Eigenstructure Assignment Based Controllers in Flexible Spacecrafts

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Abstract

A specific modal analysis performed about a flexible spacecraft leads to identify rigid modes and bending modes. One of the main problems of this kind of vehicles is the mechanical coupling existing between rigid and flexible spacecraft parts. In spacecraft with fine attitude pointing requirements it is very important to take into account possible misalignments for the whole vehicle. In designing the attitude controller it is necessary to consider the possible vibrations of the solar panels, and how they influence on the rest of the vehicle. Spacecraft in their orbits with large flexible appendages are susceptible to perturbations and non desired attitude motions. In this paper, the modelling of flexible spacecraft is taken as the preliminary phase of the controller design by the Eigenstructure Assignment (EA) method. The design process is closed by an assignment of the robustness performances of the spacecraft.

1. Introduction

One of the researchers and engineers interest reside in the way of obtaining high level of mitigation in the repercussion between rigid and deformation modes in the spacecraft. The controllers applied to this kind of vehicles may influence about such aspect, leading to an acceptable grade of decoupling in system modes. If these aspects are solved by means of specific controllers, it will be possible to get decoupled motions between the identified vehicle modes. In this way, any attitude motion around any spacecraft axe will have a minor repercussion about spacecraft appendages deformations. In the other side, any deformation on flexible appendages will have a relative repercussion on attitude movements.

From the control perspective, the election and design of the controller is one of the mayor tasks for the Attitude and Control Subsystem in any three axes controlled spacecraft. This work is focused in the design and implementation of a controller based on the called Eigenstructure Assignment (EA) Method. By application of the EA method to MIMO systems [1], and based on a good knowledge of the system, it will be possible to obtain a good decoupling between dynamical modes, such as those belong to rigid and bending modes.

The vehicle considered in this paper is a spacecraft with two parts. One of them is the rigid part of the spacecraft, where are located the spacecraft systems, and the second one is the flexible part. This part may be any external appendage such as large solar arrays. The whole vehicle is subject to internal and external perturbations. The external perturbations are those related with magnetic, solar and aerodynamic forces found in the spacecraft orbit. These forces may produce undesired vibrations on solar arrays causing misalignment in spacecraft attitude pointing. The problem falls on the necessary process to avoid any misalignment in attitude pointing when vibrations or undesired motions are produced on the solar arrays. The solution to this problem may be focused from different perspectives. It is interesting to consider that controller may mitigate or avoid this problem by a decoupling process. If the controller designed obtains an acceptable decoupling for the system modes, the interactions between the rigid and deformable parts of the spacecraft will be mitigated.

The decoupling of the system modes must be taken as a system requirement for the controller. There are some interesting techniques to be considered in the controller design to obtain suitable performances of the spacecraft. In this paper, the Eigenstructure Assignment (EA) Method is going to be explored from a double perspective. The first one is the capacity of the EA method to get an acceptable controller to meet the decoupling requirement and the second one is related with the robustness capabilities given to the system.

Taken into account these requirements, the EA method is based on design a static controller of value **K** allowing the possibility of positioning some eigenvalues in desired positions once the control loop has been closed [2]. The eigenvectors of the dynamical system are also addressed by the EA method. The system eigenvectors identify the shape of the response. Eigenstructure assignment requires the eigenvalues position and the desired closed system behaviour by a proper choice of eigenvectors. As any other controller, robustness is one of the mayor concerns of the control designers [3]. This is addressed by the EA method improving system sensitivity against non-modelled dynamics and perturbations.

2. Mathematical Model

Mathematical model is based in the developing of Newton-Euler dynamic equations, taking into account all possible perturbations affecting the spacecraft movement, together with inertia moments for rigid body, flexible panels and reaction wheels. In order to develop the mathematical model three reference frames have been taken into account. The first frame used is the Earth-Centered Inertial (ECI) reference frame centered in the Earth. The second one is an orbit reference frame, located in the mass center of the satellite. In this frame the **z**-axis is pointing to the earth center, the **x**-axis is tangential to the orbit, being the **y**-axis perpendicular to orbit plane. The attitude maneuvers are related with rotations around **x**, **y** and **z** axis, called roll, pitch and yaw respectively. These rotations are identified as attitude modes in the developing of the controller by EA method. Lastly the body reference frame is located in the center mass of the satellite coinciding with the principal axis of inertia.

Potential and kinetic energy are considered to develop the mathematical model. Also, is taken into account the environmental perturbations and the necessary forces applied by actuators. These considerations are developed by application of the Hamilton's principle to the system lagrangian:

$$\begin{aligned} \boldsymbol{L} &= \boldsymbol{E}_L - \boldsymbol{V}_L \\ \frac{d}{dt}\left(\frac{\partial \boldsymbol{L}}{\partial \dot{\boldsymbol{u}}}\right) - \frac{\partial \boldsymbol{L}}{\partial \boldsymbol{u}} &= \boldsymbol{Q}_{nc} \end{aligned} \qquad (1)$$

Being $\boldsymbol{E}_L$ and $\boldsymbol{V}_L$ the kinetic and potential system energies and $\boldsymbol{Q}_{nc}$ the non conservative applied to the system. In obtaining potential and kinetic energies must be taken into account the influence of the rigid and flexible part of the spacecraft. In order to obtain a precise mathematical model representative of the real system, also is going to be modeled the way in which solar panels describe their motion caused by external perturbations or by attitude maneuvers. The assumed modes method is used to describe the elastic displacements of solar panels respect to the body frame. The elastic deformation of them is modelled as a function of time and some generalized coordinates. The displacements of the solar panels must satisfy the geometric boundary conditions imposed to the system to avoid structural components fail. The geometrical shape of the solar panels can be considered as rectangular plates, being the modes of vibration of them considered as those taken for a clamped-free and a free-free beam. The modes of vibration of this solar panel model are represented by a function obtained by application of the Assumed Mode Method. The elastic displacements of any point on the solar panel calculated by means of this method are given by $w_i(x,y,t)$, considering as a product of functions of space coordinates *x, y* and time, together with the shape factors $\phi_i(x)$ and $\psi_i(y)$. The respective generalized coordinates associated with the elastic displacements are represented by $q_i(t)$ and $r_i(t)$:

$$w_i(x,y,t) = \sum_i \phi_i(x)\psi_i(y)q_i(t)r_i(t) \qquad (2)$$

The graphical representations of the solar panels bending are depicted in the following figures. In these figures can be confirmed that the solar panel bends in two dimensions according to the mathematical model (2).

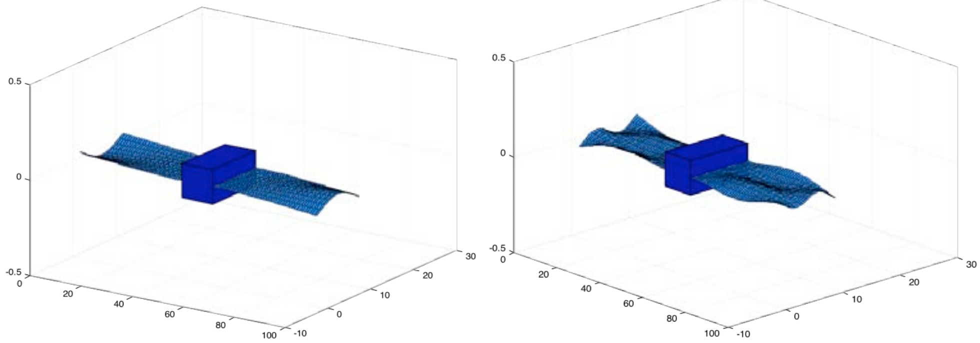
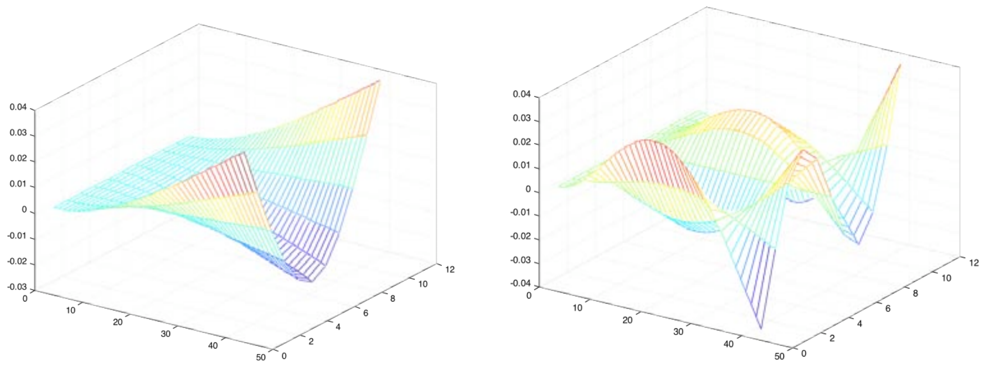


Figure 1: Graphical representation of solar panels bending.



(a) (b)

Figure 2: Elastic displacements of one solar panel: (a) mode 1-1, (b) mode 2-2.

These considerations lead to obtain a mathematical model represented by:

$$\boldsymbol{M}\ddot{\boldsymbol{u}} + \boldsymbol{G}\dot{\boldsymbol{u}} + \boldsymbol{K}_m\boldsymbol{u} = \boldsymbol{Q}_{nc} \tag{2}$$

In expression (2) the ***M*** matrix represents the mass generalized matrix, where inertia moments for rigid body, solar panels and reaction wheel actuators, the generalized moments for solar panels deformations and the generalized mass for solar panels are included. The gyroscopic matrix is ***G*** including the results for calculated inertia moment, and lastly the stiffness matrix $\boldsymbol{K_m}$ including the effects of angular movement around the Earth and the damping ratio considerations. The internal damping of the structure is integrated in the mathematical model by adding any suitable value to the mathematical model. The generalized forces $\boldsymbol{Q_{nc}}$ include those non conservative forces applied on the spacecraft with relevant repercussion on attitude movement. It must be taken into account that the action from internal actuators, such as reaction wheels, are taken into account by means of their angular moments. So, reaction wheel actions together with external actions complete the system model leading to a complete definition of $\boldsymbol{Q_{nc}}$.

The vector ***u*** represents the generalized coordinates vector. The components of this vector are chosen to match with the objective of the controller design. In order to that, the elements of the vector ***u*** can be measured by suitable sensors, such as angular rotations sensor, angular rate sensors and sensors to measure the solar panels bending:

$$\boldsymbol{u} = \begin{bmatrix} \phi & \theta & \psi & q_1 & r_1 & q_2 & r_2 \end{bmatrix}^T \tag{3}$$

The mathematical model (2) represents the non linear model of the system, in other words it is the real model. The Figure 3 shows the relation between linear and nonlinear systems. This nonlinear model must be linearized around an equilibrium point, considering in this case a fine pointing of the spacecraft to Earth. This requirement leads to the linear model of the system. The relation of the system matrix (2) with the linear model is:

$$\boldsymbol{A} = \begin{bmatrix} 0 & \boldsymbol{I}_{nxn} \\ -\boldsymbol{M}^{-1}\boldsymbol{K}_m & -\boldsymbol{M}^{-1}\boldsymbol{G} \end{bmatrix} \qquad \boldsymbol{B} = \begin{bmatrix} 0 \\ \boldsymbol{M}^{-1}\begin{bmatrix} 0_{(n-m)xm} \\ \boldsymbol{I}_m \end{bmatrix} \end{bmatrix} \qquad \boldsymbol{C} = \left[\boldsymbol{I}\right] \qquad \boldsymbol{D} = 0 \tag{4}$$

Leading to the linear system:

$$\dot{\vec{x}} = \boldsymbol{A}\vec{x} + \boldsymbol{B}\vec{u}$$
$$\vec{y} = \boldsymbol{C}\vec{x}$$
(5)

In this system the state matrix is ***A***, the control matrix is ***B*** and the output matrix is ***C***. It is assumed that the system is controllable and observable. In this system is considered a full access to the states of the system. In this case the output feedback system becomes in a state feedback system. A particularity of the system (5) it is related with the state matrix ***A.*** This matrix is considered as an ill-conditioned matrix by the nature of its elements, in particular those related with solar panels vibration. This characteristic affects the robustness of the system. The state vector is represented by the Euler angles and solar panels displacements together with their first derivatives as:

$$\bar{x} = \begin{bmatrix} \phi & \theta & \psi & q_1 & r_1 & q_2 & r_2 & \dot{\phi} & \dot{\theta} & \dot{\psi} & \dot{q}_1 & \dot{r}_1 & \dot{q}_2 & \dot{r}_2 \end{bmatrix}^T \tag{6}$$

Assuming that the system is controllable and observable and that the output of the system are the states of the system the control law required is [4][5]: $\vec{u} = -\boldsymbol{K}\vec{y} = -\boldsymbol{K}\boldsymbol{C}\vec{x}$

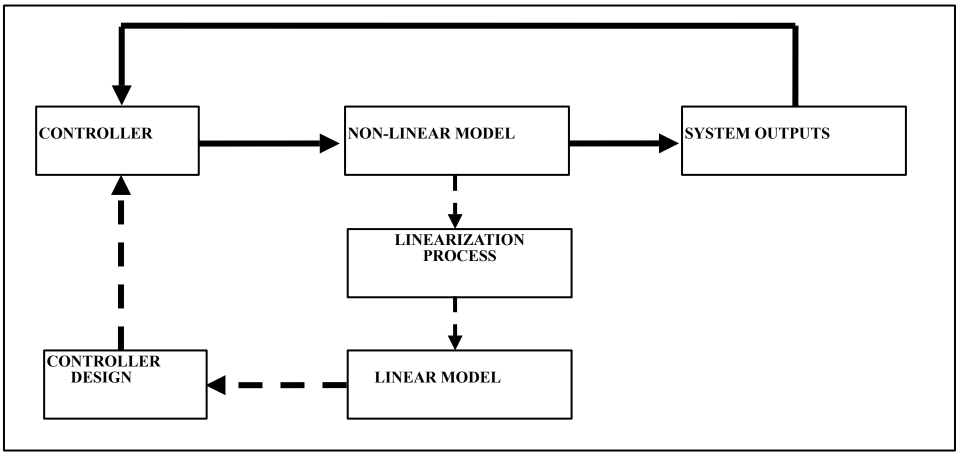


Figure 3: Linearization process.

3. System Analysis

A modal analysis performed to the system reveals the main characteristics of the system in terms of natural frequency and damping system modes [6]. By means of this analysis the orbital and flexion modes are identified and characterised. This trade off is carry out with the nonlinear system linearized around the equilibrium point.

The data taken from open loop system analysis are shown in the following table, where is possible to relate the eigenvalue with the system modes. Also damping and natural frequency values are shown, together with sensitivity of any eigenvalues and the condition number. The condition number is an indication of the system robustness. Values of the condition number close to zero indicate that the system is robust, while large condition numbers indicate poor system robustness.

Dynamics	Eigenvalues	Damping	Natural Frequency (rad/ sec)	Eigenvalue Sensitivity	Condition Number

Orbital Modes	**Roll**	±2.91e-002	±1.00e+000	2.91e-002	2.8472e+002 2.9477e+002	1156.0321
	Pitch	0 ± 3.24e-002i	-2.68e-017	3.24e-002	1.5444e+001	
	Yaw	-6.15e-004 ± 3.48e-002i	1.76e-002	3.48e-002	2.8959e+002	
Bending Modes	**Flexion**	0 ± 2.77e+001i	-1.69e-014	2.77e+001	1.3892e+001	
	Torsion	0 ± 4.00e+002i	0	4.00e+002	2.0000e+002	

Table 1: Open loop system data.

The relation of the system eigenvalues and their modes are determined by the value of the corresponding eigenvectors. The eigenvectors reveals the coupling existing in the system.

The stability of flexible spacecraft depends on the inertia moment values. This is conditioned by the spacecraft configuration that is, the position of the solar panels in respect to the main rigid body. In this system pitch and yaw eigenvalues are stable while the roll eigenvalues stand on the right semi-plane of the complex plane. The system is unstable ought to the roll eigenvalue.

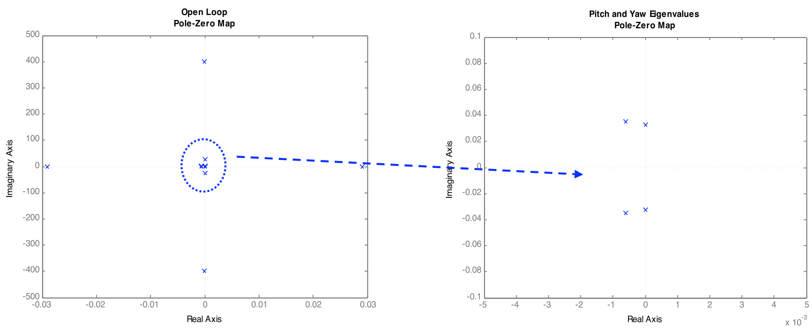


Figure 4: Open loop eigenvalues in the complex plane.

The open loop analysis has shown that the system is unstable, and the coupling existing between orbital modes and bending modes. One of the main requirements done to the controller is related with the coupling of the system. The objective is to get a whole decoupled system to avoid mechanical interactions between the rigid and flexible elements of the spacecraft.

Most attitude control problem of flexible spacecraft are concerned with require and passive damping control of the structural modes. The flexible modes in this kind of vehicles are often stabilized by means of collocated sensors and by employing rolloff or notch filters. The challenge of this paper is focused on obtaining a static controller ***K*** by application of the Eigenstructure Assignment method to obtain a suitable decoupling between structural and orbital spacecraft modes.

4. Control Design

The problem of attitude manoeuvres is submitted to the design of a regulator where attitude angles and attitude rates lead to a known reference. In rigid spacecraft this implies the movement in a coordinated way of the spacecraft structure. However, in flexible spacecraft the attitude motion to get a known reference may cause the excitation of vibration modes belong to the solar panels. The vibration modes attenuation is one of the main purposes in attitude manoeuvres in order to avoid potential material stress. This can be get by means of a controller that introduce the capability of decoupling the orbital spacecraft motion to get the referenced point, and the flexible modes introduce by the induced vibration appeared in flexible appendages, such as solar panels. The decoupling characteristic may be obtained by means of controllers design under EA method considerations.

The closed loop system is configured as a regulator. The main pointing requirement is to maintain the instruments of the spacecraft facing to the Earth. The following diagram shows this configuration.

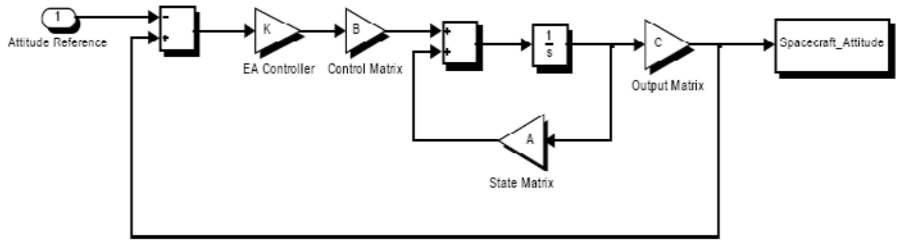


Figure 5: Block diagram of the MIMO system.

The EA method implies the values of choosing eigenvalues and eigenvectors in order to determine the close loop system performance. This implies a good knowledge of the open loop system, understanding and identifying the system modes. In order to design the controller some performance specifications must be identified. These are settling time, overshoot and response time. These system characteristics must be contained in the closed loop desired eigenvalues. The Table 2 shows the decoupling criteria taken for the controller design. In this table, the symbol "x" means a non-mater condition, while the "0" symbol indicates a whole decoupling between modes.

	Orbital Modes			Bending Modes	
	Roll	**Pitch**	**Yaw**	**Flexion**	**Torsion**
Roll	X	X	0	X	0
Pitch	X	X	X	X	0
Yaw	0	X	X	0	X
Flexion	X	X	0	X	X
Torsion	0	0	X	X	X

Table 2: Decoupling criteria.

5. Simulations

Based on the mathematical model previously presented several approximations to a final controller has been done. One of the mayor concerns is the behaviour of the bending modes (flexion and torsion) when the spacecraft is performing any attitude manoeuvre. The coupling between flexion and orbital modes may be determined by means of a system modal analysis [7]. Splitting the MIMO system in three single SISO channels is possible to understand the potential coupling. The bending modes appear in frequencies around 27 and 400 rad/sec. One of the main tasks that the controller should be done is move the corresponding eigenvalues to positions more damped, because the position of they in the complex plane lie in the imaginary axe.

The EA method requires the definition of the desirable eigenstructure, to work with the MIMO system. This requirement must be based on a good knowledge of the spacecraft dynamics and its interaction with external environment. Perturbations as magnetic, solar and aerodynamics can cause non desired attitude movements.

The EA method to design the controller works with two eigenstructure. One of them is the called desired eigenstructure, which is the closed loop system behaviour once the controller has been set up in the loop. The second one is those called the obtained eigenstructure. This eigenstructure is the actual one obtained after application of the EA method. The desired eigenstructure is characterised by the eigenvalues and eigenvectors required. Some times is not possible to obtain the eigenstructure desired, so a trial and error process is opened to obtain suitable results for the controller. The following table shows the desired and obtained eigenvalues and how they related with system modes.

Dynamic Modes	Desired Eigenvalues	Obtained Eigenvalues
Roll	-6.1457e-002±3.4821e-002i	-6.1457e-002±3.4821e-002i
Pitch	-8.6736e-02±3.2408e-002i	-8.6736e-002±3.2408e-002i
Yaw	-2.9122e-00+2i±2.9122e-00-2i	-2.9122e+000±2.0000e+000i
Flexion	-0±2.7746e+001i -0-2.7746e+001i	-6.5725e-13±2.7746e+001i
Torsion	-0±4.0000e+002i	-1.3058e-009±4.0000e+002

Table 3: Eigenstructures desired and obtained.

The performances of these eigenvalues in terms of natural frequency, damping and sensitivity are shown in the following table. Also is incorporated in the table the condition number of closed loop system. This number is higher than the open loop condition number. This implies that the closed loop system is less robust than open loop system. The Table 4 shows these data.

Dynamic Modes		Eigenvalues	Damping	Natural Frequency (rad/sec)	Eigenvalue Sensitivity	Condition Number
Orbital Modes	**Roll**	-6.1457e-002±3.4821e-002i	8.70e+03	7.06e+02	2.8267e+002	4.7928e+004
	Pitch	-8.6736e-002±3.2408e-002i	9.37e+03	9.26e+02	1.0294e+002	
	Yaw	-2.9122±2.000e+000i	8.24e+03	3.53e+04	1.0456e+003	
Bending Modes	**Flexion**	0 ±2.7746e+001i	2.37e-10	2.77e+05	2.1760e+004	
	Torsion	0 ±4.0000e+002	3.26e-08	4.00e+06	8.9076e+003	

Table 4: Eigenstructures desired and obtained.

The pole-zero map depicted in Figure 6 shows the position of the eigenvalues in the complex plane. It is interesting to highlight that some eigenvalues have been move to new positions, according to the desired eigenstructure. Some eigenvalues stay in the same position that in open loop. The torsion eigenvalues stay in the same position and the flexion eigenvalues have been damped in respect to the open loop.

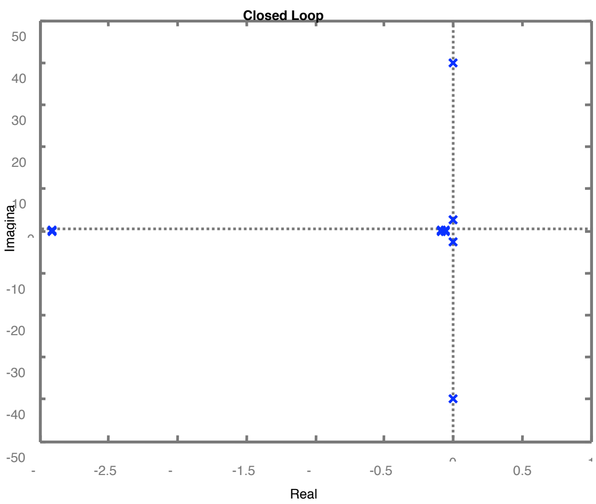


Figure 6: Closed loop eigenvalues.

In order to verify if the main requirement imposed to the system, that is the system decoupling has been obtained, a step simulation have been performed in both linear and non-linear systems. The Figure 7 shows these tests. The graph shows that the behaviour of linear and non-linear systems is similar for correction manoeuvres. The figure also shows that the decoupling have been obtained between orbital and bending modes. In this sense, a roll manoeuvre is decoupled of the flexion mode, so in the real system do not caused solar panel bending. This situation is also applicable to solar panel bending in the transverse motion. With the desired eigenstructure has been obtained a relative decoupling of the orbital modes. So any motion around any spacecraft axe induces light movement in the rest of the axes. This situation is not concern in excess, because of the time response obtained for the closed loop system.

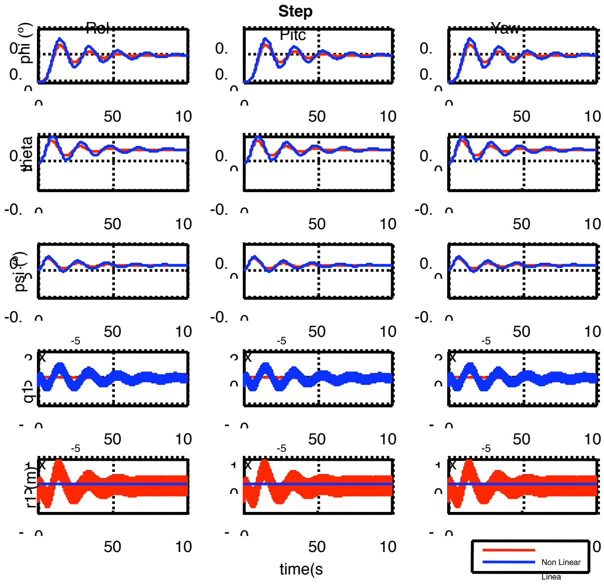


Figure 7: Linear and non-linear step responses.

An interesting possibility in designing the attitude control is based on the design of independent controllers for the three attitude channels. This consideration only is valid for the linear systems. In this paper the job hypothesis considers both linear and non-linear systems have a demonstrated coupling in orbital modes.

6. Robustness Analysis by LFT.

Spacecraft suffer changes in the parameters of its mathematical model due to the action of external disturbances. It is also necessary to consider the action of non-modelled dynamics and the uncertainty that have the parameters of the system. All these factors affect the robustness and tolerance to both internal and external disturbances. Therefore there must be a suitable method of analysis to the characteristics of the system that enables the behaviour of both high frequency and low frequency. In general there are three methods considered for the analysis of robustness of the system:

- Condition number: It is the basic robustness method employed in this job to know basically the system behaviour.
- Sensitivity functions: It is the general method based on the knowledge of the sensitivity function and the complementary sensitivity function.
- Mu-analysis: It is useful to determine robust stability, nominal performance and robust performance.

The model with their perturbations and disturbances are shown in Figure 8. In the disturbances that were observed in any system. Disturbances affect the control signal, the system output and furthermore the noise introduce in the measurements of the sensors.

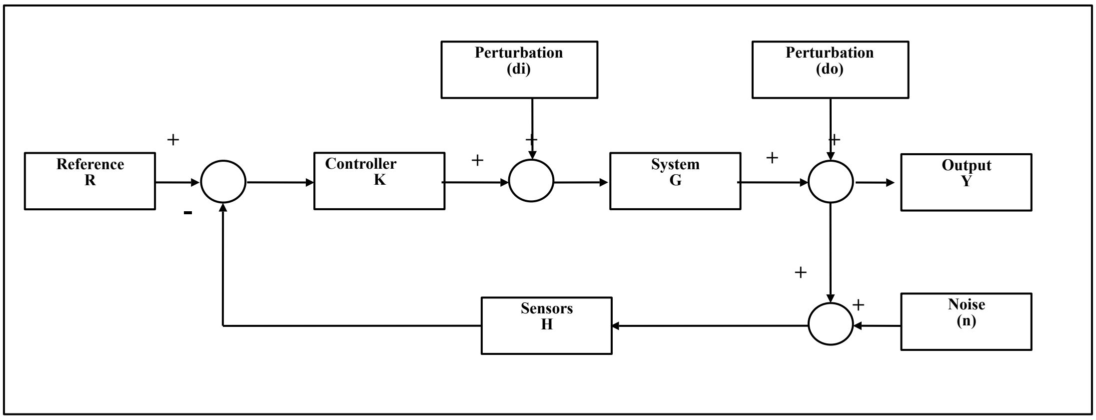


Figure 8: Disturbances affecting the system.

A linear fractional transformation (LFT) has been used to model the system with the mentioned disturbances. The following figure shows the LFT model in which Δ represents the system uncertainty. In this LFT representation the controller ***K*** has been obtained by the EA process. It must be taken into account that the EA method applied to the MIMO system can be potentially robust defining some suitable locations for the closed loop eigenvalues and selecting the eigenvector to ensure robustness.

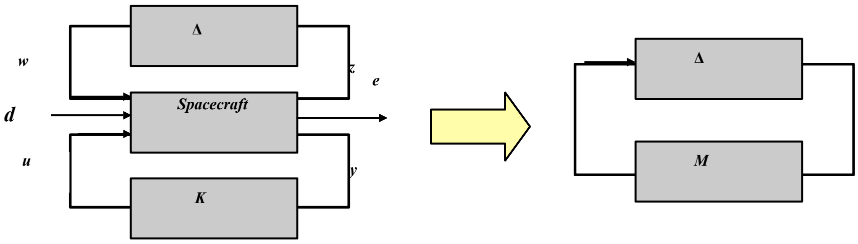


Figure 9: Linear fractional model.

The nominal performance, robust stability and robust performance are obtained by means of ser of weighting functions. These considerations lead to obtain the upper and lower bounds for all the frequencies. The Figure 10 depicts the system behaviours.

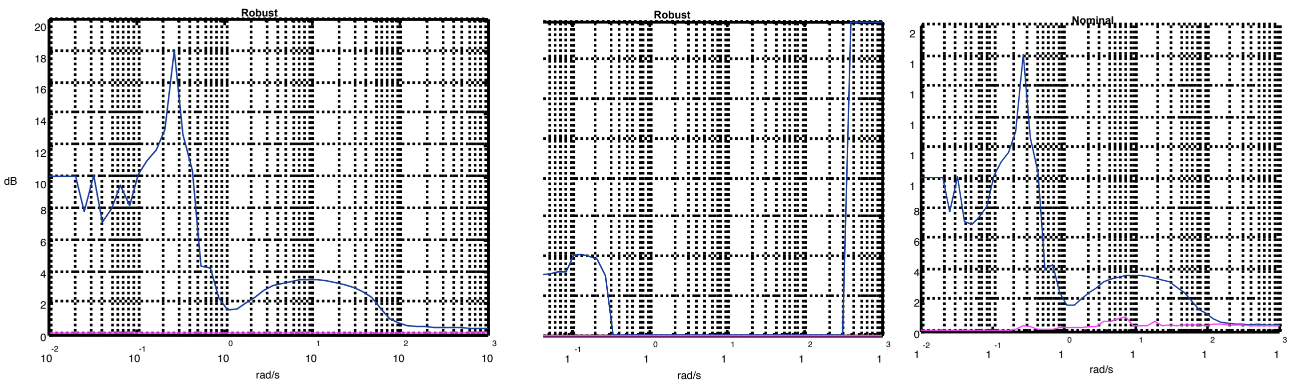


Figure 11: Robust performance.

The robustness results depicted in the figure show that at low frequencies the system has a good tracking for robust stability, nominal robustness and nominal performances. At intermediate frequencies the system shows good responses for nominal performance and robust stability, and at high frequencies the upper and low bounds of the frequency responses are not suitable.

7. Conclusions

The problem of control a flexible spacecraft in which the flexible appendages are solar panels has been exposed in this paper. The objective of the controller is to stabilize the open loop system as main premise, obtain a suitable decoupling between spacecraft modes and finally to get a robust closed system. By application of the EA method to design the controller has been obtained the stability of the system and the decoupling of system modes. The EA method does not provide intrinsic robustness to the system, so this performance has to be analysed carefully. In this paper a mu-analysis has been performed showing good system behaviour to low and intermediate frequencies. One of the advantages of the EA method is the relative simplicity of application and the possibility of obtain static controllers to meet the system requirements. The EA method is classified as a modal process in the design of controllers. So, it is very important to have a good mathematical model of system to work with precision.

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