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Turbulence in a supersonic spatial flow over a flat plate

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Abstract
In the framework of Navier-Stokes equations, it is shown that when exposed to external disturbances in the form of low-intensity harmonic waves, a linear instability is observed on a plate immersed in a supersonic gas flow, which then becomes non-linear, leading to stochastic gas movement. The parameters for the self-sustaining turbulence have been determined. The flow patterns on the plate surface and the flow structure inside the turbulent boundary layer are presented.

1. Introduction

The problem of laminar-to-turbulent transition of a supersonic gas flow with the manifestation of linear and nonlinear instability has been the subject of many studies, both theoretical research and numerical and experimental modeling [1-11]. This work, being a continuation of [9, 10], relates to a development of a turbulent flow over a plate when no external perturbations act on it. Earlier in [10], it was obtained that, when external disturbances interact in the form of harmonic waves of intensity 1-2% w_∞ with the boundary layer, a resonant amplification of waves occurs (depending on the number M, boundary layer thickness, wavelength and plate width), forming strong almost two-dimensional waves, which decay to form coherent structures. Thus external perturbations, turning into strong waves and breaking up, as if instantly shake the gas flow, after which it is turbulized according to the scenario of [3]. When the intensity of the decaying waves (pulsations) becomes small, then for a certain number Re (along the plate length) a "slow" gas is ejected from the viscous sublayer to the outer boundary of the boundary layer, whose thickness doubles (M=2). And the flow pattern remains the same, thus recalling the "established turbulence" [6]. With an increase in the Re number, the flow becomes turbulent as a result of the diffusion of the vortex structures thus formed. Here the distribution of flow parameters on the plate surface (pressure, temperature, surface friction coefficient) corresponds to the experimental data.
This can be explained as follows: in turbulent flows, the main energy is contained in large-scale vortices [3], the latter simulated precisely enough in the numerical method [12] and many others, incl. [8]. So if no stochastic movement with a natural frequency (no turbulent spots either) are formed in the flow, the principal average flow parameters practically coincide both for numerical modeling and the experiment.
Thus, as shown in [10], with sufficiently strong external disturbances up to Re $\approx 2 \times 10^6$, no ripple formation with a natural frequency occurs, since the disturbances, passing through the head shock wave, move over the boundary layer, and they impose on the flow their frequency and intensity suppressing the natural frequency.
This work shows that as the intensity of external waves decreases by an order of magnitude in the flow, a stochastic motion is formed (with its own frequency, with the formation of turbulent spots), which is even preserved when the external disturbances are "turned off", i.e. the energy that is dissipated in the turbulent region is extracted from the energy of the incident stream.

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2. Problem formulation

Fig. 1 shows how a supersonic viscous gas is flowing around a rectangular plate (where z is the longitudinal, x is transverse, y is the normal coordinates; w, u, v are the velocity components along these coordinates).

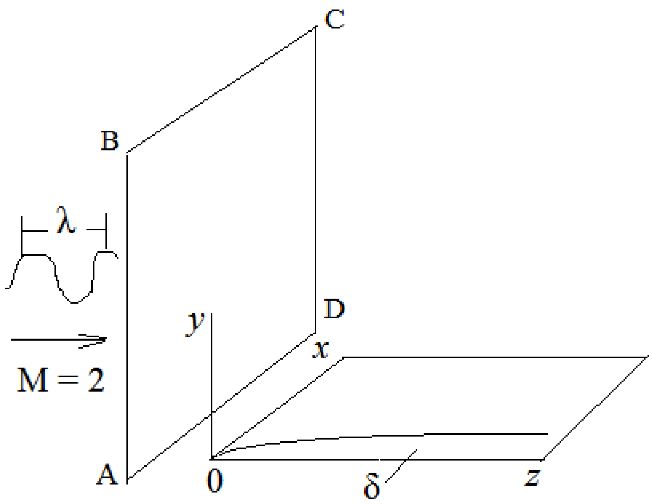

Fig. 1. Calculated area schematic

In the front section ABCD of the rectangular calculated area, the parameters for the incident gas stream are sustained. First, a quasistationary flow past a plate with a boundary layer thickness δ is established. Then, a flat harmonic wave with wavelength λ and a certain intensity is superimposed with the free-stream velocity w_∞. The Re number, determined by the length of the plate, for spatial calculation is approximately equal to 10^6. We consider a symmetric flow along the y axis, thus we are presenting the pictures for only one plate surface. The results are obtained by direct numerical simulation of the flow in the framework of the Navier-Stokes equations, without involving turbulence schemes; an explicit two-step difference scheme is used [12]. Calculation accuracy: δ in the undisturbed flow for spatial flow around a short plate with an accuracy of 10-15% coincides with the value calculated in the central longitudinal section of the plate using the approximate formula for $w_{0.99}$. The geometrical dimensions shown in the figures are related to the length of the plate; pressure p, density R are related to their values in the incident flow, and velocity components to the velocity of sound of an unperturbed flow. The maximum number of calculation points is about 1.7 x 10^7; spatial steps – 10^{-6}-10^{-5} m, time steps – 10^{-9} s.
It should be noted that initially, for spatial flows, calculations with a symmetric distribution of external disturbances are shown over the entire area of the original section ABCD (Fig. 1), then, after turbulization of a part of the flow, the external disturbances are "turned off".

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3. Calculation results

Gas flow is calculated around a heat-insulated plate with a velocity w_∞, parameters $p_\infty = 1$ atm, $T_\infty = 278$ K, $R_\infty = 1.25$ kg·s^2/m^4, $\mu_\infty = 1.72 \times 10^{-5}$ kg·s/m^2, Prandtl number Pr = 0.72 for the incident flow M=2.

3.1 Distribution of flow parameters on the plate surface.

Fig. 2a and 2b show patterns of pressure distribution for a plate in a gas flow with M=2 and an initially plane harmonic wave with a frequency of 780 kHz for times t_1 and t_2. The frequency is taken from the following condition: the maximum amplification of incident disturbances occurs at a wavelength of the order of magnitude of laminar boundary layer thickness [10].

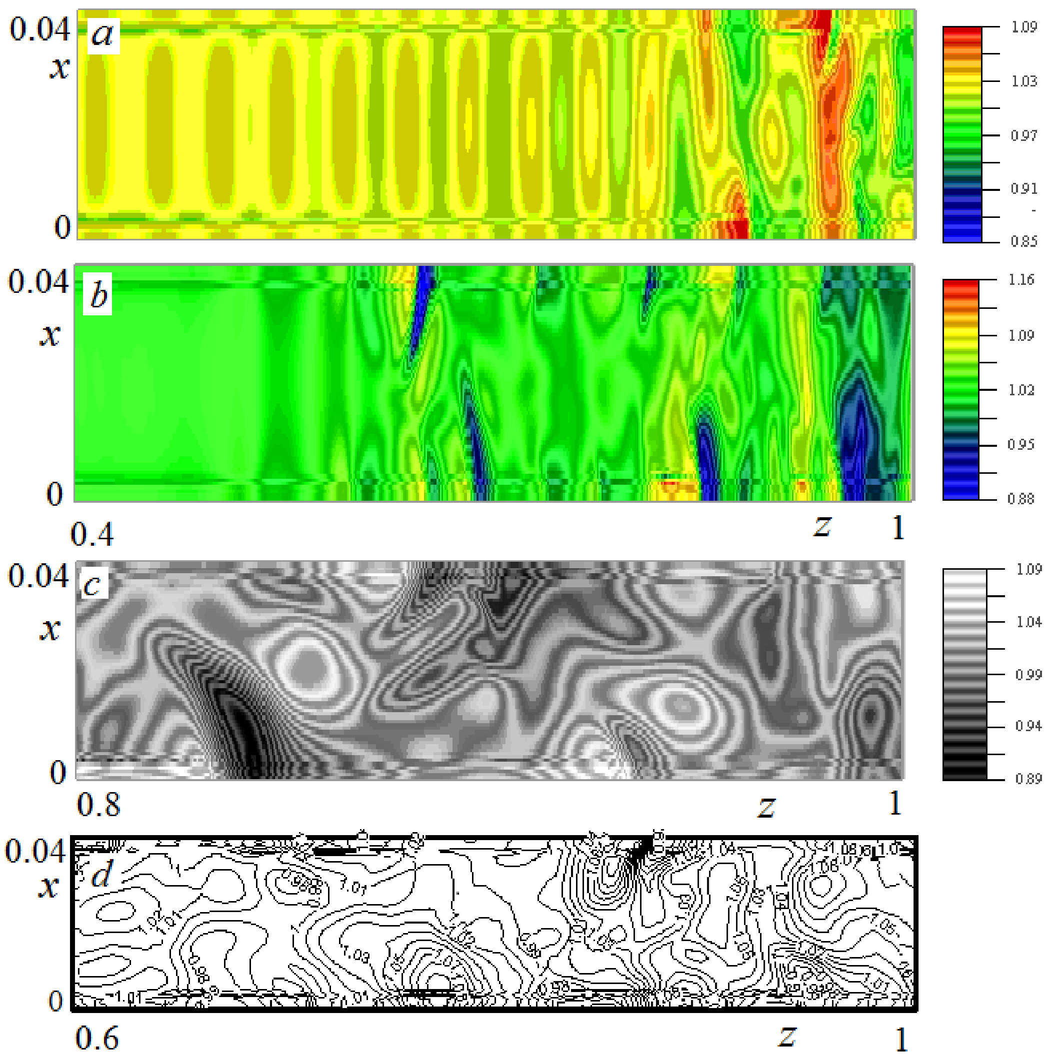

Fig. 2. Pictures of pressure fields on the plate surface in a turbulent gas flow: a) the initial stage of flow turbulization, the destruction of the initial external disturbances in the form of harmonic waves; b) the developed pattern of turbulent flow obtained without the influence of external disturbances supported by energy from the main flow; c) d) turbulent spots and lines of equal isobars on the plate.

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In Fig. 2a at the leading edge, one can see traces of an incident wave of weak intensity, which do not lead to the formation of a secondary flow in the form of longitudinal vortex structures along the sides of the plate. When the wave intensity is weak, the flow is almost quasi-two-dimensional and the transition period occurs almost linearly. Pressure and entropy increase gradually. The temperature is almost constant. This flow development picture differs significantly from the case with disturbances of a sufficiently strong intensity, which leads to the spatial distribution of flow parameters on the plate almost from its leading edge [10] and the flow parameters in the developing instability wave.
At the plate trailing edge (Re ≈ 10^6), gas flow is turbulized. The external waves here are destroyed by intense pulsations formed with their natural frequency. Spatial turbulent spots of various size appear. The development of the turbulent region at time t_2 (after 2 x 10^{-4} s) is shown in Fig. 2b, where a wide area is visible with chaotic movement of spots of size (diameter) from 0.1 mm to 1.5 mm. At the moment, the dimensions of the turbulent region have reached their maximum values, and at subsequent moments, the wave front retreats back, oscillating with a frequency of ≈ 10 kHz in the range: $0.45 < z < 0.85$ ($0.45 \times 10^6 < Re < 0.85 \times 10^6$). The speed of the front advance towards the oncoming flow from the position of Fig. 2a is currently ≈ 25 m/s. When the turbulence front retreats, on the plate surface there remain areas of weakly perturbed gas in the form of transverse waves, the length of which is equal to half the plate width. This gas behavior is explained by the fact that, in the interaction of a turbulent region with side vortices, regions of increased vorticity are formed, which are preserved during the retreat of the turbulence zone. When an oncoming gas flows around these areas, transverse waves are formed due to its deceleration.
It should be noted that at the time t_2, external disturbances in front of the plate are "disconnected", and at this moment there are no traces of external waves at the plate leading edge right up to the turbulent region. A self-sustaining turbulence is realized here, when the energy dissipated in the boundary layer is compensated by the energy coming from the main flow. A more detailed picture of the flow in the turbulent zone at the plate trailing edge is shown in Fig. 2c and with isobar values – in Fig. 2d.
During flow turbulization, changes occur in the behavior of the gas parameters along the plate surface. Fig. 3 shows their instantaneous values along the plate in the central longitudinal section: entropy s (curve 1), temperature T (2), pressure p (3), surface friction coefficient c_f (4).

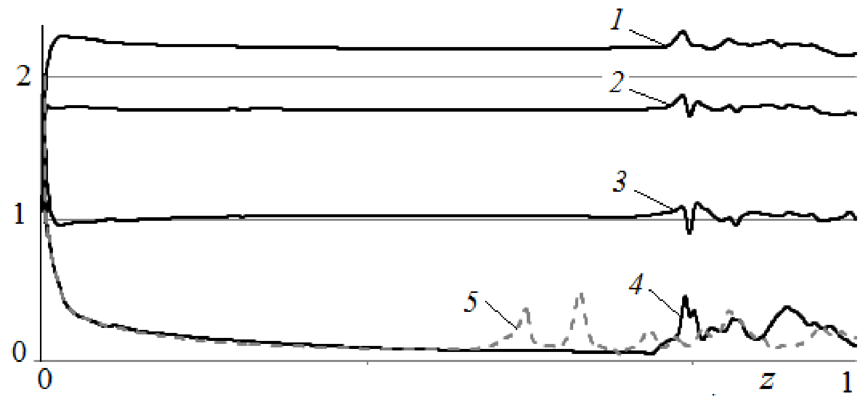

Fig. 3 Behavior of gas parameters: s (curve 1), T (2), p (3), surface friction coefficient c_f (4) along the plate in the laminar and turbulent regions. Curve 5 corresponds to c_f at time t_2.

It can be seen that the values of all parameters increase during flow turbulization, which corresponds to experimental [1,4] and numerical data [8]. Curve 5 shows the distribution of c_f at time t_2. The same behavior at a given time is observed with other parameters.
The behavior of the gas parameters on the plate is directly related to the pulsating gas above the plate.

3.2 Gas flow above the plate within the boundary layer

Fig. 4a shows the instantaneous density distribution in the central longitudinal section of the calculated field. One can see a laminar boundary layer (at t_2), turning into a turbulent one consisting of various pulsating regions. Some of these perturbed regions tend to escape from the zone of the boundary layer into the oncoming supersonic flow, which leads to the formation of inclined waves beyond the boundary layer. The angle of inclination of the waves should be determined by the intensity (speed) with which the gas passes the outer boundary of the boundary layer.

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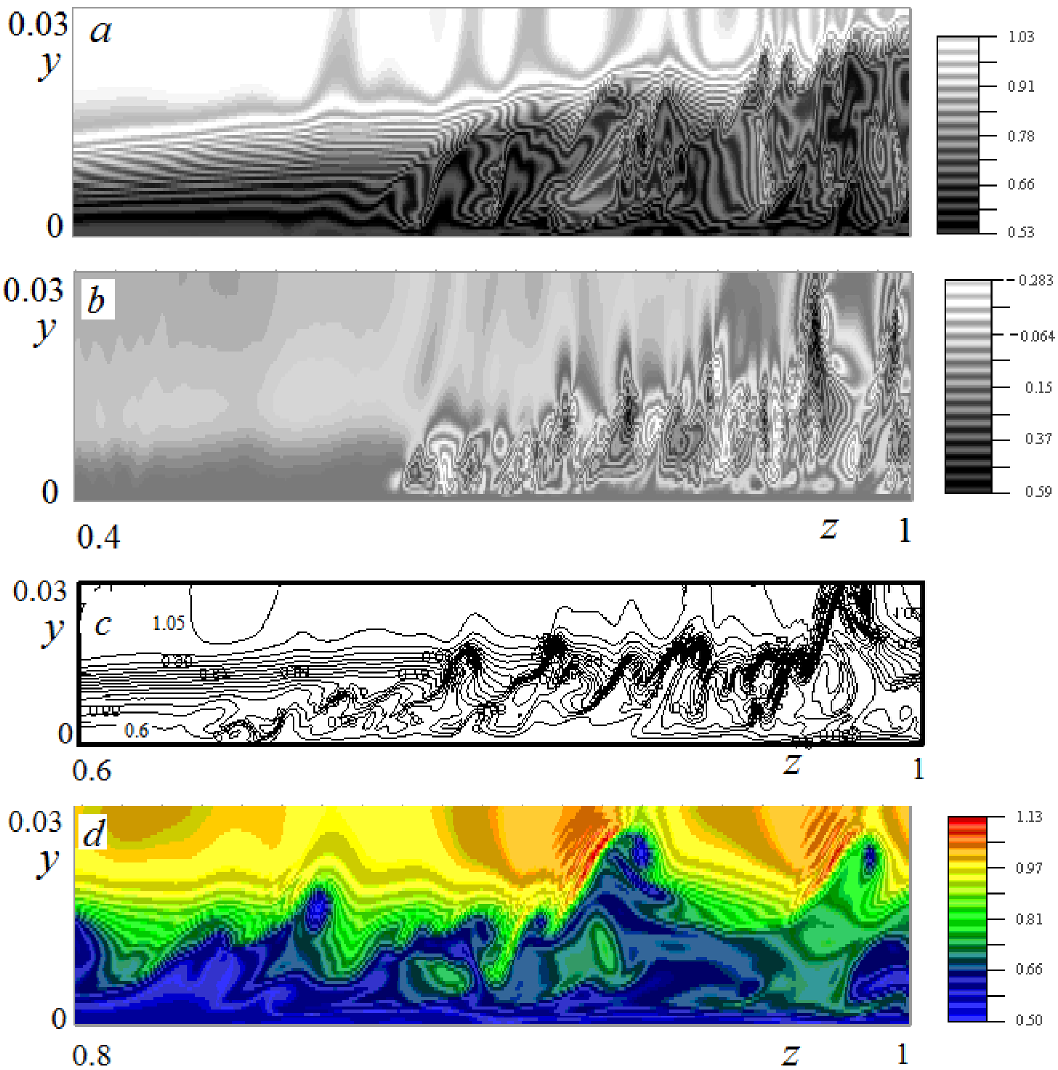

Fig. 4 Flow patterns inside the boundary layer (instantaneous values in the longitudinal central section): a) density field, b) velocity values normal to the plate surface, c) quasi-periodic emissions of a viscous fluid from the near-wall region – burstings, d) radiation of entropy waves in the interaction of the bursting with the outer boundary of the boundary layer.

If very weak disturbances propagated from the boundary layer, Mach waves would form with an inclination angle of 30°. Yet the calculations give the angle of inclination of the waves as ≈ 45°-70°. To clarify the angle of the generated waves in Fig. 4b, in the central longitudinal section, the isosurfaces of the pulsations of the velocity component v normal to the plate surface are given. The dimensionless component v varies in the range of -0.3÷0.6, and it takes on its maximum value on the outer boundary of the boundary layer at the burstings' vertices – quasiperiodic emissions of viscous fluid from the near-wall region of the flow (Fig. 4, c, d). Taking into account the gas release rate from the boundary layer, one can estimate the angle of inclination of the waves as arcsin 0.8 ≈ 54° using the formula for the Mach line inclination. Considering that the generated waves are not of zero intensity, the value of this angle should be greater. Thus, it was shown in the work that when a flow is turbulized, waves with a tilt angle of ≈ 45°-60° are generated. The smallest slope is obtained for a wave that is formed from a moving front of the turbulence zone.
The introduction of oblique waves in the works of experimental [4-6] and numerical modeling [7, 8, 12] contributes to the flow turbulization, although in nature they arise themselves as a response of an external flow to pulsations of a turbulent flow inside the boundary layer.

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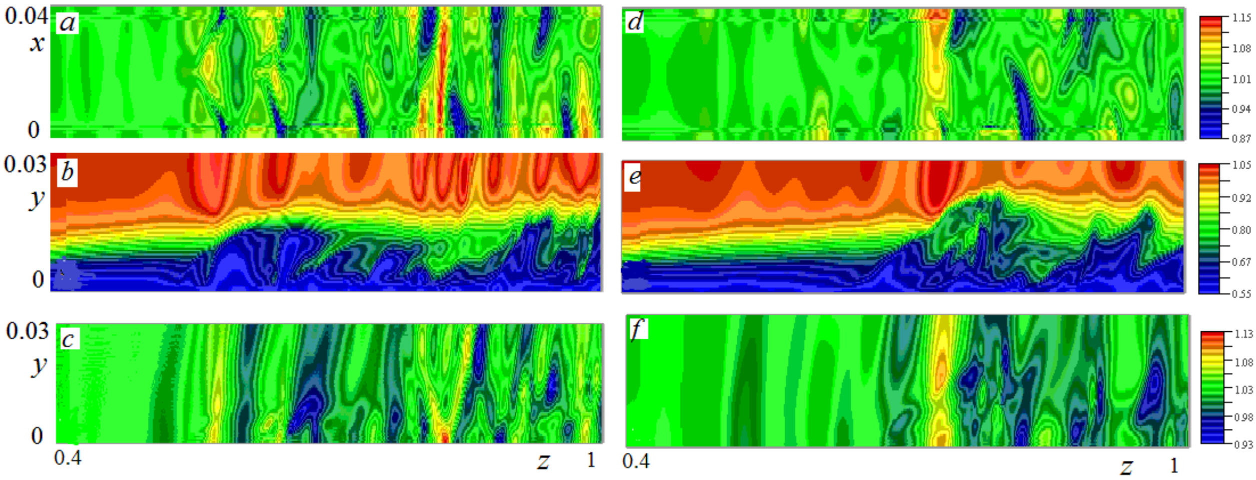

Fig. 5 The plate flow-around pattern for two points in time: a) and d) are the pressure fields on the plate surface; b), e) and c), f) are the density and pressure distributions in the longitudinal central section of the calculated field.

The dynamics of the development of turbulent flow for two points in time is shown in Fig. 5 in the form of pressure distribution on the plate surface (a, d), density in the longitudinal central section of the calculated field (b, e), pressure in the longitudinal central section (c, f). The figure shows how far the front of turbulence retreated in time ≈ 10^{-5} s. When gas flow becomes turbulent, a change in the velocity profile occurs in the boundary layer. Fig. 6 shows instantaneous values of the longitudinal velocity for two points a and b (step 10^{-6} s) in the boundary layer's cross sections. The curves in figures 1-4 correspond to the velocity profiles in sections, where the flow is laminar (z = 0.2), transitional (0.4) and turbulent (0.8, 0.95).

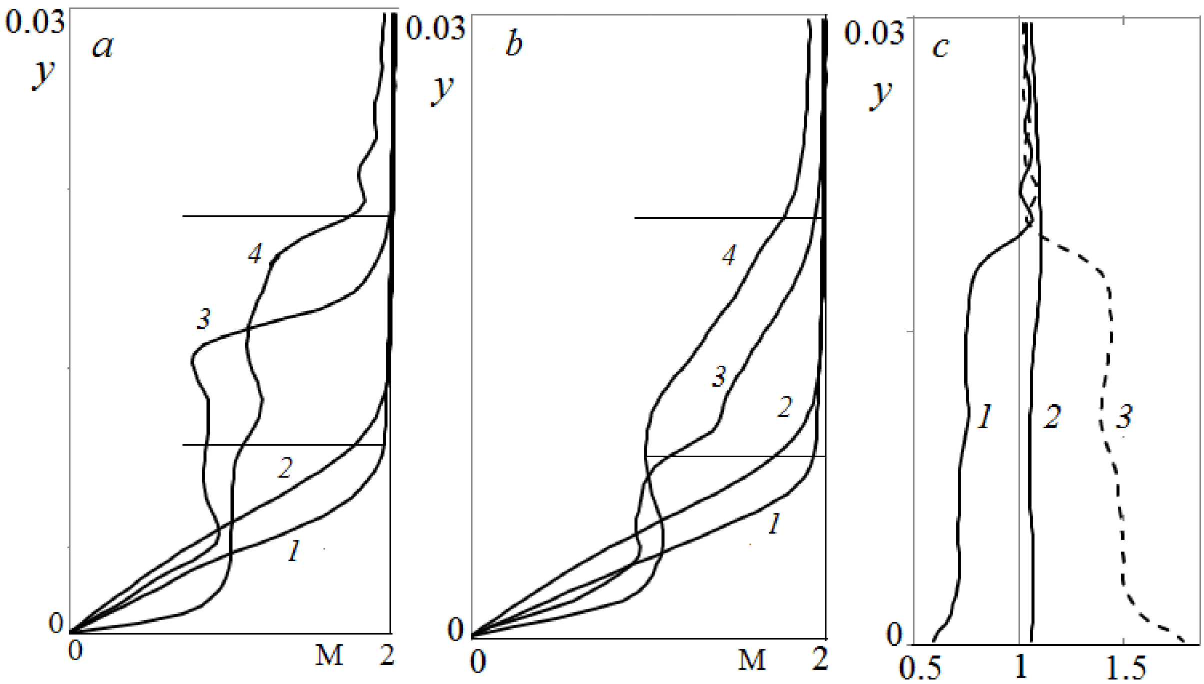

Fig. 6 Behavior of flow parameters along the boundary layer cross section: a) and b) longitudinal velocity profiles for two points in time; curves 1-4 correspond to velocity profiles in sections, where the flow is laminar (z = 0.2), transitional (0.4) and turbulent (0.8, 0.95); horizontal lines – boundary layer thickness δ (both laminar and turbulent), c) density (1), pressure (2) and temperature (3).

The analysis shows that, due to nonlinear amplification of disturbances, the laminar flow develops into a transient and then into a turbulent state, which is characterized by a more "filled" velocity profile. The horizontal lines in the figures corresponding to the thickness of the laminar and turbulent layers indicate that the δ for the turbulent layer is twice as large. This corresponds to the experimental data [2] and the numerical calculations [10].

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Fig. 6 shows that both in laminar and turbulent flows, the velocity varies linearly in the viscous sublayer region. With distancing from the plate, this behavior is observed in turbulent flow regimes (curves 3, 4) up to numbers M=0.8-1, i.e. in the subsonic region. Depending on the Reynolds number (curves 3, 4), the velocity profile fill can be less for different times than for laminar flow. This means that the thickness of the viscous sublayer in a turbulent flow varies with time due to the ejection of a part of gas from the plate surface in the form of bursting.
The distribution of density values (curve 1), pressure (2) and temperature (3) is shown in Fig. 6, with cross section z = 0.95. Analysis shows that entropy waves propagate from the turbulent boundary layer to the external flow, which are visible in Fig. 4d at the time of the interaction of bursting with the outer boundary of the boundary layer. These waves are associated with the frequency of emissions of a fluid and therefore are irregular. So on figures 6a and 6b it can be seen that curve 4 outside the boundary layer demonstrates different behavior for different points in time.

3.3 Distribution of flow parameters on the outer boundary of the boundary layer

The bursting is an ejection of fluid from the plate surface. Fig. 6c makes it clear that towards the outer boundary of the boundary layer the gas is emitted with the parameters corresponding to the values at the boundary of the viscous sublayer, that is the gas is less dense and hotter than in the free-stream flow.
Fig. 7 shows the fields of the values of the vorticity component normal to the plate surface in the cross sections $y_1 = 1.5\delta$ (a) and $y_2 = 2\delta$ (b) and densities (c) and (d), respectively. It can be seen that, at a height of y_1, complex patterns and vorticity densities are realized in a turbulent flow. There are areas of lower density (hotter gas) than on the surface of the plate. Above the turbulent region in cross section y_2, the vertices of the bursting, consisting of small-scale eddies, are visible.

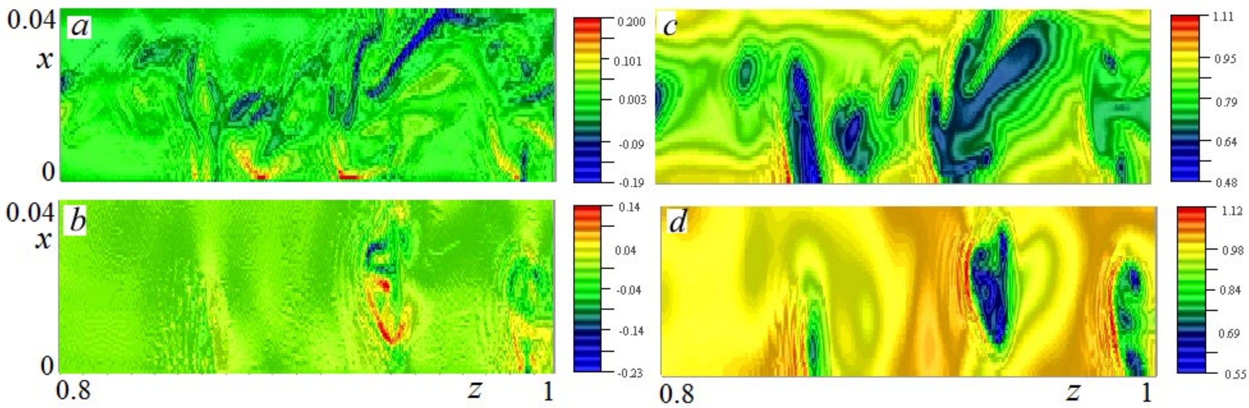

Fig. 7. Behavior of the vorticity component normal to the plate surface in the cross sections y = 1.5δ (a) and y = 2δ (b) and density (c) and (d), respectively.

3.4 Pulsations and discussion of results

Direct numerical simulation provides a huge amount of information, the value of which is the average flow values for comparing these results with solutions of problems obtained by other methods.

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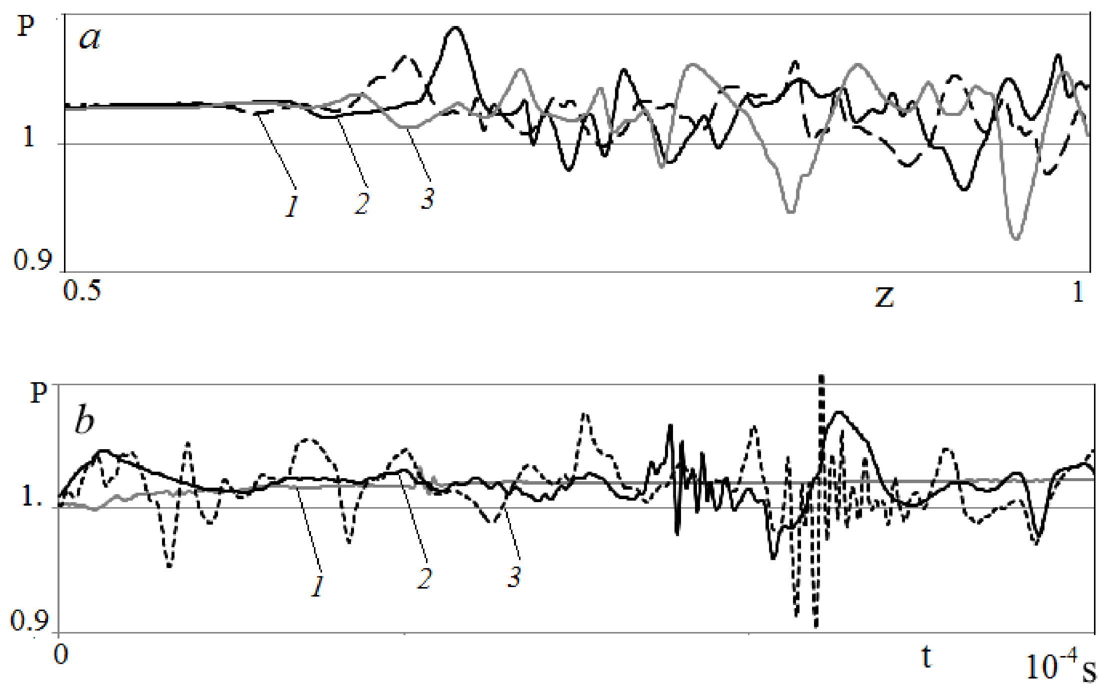

Fig. 8 Pulsations of pressure in a turbulent flow: a) instantaneous values of pressure on the plate for three points in time (step 1.5 x 10^{-5} s), b) the behavior of the pulsations at three points (z = 0.2, 0.8, 0.95) in time.

Fig. 8a shows the values of pressure pulsations along the plate in the turbulent part of the flow for three time points with a step of 1.5 x 10^{-5} s. These pulsations are born without external disturbances. The behavior of the pulsations at three points (z = 0.2, 0.8, 0.95) in time is shown in Fig. 8b. The frequency of the pulsations arising from the turbulence of the gas flow is in the range of 300 kHz - 3000 kHz. The calculation of the average values of pressure on the plate surface at the points z = 0.2, 0.7, 0.95 gives the corresponding results: 1.015, 1.052, 1.017. It can be seen that at the point with the coordinate z = 0.7, where the front of the turbulent region is located, the pressure is higher than in the surrounding gas. This causes the gas to move against the oncoming free-stream flow.
Detailed flow pattern data allow us to compare the fundamental laws of turbulence. So the local turbulence similarity property: $v_n/\Delta w \sim (n/L)^{1/3}$ [3], where n/L is the ratio of the ripple scale (size of small vortices) to the thickness of the turbulent boundary layer $L \approx 0.1$, v_n is the order of magnitude of the ripple velocity (normal to the plate) ≈ 0.4 (Fig. 4b), Δw – changes in the average velocity in the turbulent region ≈ 1. With this choice of average values of the velocity pulsations and the magnitude of the small vortices, the similarity property is reduced to 0.4 ~ 0.46. Given that some average values were chosen, it can be assumed that the similarity property holds.
The analysis of the results obtained by numerical methods for the flow of a supersonic gas around a plate with M=2 shows that the initial stage of transition and the flow turbulization strongly depend on the intensity of the external waves. At a sufficiently strong intensity of external disturbances in the form of harmonic waves, internal pulsations are "blocked", the cross-sectional size of the resulting large-scale longitudinal vortices increases in the process of diffusion as Re grows. And as a result of the interaction of two lateral longitudinal vortices, gas flow is turbulized. Thus in numerical simulation, this process substantially depends on the width of the plate. The dependence of the maximum growth of disturbances on the plate width is shown in [10].
When the plate is exposed to external disturbances of low intensity, stages of linear instability are observed, which later become nonlinear, leading to flow turbulization.

4. Conclusions

In the framework of the non-stationary Navier-Stokes equations, the problem of gas flow turbulization in a supersonic flow over a heat-insulated flat plate is solved, up to the scale when pulsations at natural frequencies occur. It is shown that the resulting turbulent flow is realized without the influence of external disturbances. The flow patterns on the plate surface and also the flow structure inside the turbulent boundary layer are presented. The connection of the resulting oblique waves with the outbursts of liquid from the surface of the plate in the form of bursting is explained. The implementation of the similarity of local turbulence is noted.

Acknowledgements

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