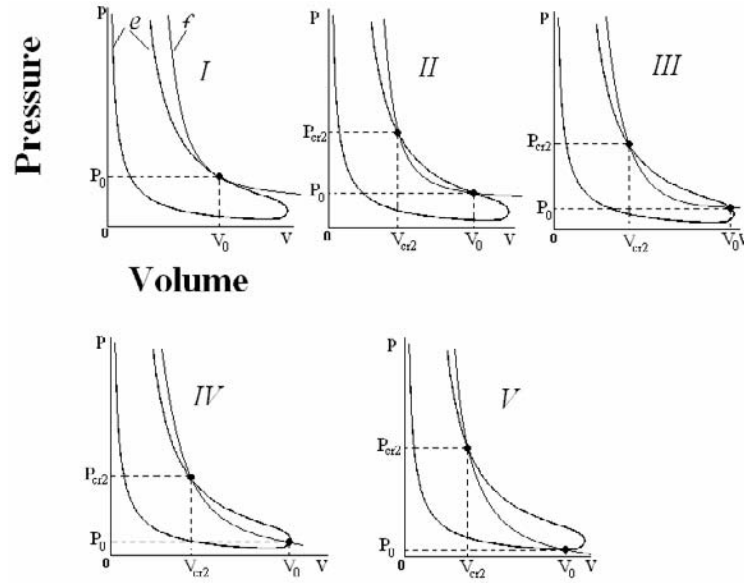


Fig. 3. Frozen (*f*) and equilibrium (*e*) shock adiabats in fields of nonequilibrium I–V. The equilibrium adiabat has two branches and meets with the frozen adiabat in the point (P_{cr2}, V_{cr2}) . The point (P_0, V_0) corresponds to an initial state before the shock wave front. The initial point (P_0, V_0) on the equilibrium adiabat moves from upper branch to lower branch with increase in the nonequilibrium degree.

considered. One corresponds to total equilibrium of the final states of the gas and, therefore, is called the equilibrium adiabat. The other, referred to as “frozen,” assumes that the relaxation processes do not proceed at all. These adiabats can be obtained from the general Rankine-Hugoniot expression

$$\varepsilon_0 - \varepsilon_1 + \frac{1}{2}(V_0 - V_1)(P_0 + P_1) = 0,$$

where subscripts 0 and 1 correspond to stationary states before and after the shock front, $V = 1/\rho$ is the specific volume, ε is the specific inner energy.

The frozen adiabat corresponds to $\varepsilon_0 = C_{V\infty}T_0 + E_{v0}$, $\varepsilon_1 = C_{V\infty}T_1 + E_{v0}$, where $T = MPV$, from which it follows [20]

$$\frac{P_1}{P_0} = \frac{(\gamma_\infty + 1)V_0 - (\gamma_\infty - 1)V_1}{(\gamma_\infty + 1)V_1 - (\gamma_\infty - 1)V_0}.$$

The equilibrium adiabat corresponds to

$$\varepsilon_0 = C_{V\infty}T_0 + E_{v0} = (C_{V\infty} + S_0)T_0 + E_e(T_0), \quad \varepsilon_1 = C_{V\infty}T_1 + E_{v1} = (C_{V\infty} + S_1)T_1 + E_e(T_1).$$

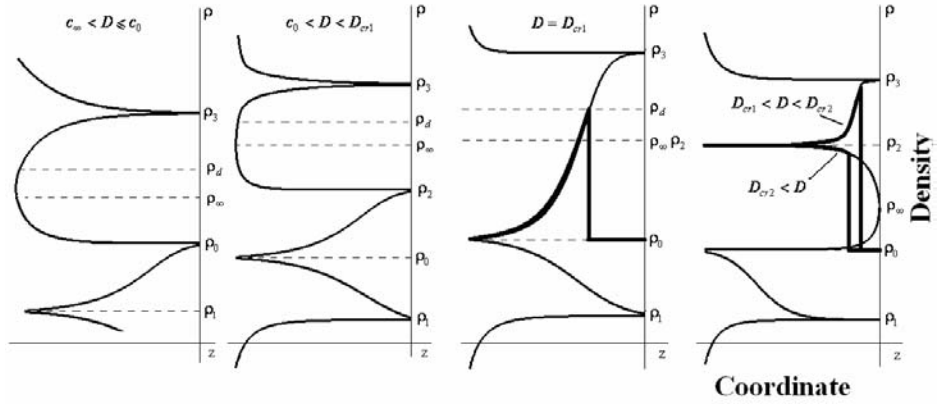


Fig. 4. Integral curves of Eq. (2) in field $S_{thr} < S < S_n$ and shock wave structures at $D = D_{cr1}, D_{cr1} < D < D_{cr2}, D > D_{cr2}$. Stationary states $\rho_i, i = 1, 4$ correspond to $A(\rho) = 0$, the inflection point ρ_∞ corresponds to $B(\rho) = 0$.

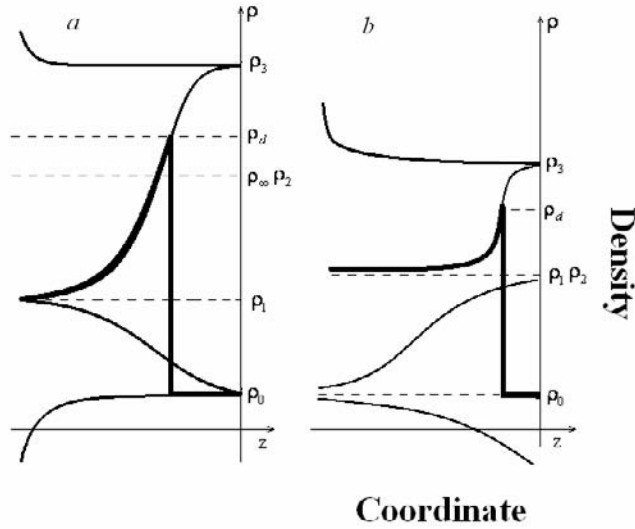


Fig. 5. Integral curves of Eq. (2) and shock wave structures in field $S_n < S$:
a $S_n < S < S_e, D = D_{cr1}$, b $S > S_e, D = D_t$.

For Landau-Teller dependence $\tau_v(T, \rho) \sim \exp(b/\sqrt[3]{T})/\rho\sqrt{T}$ and an equilibrium vibrational energy in a harmonic - oscillator form

$$E_e = \frac{T_k}{\exp\left(\frac{T_k}{T}\right) - 1},$$

where b and T_k are constants, we obtain [19]

$$C_{V\infty}(P_0V_0 - P_1V_1) + \frac{1}{2}(V_0 - V_1)(P_0 + P_1) + S_0P_0V_0 \left[1 - \sqrt{\frac{V_1P_0}{V_0P_1}} \exp\left(\frac{b}{\sqrt[3]{MP_1V_1}} - \frac{b}{\sqrt[3]{MP_0V_0}}\right) \right] +$$

$$\frac{\theta}{M} \left\{ \left[\exp\left(\frac{\theta}{MP_0V_0}\right) - 1 \right]^{-1} - \left[\exp\left(\frac{\theta}{MP_1V_1}\right) - 1 \right]^{-1} \right\} = 0.$$

For $S_0 \neq 0$, the equilibrium adiabat has two branches with two asymptotes $P \rightarrow \infty$ (Fig. 3). There is the point (P_{cr2}, V_{cr2}) where the frozen and equilibrium adiabats meet. With increase in the nonequilibrium degree, the initial point (P_0, V_0) on the equilibrium adiabat moves from the upper branch to the lower branch. In the field IV, where $\tilde{\Psi}_0 < 0$, the equilibrium adiabat has a convexity near the initial point (P_0, V_0) .

3. SHOCK WAVE STRUCTURES. BIFURCATION DIAGRAM

In [19], we reduced system of equations (1) for stationary waves propagating with the speed D to one equation

$$\frac{d\rho}{dz} = -\frac{\rho\{[E_e(\rho) - E_v(\rho)]/\tau_v(\rho) + Q\}}{\rho_0 D(dE_v/d\rho)} \equiv \frac{A(\rho)}{B(\rho)} \quad (2)$$

where

$$z = x - Dt, \quad E_v(\rho) = E_{v0} + M[C_{P\infty} \frac{P_0}{\rho_0} + \frac{D^2}{2} - \frac{C_{P\infty}}{\rho}(P_0 + \rho_0 D^2(1 - \frac{\rho_0}{\rho})) - \frac{1}{2}(\frac{\rho_0 D}{\rho})^2].$$

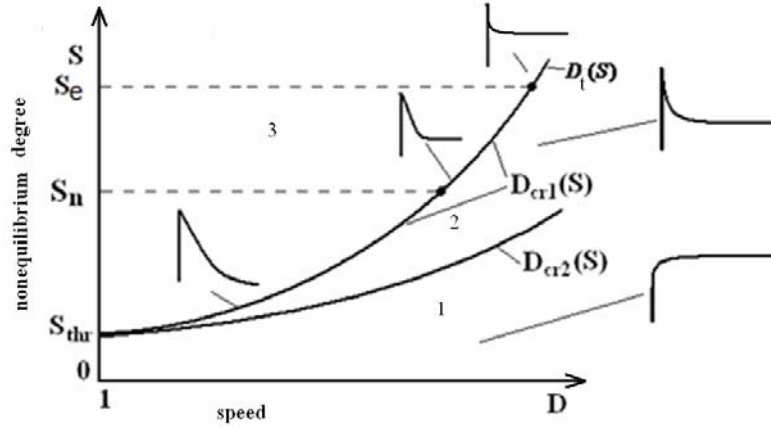


Fig. 6. Bifurcation diagram[24].

The shock wave structure after the sharp front ρ_d , which is equal to

$$\frac{\rho_d}{\rho_0} = \frac{(\gamma_\infty + 1)D^2}{(\gamma_\infty - 1)D^2 + 2c_\infty^2},$$

was obtained using the numerical solution of Eq. (2). Integral curves and possible stationary wave solutions of Eq. (2) are shown in Figs. 4,5.

All results can be presented in the bifurcation diagram (Fig.6).

Here, the implicit forms of boundaries D_{cr1}, D_{cr2} are

$$S = \frac{\tilde{T}_k / [\exp\{\frac{\tilde{T}_k}{\tilde{T}_2}\} - 1] - \tilde{T}_k / [\exp\{\tilde{T}_k\} - 1]}{1 - \exp\{\tilde{b} / \sqrt[3]{\tilde{T}_2} - \tilde{b}\} [(\gamma_\infty - 1)\tilde{D}_{cr2}^2 + 2] / \sqrt{\tilde{T}_2} (\gamma_\infty + 1)\tilde{D}_{cr2}^2},$$

$$\tilde{T}_2 = \frac{[(\gamma_\infty - 1)\tilde{D}_{cr2}^2 + 2][2\gamma_\infty\tilde{D}_{cr2}^2 - (\gamma_\infty - 1)]}{(\gamma_\infty + 1)^2 \tilde{D}_{cr2}^2},$$

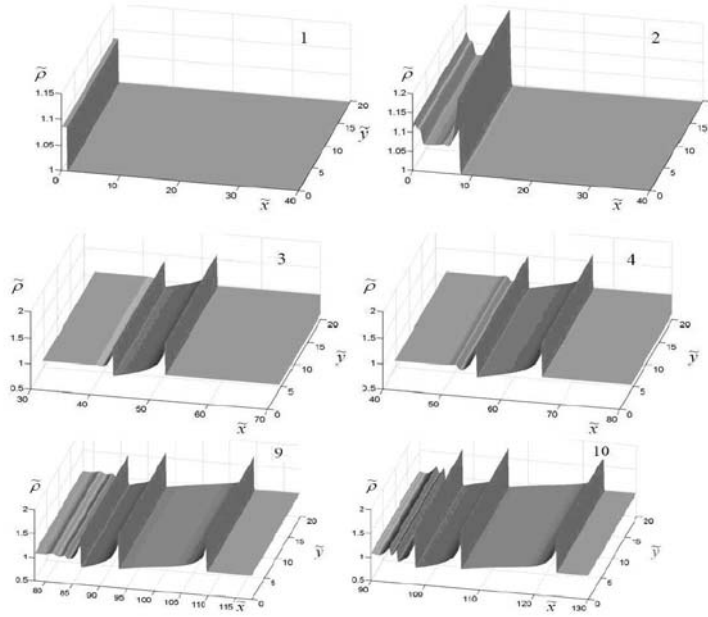


Fig. 9. The splitting of the shock wave front for the 2D geometry

The obtained shock wave structures can be easily explained with help of shock adiabats (Fig. 7). Here, stationary states $\rho_i = 1/V_i, i = 1, 4$ correspond to points of intersection of the chord drawn from initial point $0(P_0, V_0)$ and the equilibrium adiabat, state ρ_d corresponds to point of intersection of this chord with the frozen adiabat. As an example, we consider the nonequilibrium field II ($S_{thr} < S < S_n$), where dispersion is negative. The inclination of the chord drawn from initial point $0(P_0, V_0)$ to a crosspoint of adiabatic curves $A_{cr2}(P_{cr2}, V_{cr2})$ defines the critical velocity of a shock wave $D_{cr2} = V_0 \sqrt{(P_{cr2} - P_0)/(V_0 - V_{cr2})}$. For the shock wave velocity $D > D_{cr2}$, a corresponding chord inclination is greater (Fig. 7a, chord 01'2'). Here, the shock wave structure is typical of relaxing media with the positive dispersion, because the point of intersection between the chord and the frozen adiabat is to the right of the point of intersection with the equilibrium adiabat. Therefore, the medium is first rapidly compressed to the value specified by the point 1' of intersection between the corresponding chord and the frozen adiabat and then is gradually compressed to the final state 2' specified by the intersection of the chord with the equilibrium adiabat. The related shock wave pattern is shown in Fig. 7b.

The shock wave with $D = D_{cr2}$ has step-wise form with the amplitude $P_{cr2} - P_0$. For the shock wave velocity $D_{cr1} < D < D_{cr2}$, the shock wave structure changes. In this range, the frozen adiabat is to the left of the equilibrium one (Fig. 7a, chord 02''1''). Here, the fast compression determined by the intersection of the corresponding chord with the frozen adiabat is followed by the gradual expansion to the final state 1'' specified by the intersection between the chord and the equilibrium adiabat.

As well known, the shock wave becomes unstable if the velocity of sound propagating behind the shock wave front is less than the shock wave velocity. In the present paper we show, that this condition coincides with $D < D_{cr1}$ for $S_{thr} < S < S_e$ and $D < D_t$ for $S > S_e$ (region 3 in Fig. 6). The velocity D_t is an analog of the Chapman-Jouguet velocity in the detonation theory. For $D = D_{cr1}$ or $D = D_t$, the stationary wave has form of the pulse with amplitude ρ_d (Fig. 4) or the wave with the non-zero asymptote (Fig. 5).

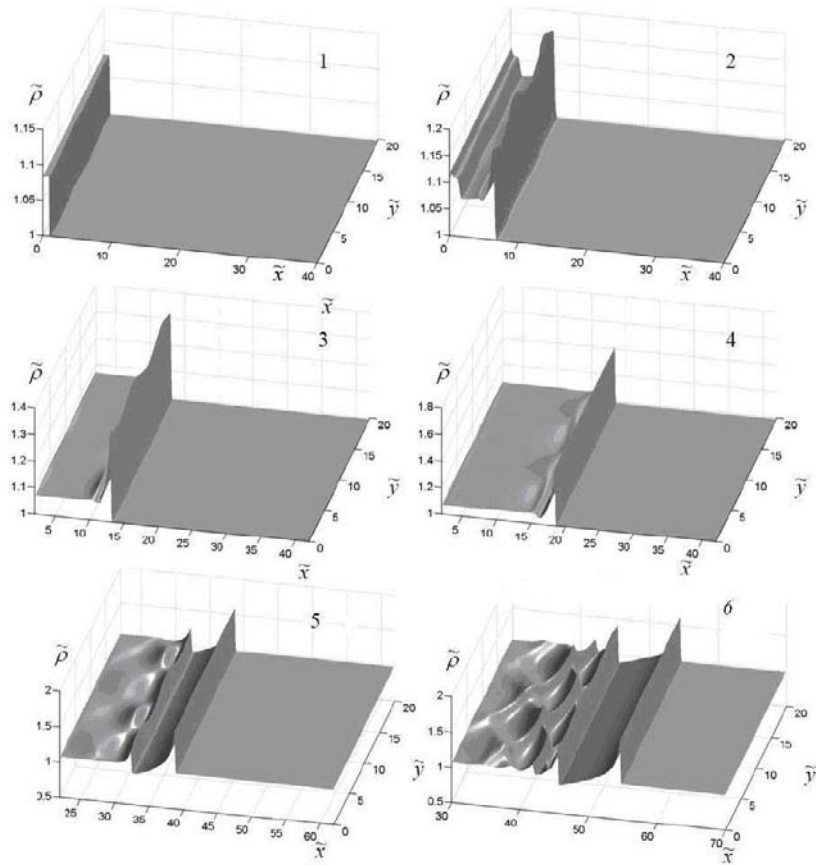


Fig. 10. The stability of the flat front of the autowave structure for the 2D geometry. The initial transverse sinusoidal perturbation don't change the self-sustaining structure.

4. NONSTATIONARY EVOLUTION OF WAVES WITH $D < D_{cr1}, D_t$

We have investigated the nonstationary evolution of the step-like perturbations numerically. For unstable region 3 of S - D diagram, two self-sustaining structures (the pulse or the wave with non-zero asymptote) were obtained as result of such evolution. Their form, amplitude and velocity do not depend on the amplitude of the initial perturbation.

For $S_{thr} < S < S_n$, the unstable waves with $D < D_{cr1}$ disintegrated into sequence of self-sustaining pulses of equal amplitude propagating with the velocity $D = D_{cr1}(S)$ (Fig. 8a).

For $S_n < S < S_e$, the unstable waves disintegrated into sequence of self-sustaining waves with non-zero asymptote and similar velocity $D = D_{cr1}(S)$ (Fig. 8b,9-11). The formation of the sequence of self-sustaining waves at $S_{thr} < S < S_e$, is connected with the acoustical activity conservation behind the front. Oppositely, in a case $S > S_e$, the unstable waves transforms to the single self-sustaining wave propagating with velocity $D = D_t(S)$ (Fig. 8 c).

The stability of the autowave structures to the transverse disturbances are shown (Fig.10).

5. CONCLUDING REMARKS

New acoustical properties of the stationary nonequilibrium media change the shock wave structure. The shock wave structure depends strongly on the nonequilibrium degree S and on the shock wave speed D . We have shown the existence of three regions with qualitatively different shock wave structures in S - D bifurcation diagram. The boundaries of these regions $D_{cr1}(S)$, $D_{cr2}(S)$ were

obtained analytically. In field of nonequilibrium $S < S_{thr}$, the shock wave structure is similar to this structure in equilibrium media with $S=0$. In fields of nonequilibrium $S > S_{thr}$ for the shock wave speed $D > D_{cr2}$, the fast compression is followed by the gradual compression to the final state. For $D_{cr1} < D < D_{cr2}$, the fast compression is followed by the gradual expansion to the final state. For the boundary $D = D_{cr2}(S)$, the shock wave is step-wise. For $D < D_{cr1}$ or $D < D_t$, the shock wave is unstable. In field $S_{thr} < S < S_n$, unstable waves with $D < D_{cr1}$ disintegrate into sequence of solitary self-sustained pulses, propagating with the boundary speed $D = D_{cr1}(S)$. In the strongly nonequilibrium medium, we obtain the self-sustained wave with the non-zero asymptote, propagating with the boundary velocity $D = D_{cr1}(S)$ or $D = D_t(S)$.

This work was partially supported by the research program GR 01200805605, Program "Development of scientific potential of the High School (2009 – 2010 rr.)" (project 2.1.1/309), grants of the RFBR (projects No. 07-01-96608r-povoljje-a & 09-01-08095-3).

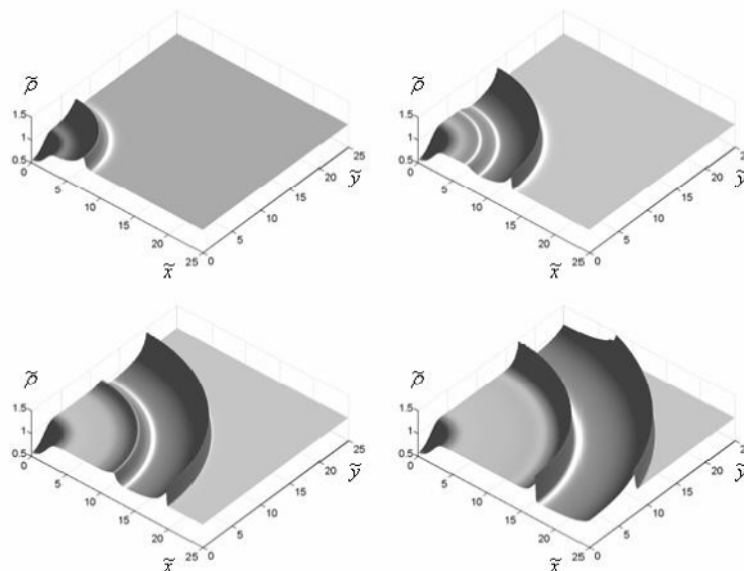


Fig. 11. The splitting of the shock wave front for the cylindrical geometry.

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