# Collision avoidance multi-vehicle distributed cooperative guidance strategy with high performance communication

Xi Zhang, Shui Yongtao, Wang Yonghai, Wang Liyan, Zhang Xuejiao Beijing Institute of Space Long March Vehicle, Beijing 100076 No.1, South Dahongmen Road, Fengtai District, Beijing, China calt14 zhangx3@163.com

#### Abstract

In order to achieve cooperative saturation attack of multi-vehicle, a three-dimensional cooperative guidance strategy with collision avoidance is proposed. Based on distributed De Bruijn communication topology, the states of vehicles are only needed to be communicated with vehicles in adjacent topology set, which results in characteristics of scalability and flexibility. Adopting the network synchronization principle, guidance can make the state of vehicles converge to target asymptotically. Considering collision avoidance, a synchronized strategy regarding safe distance control is used to prevent trajectory from collision. The simulations verify that this guidance law can complete the collision avoidance cooperative attack effectively.

#### 1. Introduction

With the improvement of the capability of modern concentrated vehicle defense system, the penetration of a single vehicle has can hardly cope with the complex combat environment. To improve the striking ability of vehicles and penetration efficiency, cooperative attack of multi-vehicle is an effective way<sup>[1-4]</sup>. By building the communication network of multi vehicles, the impact time of vehicles will tend to be consistent and the cooperative attack can be achieved, Moreover, compared with the single vehicle with the same total equivalent charge, cooperative attack can greatly increases the damage effect<sup>[5]</sup> as a result of aggregation effect produced by simultaneous multi-point explosion. As the key technology in cooperative attack, cooperative guidance law has been widely studied.

In order to achieve the cooperative attack, a time-varying coefficients guidance  $law^{[6]}$  is designed by adjusting the coefficient by the difference of the remaining time based on the communication network. Ref. [7] has a leader-follower strategy and follower approaches hypothetical virtual point by the state controller, which can achieve coordinated control of attack time and attack angle. Ref. [7,8] belongs to centralized cooperative guidance law, which has high requirements for communication network and low robustness. In order to decrease the demand of communication network, Ref. [9] design a finite time cooperative guidance based on the sliding mode control. However, it is hard to apply in the practical. A guidance law based on the optimal control<sup>[10,11]</sup> is designed to achieve space cooperative attack rather than time consistent. Using the network synchronization theory, a three-dimensional guidance law<sup>[12]</sup> is proposed. Then a backsteeping method is applied to control the attitude of vehicle, which is difficult to achieve in practical engineering.

At present, few foreign scholars have studied the three-dimensional cooperative guidance law considering both collision and communication topology. Inspired by the Ref. [1-12], the distributed cooperative guidance strategy with self-avoidance collision is proposed for multi-vehicle salvo attack, which is more practical to apply. Firstly, an undirected topology communication network is designed using the De Bruijn network, which has advantages of fault-tolerance, scalability and flexibility. Then, a distributed cooperative guidance law based on the network synchronization theory is designed. The safe distance control takes effect when approaching the target, which can ensure the self-collision avoidance of vehicles. And the convergence proof is given by Hurwitz theorem. Finally, the numerical simulations are performed that verify the effectiveness of the guidance law.

## 2. Description

#### 2.1 Method of control

In nature, the gregarious animals can communicate with each other and allocate resources to accomplish some challenging task, for instance, wild geese migration and ant colony foraging as shown in Figure 1. Inspired by nature, distributed control inherits the collaboration performance of social animals and has been widely applied in flight vehicle. The cooperative guidance of vehicle belongs to the distributed control of multi-agent system, which adopts the graph topology theory and network consensus principle of distributed control<sup>[13]</sup>. The vehicle is regarded as the unit of multi-agent system. Distributed control is characterized by the agent unit and topological network. All the units follow the rules of topology network and tend to be consistent in coordinating data through information sharing and interaction. Thus the units can complete task independently and collaboratively, which results in character of strong adaptive and scalability.



Figure 1: Large-scale unmanned aerial vehicle cluster in cooperative battle field

The control strategy of information interaction generally include centralized control and distributed control. Centralized control requires sufficient global information and do not have communication among the units. Distributed control takes the coupling and synergy among vehicles into account, so it can collaboratively achieve the overall goal by information exchange and the redistribute the resources. Thus distributed control has gradually replaced centralized control with its unique advantages of strong adaptability and easy maintenance.

## 2.2 Communication topology network

As shown in Figure 2, communication topology network consists of undirected and directed graph. There is no direction in information exchange in undirected graph and if any two nodes are path-connected, the undirected graph is fully connected. If vehicles are edge-connected, they are adjacent members in their neighbourhood. The directed graph takes the practical application into consideration, so the information exchange between any two nodes have directions.



Figure 2: Directed graph (a) and undirected graph (b)

In the topological structure, each node represents a vehicle in combat. The edges connecting nodes represent the information exchange between two members and the arrow direction in the directed graph is the information flow direction. In this paper, undirected topology represented by *G* is adopted to develop the communication structure among vehicles information interaction<sup>[14]</sup>. Topology graph *G* denoted as  $G = (v, \varsigma, A)$  is composed of nodes and edges connecting nodes. The nodes are denoted as  $v = \{v_1, v_2, \dots, v_n\}$  and the set of edges are the communication link among the vehicles denoted as  $\varsigma \subseteq v \times v$ . For  $\forall i, j = 1, 2, \dots, n$ , and  $i \neq j$ ,  $(v_i, v_j) \in \varsigma$  is represented that the vehicle  $v_i$  can acquire the related information of vehicle  $v_j$ , then there is an edge connecting  $v_i$  and  $v_j$ . *A* is weighted adjacent matrix denoted as  $a = [a_{ij}] \in \mathbb{R}^{n \times n}$ . If vehicle  $v_i$  has communication with vehicle  $v_j$ , thus  $v_j$  is in the neighbourhood of  $v_i$  denoted as  $v_j \in \Omega_i(t)$  and  $(v_i, v_j) \in \varsigma$ ,  $a_{ij} > 0$ . If vehicle  $v_i$  do not have any interaction, then  $a_{ij} > 0$ . Generally assumed that nodes are not connected to themselves, which denotes as  $a_{ii} = 0$ . Undirected graph requires *A* to be a symmetric matrix with zero main diagonal elements, which denotes as  $a_{ij} = a_{ji} > 0$ . By contrast, matrix *A* is not necessarily symmetric in the directed graph. The path of graph is a finite sequence of nodes  $v_1, \dots, v_k$ .

$$L_{A} = diag\left\{\sum_{j=1}^{n} a_{1j}, \cdots, \sum_{j=1}^{n} a_{nj}\right\} - A$$
(1)

The element of  $L_A = [l_{ji}]_{n \times n}$  is given by

$$l_{ij} = \begin{cases} -a_{ij}, & i \neq j \\ \sum_{j=1}^{n} a_{ij}, i = j \end{cases}$$
(2)

Note that the matrix L is symmetric positive semidefinite in undirected graph but this performance is not applicable for directed graph.

#### 2.3 Equation of motions

Assuming the scenario that n vehicles attack a stationary target simultaneously, a three-dimensional vehicle cooperative combat model is given as shown in Figure 3. In order to analyse the problem, the vehicle and the target are regarded as point mass. Moreover, the vehicle dynamics is not considered. Then the equation of motions will serve for the guidance design and analysis in this paper.



Figure 3: Vehicles guidance geometry

where *T* and  $M_i$  represent the target and combat vehicle respectively,  $\theta_i$  and  $\psi_i$  denote the flight path angle and flight heading angle,  $X_o - Y_o - Z_o$  denotes the launching coordinate,  $X_M - Y_M - Z_M$  represents the body coordinate system and *LOS* is the distance between the vehicle and the target.

The dynamics equations of vehicles in this cooperative engagement are defined as follows

$$\begin{cases} m\frac{dv}{dt} = G_x + F_x \\ mv\frac{d\theta}{dt} = G_y + F_y \\ -mv\cos\theta\frac{d\psi}{dt} = G_z + F_z \end{cases}$$

$$\begin{cases} \frac{dx}{dt} = v\cos\theta\cos\psi \\ \frac{dy}{dt} = v\sin\theta \\ \frac{dz}{dt} = -v\cos\theta\sin\psi, \quad i = 1,...,n \end{cases}$$
(3)

where (x, y, z) denotes the position of vehicle, v is the velocity of vehicle,  $G_{x,y,z}$  are the components of gravity in ballistic coordinate system,  $F_{x,y,z}$  are the components of all external forces except gravity in ballistic coordinate system. The initial conditions of vehicles are expressed as

$$x_{i}(t_{0}) = x_{0} \quad y_{i}(t_{0}) = y_{0} \quad z_{i}(t_{0}) = z_{0}$$
  

$$v_{i}(t_{0}) = v_{0} \quad \theta_{i}(t_{0}) = \theta_{0} \quad \psi_{i}(t_{0}) = \psi_{0}$$
(4)

Where  $t_0$  denotes the initial time of cooperative operations.

## 3. Cooperative guidance law design

#### 3.1 Cooperatove guidance strategy

Assuming a scenario that a multi-agent system consisting of n vehicles salvo attack a target. Because the initial conditions of vehicles including LOS, flight angle and velocity are different, it is impossible to guarantee the same remaining impact time for the vehicle to reach target without any correction. Thus a cooperative guidance strategy is designed in this paper. By using the interactive information to generate guidance instructions, each vehicle can adjust its trajectory angle and velocity so that each vehicle can arrive at the target simultaneously.

The distributed cooperative guidance can be regarded as the problem of cooperative control according to the consensus theory. The consensus theory can make all state variable of vehicles converge gradually based on the distributed network synchronization algorithm. Therefore the cooperative control<sup>[15]</sup> of continuous multi-vehicle system is given

$$\dot{x}_{i} = f(x_{i}, t) + u_{i}, \quad i = 1, ..., n$$
  
$$u_{i} = \sum_{j \in \Omega_{i}} a_{ij}(x_{j} - x_{i})$$
(5)

where  $x_i \in \mathbb{R}^n$  denotes the state information of vehicle *i*,  $u_i$  is the control variable of system,  $a_{ij}$  is defined as the component of adjacent matrix of vehicle *i* and  $\Omega_i$  is the communication set of vehicle *i*.

If undirected topology communication networks are connected, vehicle i and vehicle j will satisfy the equation (5) that impact time and state of position in three-direction will converge asymptotically. Therefore, multi-vehicle system realizes cooperative guidance.

$$\begin{cases}
\lim_{t \to \infty} [x_i(t) - x_T(t)] = 0 \\
\lim_{t \to \infty} [y_i(t) - y_T(t)] = 0 \\
\lim_{t \to \infty} [z_i(t) - z_T(t)] = 0, \quad i = 1, ..., n
\end{cases}$$
(6)

Note that the design of control variable and communication topology set is the important part to realize cooperative guidance.

#### 3.2 De Bruijn communication graph

Based on the analysis of communication network, vehicles and target are considered as the nodes in the topology graph which can be more convenient to describe the communication relationship of multi-agent. Assuming that the position and velocity of vehicles and target can be measured, undirected De Bruijn graph with fixed topology is used to develop the communication graph of vehicles. In the 1980s, Pradhan<sup>[16]</sup> first used De Bruijn network to design the topology of parallel and distributed systems. At present, De Bruijn network has been widely used in many fields including VLSI(Very Large Scale Integration) design. Thus, the node interaction connections of topology network can be decided by the De Buijn network.

The undirected De Bruijn network can be described by  $G = (v, \varsigma, A)$ , where v is the set of nodes and  $\varsigma$  is the set of edges. Supposing there are *n* vehicles attack the target cooperatively, thus there are *n* nodes in the network. And the vehicle number matches the serial number of node *i*. The degree of node *i* is denoted as  $d_i$  ( $0 \le i \le n - 1$ ), which means the number of edges connected to node *i*. The number *n* of nodes is defined as the power exponent form  $n=r^m$ , which can obtain network interactions with different characteristics by choosing the value of *r*. Note that the network becomes a binary topology graph when the *r* equals 2. The form of node *i* is described as a *m*-bit binary number as the form of  $(i_{m-1}, i_{m-2}, \dots, i_0)$ . The form of node *j* is similar to node *i*, which the form is  $(j_{m-1}, j_{m-2}, \dots, j_0)$ . For any two nodes *i* and *j* in graph *G*, There are connections between node *i* and node *j* when follows the rules<sup>[17]</sup> as shown in Eq. (7)

$$i_{w} = \begin{cases} j_{w-1}, 1 \le w \le m-1 \\ j_{w+1}, 0 \le w \le m-2 \end{cases}$$
(7)

Figure 4 shows a binary(r=2) network structure with 16 nodes( $16 = 2^4$ ). Based on the previous studies, De Bruijn network has three important properties to apply. Now some definitions<sup>[18,19]</sup> are stated firstly.



Figure 4: A binary 16 nodes Bruijn topology network

Definition 1. The node disconnected path is denoted by  $c_{ij}$  ( $0 \le i \le n - 1$ ), which indicates that the path between *i* and *j* do not have any common nodes. And there must be  $c_{ij} \ge r$  in graph *G*.

Definition 2. The number of total nodes connections of G is  $nr - (r^2 + r)/2$ .

Definition 3. The maximum distance between nodes of G is denoted as k = m.

Definition 4. The average distance between node<sup>1</sup> of G is  $k_v = m - 1$ .

Definition 1 declares that distributed De Bruijn network has fault tolerance. There are also two different paths for any node in the network when the number of nodes can be represented in binary as shown in Figure 5. The degree of node represents the number of backups of links in communication, which can keep formal communication if any vehicle fails in the attack. In practical application, the vehicles in the group are vulnerable to attack. Thus, the property of good fault tolerance can improve the engineering applicability. Definition 2 shows that the number of network connections increases linearly with the size of the network. If the number of nodes change from n to 2n, the number of connections will increase by two times instead of factorial growth in traditional connection, which improve the applicability of large-scale systems and reduce economic costs. Definition 3 and Definition 4 indicates that there is a logarithmic relationship between the maximum distance and the scale of the network, which ensures the high communication efficiency of the De Bruijn graph when the network scale is large.

#### 3.3 Self-avoidance collision cooperative guidance law design

Considering the later process of cooperative attack, the state information of vehicles converge gradually and subsequently, collisions among multi vehicles may occur before arriving the target. Thus, a cooperative guidance law with self-avoidance collision control is designed in this section. According to the predetermined safety distance criterion, adjusting variables in three-direction are added to the state information to realize self-avoidance collision. The distributed guidance law inherits the advantages of distributed system regarding communication efficiency, which the vehicle only need to communicate with others in the neighbourhood instead of all vehicles in the whole network. Therefore, the communication efficiency and engineering fitness has been greatly improved. Based on the De Bruijn graph, the communication topology neighbourhood  $\Omega_i$  of  $M_i$  is confirmed and it can determine which information of vehicles is needed to acquire in the  $\Omega_i$ . Thus, the structure and partial coefficients of matrix  $L_A = [l_{ji}]_{n \times n}$  can be decided. Adopted the network synchronized theory, the state of vehicles is uniformly convergent by constructing and stabilizing the error state system. And the state information is automatically determined by the vehicle which belongs to  $\Omega_i$ . Note that the state of vehicle can converge to target asymptotically in a short time which do not need to the impact time imprecisely.

In the situation of cooperative attack on stationary target, the velocity command in x-y-z direction is given as Eq. (8) in inertial frame. The motion model is built in the inertial coordinate system, and the x-axis is defined as pointing to the target. And all vehicles are moving towards the target under the cooperative guidance law. Thus, there is no distance control restriction in x direction and only set the safe distance control to the direction of y and z motion, which can

keep flight collision free among vehicles. The safe distance control is removed when they are approach the target. The positions of  $M_i(i=1,2,...,n)$  and target *T* are denoted as  $(x_i, y_i, z_i)$  and  $(x_T, y_T, z_T)$  respectively. The velocity in three-direction is described as  $(v_{xi}, v_{yi}, v_{zi})$ . The matrix B represents whether  $M_i$  vehicle can obtain the information of target, which is denoted as  $B = \text{diag}\{b_1, b_2, \dots, b_n\}$ .

$$v_{xi} = \sum_{j \in \Omega_i} l_{ij} [x_j(t) - x_i(t)] + b_i [x_T(t) - x_i(t)]$$
(8)

$$v_{yi} = \begin{cases} \sum_{j \in \Omega_i} l_{ij} [y_j(t) - y_i(t) - dy_{ij}] + b_i [y_T(t) - y_i(t) - dy_{iT}], \ \delta > \delta_0 \\ \sum_{j \in \Omega_i} l_{ij} [y_j(t) - y_i(t)] + b_i [y_T(t) - y_i(t)], \ \delta \le \delta_0 \end{cases}$$

$$v_{zi} = \begin{cases} \sum_{j \in \Omega_i} l_{ij} [z_j(t) - z_i(t) - dz_{ij}] + b_i [z_T(t) - z_i(t) - dz_{iT}], \ \delta > \delta_0 \\ \sum_{j \in \Omega_i} l_{ij} [z_j(t) - z_i(t)] + b_i [z_T(t) - z_i(t)], \ \delta \le \delta_0 \end{cases}$$
(9)

where  $\delta = \sqrt{\sum_{i=1}^{n} [x_T(t) - x_i(t)]^2}$  is denoted as the distance between  $M_i$  and target in *x*-direction,  $\delta_0$  is the small positive constant number based on the operational requirements in practical,  $dy_{ij}$  and  $dz_{ij}$  are the preset safe distance value between  $M_i$  and  $M_j$ . Similarly,  $dy_{iT}$  and  $dz_{iT}$  are the predetermined safe distance between target and  $M_i$ .

**Theorem:** By adjusting the weight coefficients of  $l_{ij}$  and  $b_i$ , matrix *L-B* can satisfy the requirements of Hurwitz criterion<sup>[20]</sup>. Therefore the position of all vehicles will converge to target and the guidance can avoid collision among vehicles in both y and z-direction when the distance  $\delta$  in x-direction is greater than  $\delta_0$ .

**Proof**: The convergence of guidance strategy model in x-y-z direction is given as follows.

The collaboration of two directions y and z can be realized in two stages. The proof of x direction is equal to the first stage. The proof phase of y and z directions is similar, therefore take y-direction as an example to illustrate. To realize the collision avoidance, the synchronization algorithm with a safe distance is adopted when the vehicles and the target are far apart. The safe distance is then eliminated when they become close.

*Stage1*: If  $\delta > \delta_0$ , the vehicle and target is a greater distance apart, the y-direction guidance command is as

$$v_{yi} = \sum_{j \in \Omega_i} l_{ij} [y_j(t) - y_i(t) - y_{ji}] + b_i [y_T(t) - y_i(t) - y_{Ti}]$$
(10)

Then Eq. (10) is transformed as

$$v_{yi} = \sum_{j \in \Omega_i} l_{ij} [(y_j(t) - y_T(t) - y_{jT}) - (y_i(t) - y_T(t) - y_{iT})] - b_i [y_i(t) - y_T(t) - y_{iT}]$$
(11)

Define the state error of the predetermined safe distance between the vehicle and the target as

$$e_{y_i}(t) = y_i(t) - y_T(t) - y_{iT}$$
(12)

Eq. (12) can be expressed as

$$\dot{e}_{yi}(t) = \sum_{j \in \Omega_i} l_{ij} [e_{yj}(t) - e_{yi}(t)] - b_i e_{yi}(t)$$
(13)

Then, we can obtain

$$\dot{e}_{v}(t) = (L - B)e_{v}(t)$$
 (14)

When the matrix *L-B* meets the Hurwitz theorem as  $t \to \infty$ , the state error  $e_y(t)$  will tend to be zero gradually. Therefore, the state of all the vehicles and the target in *y*-direction will tend to be the preset safe distance asymptotically, which the collision self-avoidance can be achieved. The theorem proof is similar to the *x*-direction state.

*Stage2*: When  $\delta \leq \delta_0$ , the vehicle is close to target in *x*-direction,  $M_i$  will switch the cooperative strategy in y-direction which is similar to the guidance command of *x*-direction. Therefore, the state error between  $M_i$  and target in *y*-direction will converge to zero gradually as Eq. (15), which the vehicles will arrive the position of target cooperatively.

$$e_{y_i}(t) = y_i(t) - y_T(t) \to 0$$
 (15)

Specially, considering the condition that  $\delta > \delta_0$ ,  $M_i$  is far from target so the collision among vehicles will not happen. Therefore, the predetermined safe distance  $dy_{iT}$  and  $dz_{iT}$  between vehicles and target can be set as zero, which will not affect the proof process of stage 2. Based on the distributed network synchronized guidance algorithm of Eq. (8) and Eq. (9), multi-vehicle can accomplish cooperative attack with collision self-avoidance and three components of velocity can be obtained. By adjusting the coefficients of the matrix *L* and *B*, the state convergence rate can be adjusted. By tracking the velocity guidance command in three-direction under the inertial system, cooperative guidance strategy can be achieved. Based on the kinematics model and transformation of coordinate system, the relations of velocity, flight path angle  $\theta_i$  and flight heading angle  $\psi_i$  are given as Eq. (16).

$$\begin{cases} v_{xi} = v_i \cos \theta_i \cos \psi_i \\ v_{yi} = v_i \sin \theta_i \\ v_{zi} = -v_i \cos \theta_i \sin \psi_i \end{cases}$$
(16)

where  $v_{xi}$ ,  $v_{yi}$ , and  $v_{zi}$  are three-direction velocity in inertial coordinate system.

Three-direction cooperative guidance commands given by Eq. (8) and Eq. (9) are substituted into the kinematics relations of Eq. (16). Therefore, the command of total velocity, flight path angle and flight heading angle are obtained as Eq. (17) and the property of self-avoidance collision is acquired.

$$\begin{cases} v_{i} = \sqrt{(v_{xi})^{2} + (v_{yi})^{2} + (v_{zi})^{2}} \\ \theta_{i} = \arctan(\frac{v_{yi}}{\sqrt{(v_{xi})^{2} + (v_{zi})^{2}}}) \\ \psi_{i} = \arctan(\frac{v_{zi}}{v_{xi}}) \\ \end{cases}$$
(17)

Thus, the cooperative guidance strategy can be equated with the tracking ideal guidance command of total velocity, flight path angle and flight heading angle shown in Eq. (17). The vehicles are guided to the target cooperatively under the guidance command.

# 4. Numerical simulation

To evaluate the effectiveness of the proposed guidance law, simulations are performed as follows. Based on the analysis of previous sections, a three-dimensional engagement problem is considered. Suppose that four vehicles salvo attack a stationary target at (0, 0, 3500) under the distributed cooperative guidance strategy. The initial conditions of each vehicle are shown in Table 1, which the state information and flight angle are different from each other. The initial speed of all the vehicles are the same as 1200m/s. Based on the rule of undirected De Bruijn graph, the communication topology structure of four vehicles are shown in Figure 5. There are four nodes in the topology network represented in binary form ( $4 = 2^2$ ). Note that the serial number of nodes is from zero to three, which is one less than the number of vehicle. The serial number of four nodes are 00,01,10 and 11 respectively and the connections of nodes is acquired on the basis of the rule of De Bruijn graph. Thus, the connections among vehicles are described by communication topology neighborhood  $\Omega_i$  are given in the Table 1. The weighted laplacian matrix  $L_A$  is given as follows

$$L_{A} = \begin{bmatrix} -0.074 & -0.037 & -0.037 & 0\\ -0.037 & -0.074 & -0.037 & -0.037\\ -0.037 & -0.037 & -0.074 & -0.037\\ 0 & -0.037 & -0.037 & -0.074 \end{bmatrix}$$
(18)

In practical simulations, the coefficients of  $L_A$  are confirmed by the requirements of trajectory. And the matrix *B* is defined as  $B = \text{diag}\{0.36, 0.33, 0.34, 0.36\}$ . The eigenvalues of matrix  $L_A - B$  are all negative which meets the Hurwitz theorem. Thus, the state error  $e_{x,y,z}(t)$  will tend to be zero gradually as  $t \to \infty$ .

Table 1: Initial conditions of four vehicles

	<i>M</i> <sub>1</sub>	<i>M</i> <sub>2</sub>	<i>M</i> <sub>3</sub>	$M_4$
$(x_i, y_i, z_i) / m$	(-980,1350,500)	(-870,1180,900)	(-1100,-1205,800)	(-1500,-500,-200)
$\theta_i/{\rm deg}$	-26.29	-21.87	19.51	9.62
$\psi_i/deg$	-65.04	-64.21	-60.20	-56.24
$\Omega_i$	$M_2 M_3$	$M_1 M_3 M_4$	$M_1 M_2 M_4$	$M_2 M_3$



Figure 5: four vehicles De Bruijn communication network

Numerical simulations are carried out to verify the cooperaitve guidance law. The trajectory in three-dimensional of all vehicles salvo attack the target is shown in Figure 6 where vehicles are denoted by different types of lines. Assume that vehicles are lunached at various initial location and arrive at the target under the cooperative guidance law. It is

#### Xi Zhang, Shui Yongtao, Wang Yonghai, Wang Liyan, Zhang Xuejiao

obvious that cooperative attack is achieved within 16.4 seconds and there is no interaction between the flight paths when approaching the target. And the trajectory is convergent gradually. The predetermined safe is cancelled when the vehicles are approaching the target. Thus the simulation results verify the efficiency and self-avoidance collision of cooperative guidance law. The distance R between the target and vehicles is given in Figure 7, which is convergent asymptotically. The miss distance is controlled within 2 meters. However, the multi vehicles under traditional PNG(Proportional Navigation Guidance) law<sup>[20]</sup> do not have the convergence trend regarding distance R and any state information. In contrast, PNG law is unable to implement cooperative attack.



Figure 6: Trajectory of multi-vehicle cooperative attack



Figure 7: Relative distance between vehicles and target

Based on the synchronization principle, the three-direction position state error shown in Figure 8 tends to be zero gradually, which satisfies the requirements of the theorem. By adjusting the coefficients of the matrix L and B, the state information of vehicles in all directions has fast convergence to the target within 8 seconds. Thus, the efficiency of cooperative attack is guaranteed under the cooperative guidance law. Three-direction velocity of vehicles is given in Figure 9. According to the performance of consensus theory, velocity of vehicles is keeping the same trend to converge to the velocity of target gently, which meets the requirements of cooperative guidance. In order to prove the fault tolerance of communication network, simulations are carried out assuming that  $M_4$  is intercepted and quit the

communication network. The position state error of  $M_1$ ,  $M_2$ ,  $M_3$  is shown in Figure 10. It is clearly that the remaining vehicles still keep formal communications when  $M_4$  is intercepted. The state error converge gradually to zero within 18 seconds, which is similar to the situation of four vehicles. Moreover, the three-direction velocity in Figure 11 also keeps the same trend of the convergence. Based on the consensus theory, cooperative guidance law enable the vehicles to closely track the state of the target. Therefore, the simulations demonstrate the distributed communication network has good properties of fault tolerance and high robustness.



Figure 8: Three-direction position state error



Figure 9: Three-direction velocity of vehicles



Figure 10: Three-direction position state error when  $M_4$  quits



Figure 11: Relative distance between three vehicles and target

## 5. Concluding remarks

A distributed cooperative guidance strategy with self-avoidance collision is proposed to achieve multi-vehicle cooperative attack in three-dimensional space. Based on the advantages of distributed system, communication efficiency and applicability for large-scale has been greatly improved. Both theoretical analysis and simulation results verify that the cooperative guidance law can guarantee the same impact time of all vehicles and make the state converge asymptotically, even when one vehicle in the group is intercepted and withdraw from the communication network. The distributed guidance law can achieve cooperative attack at different initial conditions. Also the simulations show that

the safe distance control is effective to ensure self-avoidance collision as approaching the target. Considering the impact of communication environment on communication capability in practical application, the cooperative guidance strategy can provide a reference for the cooperative attack of multi-vehicle when the environment is complex. And the communication network can support large-scale cooperative operations.

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