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Fast Homotopy Method for Binary Asteroid Landing Trajectory Optimization

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Abstract

The technique of trajectory design is important for binary asteroid landing. In this paper, we adapt the fast homotopy method developed for landing trajectory optimization at a single asteroid, and extend it to binary asteroid landing problems. The dynamical equations binary asteroid landing is presented and the fast homotopy method is overviewed. The advantage of this method is that the difficult bang-bang control problem is connected to an analytically solvable optimal control problem via two homotopy process. The solution speed is extremely high. Both the effectiveness and efficiency of this method are validated in the numerical simulation.

1. Introduction

Asteroid explorations have attracts space agencies and researchers over the past two decades. Thus far, five asteroid missions have been launched. In 2018, two spacecraft, Hayabusa2 and OSIRIS-REx, just arrived their target asteroids and got attention of people around the world.

Asteroid landing is a major proximity operation in asteroid missions. Due to the importance of saving onboard fuel and the chanllenge of dealing with the irregular graivty near asteroids, trajectory optization for asteroid landing has been extensively studied in past decades [1] - [4]. The used method includes both indirect methods [1] and direct methods [2]. Rencetly, convex optimization based trajectory design has also been extended to solving optimal asteroid landing trajectory efficiently [3] [4]. Besides, a fast homotopy method proposed in [1] also enables solving optimal descent trajectory rapidy in an indirect approach. In this method, the solution process starts from analytical initial costates to avoid the common initial costate problem. However, These studies [1] - [4] are all dealing with single asteroid problems.

Binary asteroids are an important class of asteroids and have great scientific value. The observed binary asteroid 1994 KW4 [5] shows complex dynamics near it [6] - [8]. This paper concerns the descent and landing phases which are the critical steps in a binary-asteroid sample return mission. Due to complex gravitational field of binary asteroids, solving the fuel optimal landing trajectory efficiently is challenge. In this paper, we employ the recent fast homotopy method [1] developed for optimization of single asteroid landing trajectories, and extend its applicability for binary asteroid landing trajectory optimization. The solution algorithm is developed based on the binary asteroid model. In the simulation, the fast homotopy method will be applied to the binary asteroid 1994 KW4 with a simplified sphere-sphere model.

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2. Minimum-fuel descent trajectory optimization problem

2.1 Dynamical equations

The problem of landing a spacecraft on the surface of a binary asteroid is considered. A schematic diagram of a binary asteroid system is shown in Figure 1, where *Alpha* and *Beta* refers to the primary and second bodies of the system, respectively. The mass of the bodies is M_1 and M_2 and the mass of the spacecraft m is regarded as negligible comparing to bodies. The distance between the centroids of *Alpha* and *Beta* is d. Besides, the angular velocity of the binary asteroid system and *Alpha* is denoted as ω and ω_α, respectively, as shown in the figure.
Before giving the dynamical equations, a rotating coordinate system o-xyz is built, where the origin o locates at centroid of the system, axis x is aligned with the line connecting the centroids of *Alpha* and *Beta*, axis z is aligned with the angular velocity and axis y completes the right-hand system. Neglecting the perturbations, the equation of motion for the spacecraft is [1] [7]

$$\ddot{\boldsymbol{\rho}} + 2\boldsymbol{\omega}\times\dot{\boldsymbol{\rho}} + \boldsymbol{\omega}\times(\boldsymbol{\omega}\times\boldsymbol{\rho}) = \frac{\partial U}{\partial \boldsymbol{\rho}} + \frac{T_{\max} u \boldsymbol{\alpha}}{m} \quad (1)$$

where $\boldsymbol{\rho} = [x, y, z]^{\mathrm{T}}$ is the position vector of the spacecraft, $T_{\max}$ is the maximum magnitude of the thrust, $u \in [0, 1]$ is the thrust ratio, $\boldsymbol{\alpha}$ is the direction of the thrust, and U is the gravitational potential, which is

$$U = U_{Alpha} + U_{Beta} \quad (2)$$

where U_{Alpha} and U_{Beta} are the gravitational potentials due to the two bodies. Generally, Eq. (2) is the time variant when ω_α is not equal to ω. But, Eq. (2) becomes time invariant when *Alpha* and *Beta* are both spheres and the gradient of U is [9]

$$\begin{cases} \dfrac{\partial U}{\partial x} = GM\left[-\dfrac{1-\mu}{\rho_1^3}(x+\mu d) - \dfrac{\mu}{\rho_2^3}\left(x+(\mu-1)d\right)\right] \\ \dfrac{\partial U}{\partial y} = GM\left[-\dfrac{1-\mu}{\rho_1^3}y - \dfrac{\mu}{\rho_2^3}y\right] \\ \dfrac{\partial U}{\partial z} = GM\left[-\dfrac{1-\mu}{\rho_1^3}z - \dfrac{\mu}{\rho_2^3}z\right] \end{cases} \quad (3)$$

where M is the sum of M_1 and M_2 and μ is a mass ratio as

$$\mu = \frac{M_1}{M_1 + M_2} \quad (4)$$

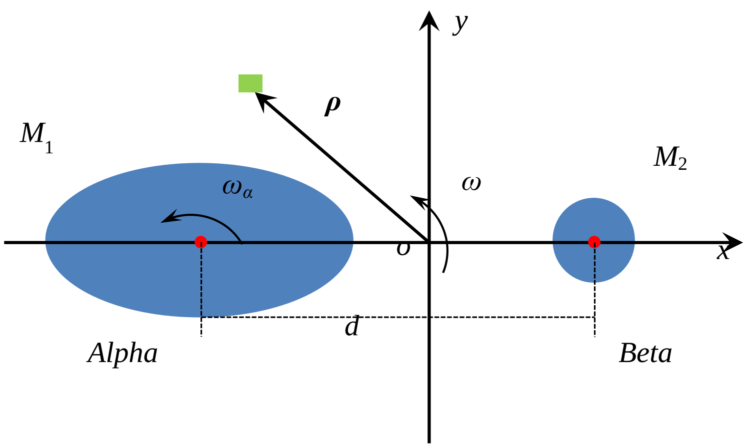

Figure 1: Schematic diagram of a binary asteroid system

DOI: 10.13009/EUCASS2019-404

For the trajectory optimization development, Eq. (1) is written as

$$\dot{\boldsymbol{\rho}} = \boldsymbol{v} \tag{5}$$

$$\dot{\boldsymbol{v}} = -2\boldsymbol{\omega}\times\boldsymbol{v} - \boldsymbol{\omega}\times(\boldsymbol{\omega}\times\boldsymbol{\rho}) + \frac{\partial U}{\partial \boldsymbol{\rho}} + \frac{T_{\max} u \boldsymbol{\alpha}}{m} \tag{6}$$

where $\boldsymbol{v} = [v_x, v_y, v_z]^T$ is the velocity of the spacecraft. The mass variation equation is [1] [3]

$$\dot{m} = -\frac{T_{\max} u}{I_{sp} g_0} \tag{7}$$

where I_{sp} is the thrust specific impulse, and g_0 = 9.80665 m/s is the standard acceleration of the gravity at the sea level.

2.2 Minimum-fuel descent control problem

The minimum-fuel descent trajectory optimization problem is equivalent to the minimum-fuel descent control problem. To achieve minimum-fuel cost, the performance index is chosen as

$$J_0 = \lambda_0 \int_0^{t_f} u dt \tag{8}$$

where λ_0 is a positive parameter and t_f is the final time. With known initial states and soft landing purpose, the boundary constraints are defined, as

$$\begin{cases} \boldsymbol{r}(0) = \boldsymbol{r}_0 \\ \boldsymbol{v}(0) = \boldsymbol{v}_0 \\ m(0) = m_0 \end{cases}, \quad \begin{cases} \boldsymbol{r}(t_f) = \boldsymbol{r}_f \\ \boldsymbol{v}(t_f) = \boldsymbol{0} \end{cases} \tag{9}$$

According to [1], the derived optimal control direction and magnitude based on the Pontryagin's Minimum Principle are

$$\boldsymbol{\alpha}^* = -\frac{\boldsymbol{\lambda}_v}{\|\boldsymbol{\lambda}_v\|} \tag{10}$$

$$u^* = \begin{cases} 0 & \text{if } \rho_0 > 0 \\ 1 & \text{if } \rho_0 < 0 \\ [0,\ 1] & \text{if } \rho_0 = 0 \end{cases} \tag{11}$$

where $\boldsymbol{\lambda}_v$ is the velocity costate and the switching function ρ_0 is expressed by

$$\rho_0 = \lambda_0 - \frac{T_{\max}}{I_{sp} g_0}\lambda_m - \frac{T_{\max}}{m}\|\boldsymbol{\lambda}_v\| \tag{12}$$

Besides, the equation of the costate variables are [1]

DOI: 10.13009/EUCASS2019-404

$$\begin{cases} \dot{\boldsymbol{\lambda}}_r = \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{\lambda}_v) - \dfrac{\partial}{\partial \boldsymbol{r}} \left(\boldsymbol{\lambda}_v \cdot \dfrac{\partial U}{\partial \boldsymbol{r}} \right) \\ \dot{\boldsymbol{\lambda}}_v = -\boldsymbol{\lambda}_r + 2\boldsymbol{\lambda}_v \times \boldsymbol{\omega} \\ \dot{\lambda}_m = -\dfrac{T_{\max} u^*}{m^2} \|\boldsymbol{\lambda}_v\| \end{cases} \tag{13}$$

where $\boldsymbol{\lambda}_r$ and $\boldsymbol{\lambda}_m$ are the position and mass costates. Due the free mass the final time, the following transversality condition can be obtained:

$$\lambda_m(t_f) = 0 \tag{14}$$

The key of the minimum-fuel descent control problem is to find a set of initial costates, with which the system (5) - (7) and (13) satisfies the boundary conditions (9) and (14).

3. Overview on the Fast homotopy method

Because the optimal control (11) is a discontinuous function, solving the initial costates by a shooting method is challenge. In order to solve the minimum-fuel descent control problem rapidly, a fast homotopy method is developed in [1]. In this section, we give a brief introduction about this method. In this method, the minimum-fuel descent control problem is denoted as *Prob0*.

First, *Prob0* is connected to an energy-optimal control problem (denoted as *Prob1*) via a homotopy of performace index which is

$$J_1 = \lambda_0 \int_0^{t_f} \left[(1-\varepsilon_1) u + \varepsilon_1 u^2 \right] dt \tag{15}$$

where ε_1 is a homotopy parameter, ranging from 0 to 1. The result function of the optimal control becomes continuous if only ε_1 is not equal to zero. However, the initial state still needs to be guessed.

Second, *Prob1* is further connected to a gravity-free energy-optimal control problem (denoted as *Prob2*) via a homotopy of dynamics. In *Prob2*, the performance index is chosen as

$$J_2 = \lambda_0 \int_0^{t_f} u^2 dt \tag{16}$$

and homotopy of dynamics is

$$\dot{\boldsymbol{v}} = \varepsilon_2 \boldsymbol{G} + \frac{T_{\max} u \boldsymbol{\alpha}}{m} \tag{17}$$

where ε_2 is also a homotopy parameter, ranging from 0 to 1, and $\boldsymbol{G}$ is the generalized gravity, which is defined by

$$\boldsymbol{G} = -2\boldsymbol{\omega} \times \boldsymbol{v} - \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{r}) + \frac{\partial U(\boldsymbol{r})}{\partial \boldsymbol{r}} \tag{18}$$

Third, an approximate solution for *Prob2* with $\varepsilon_2 = 0$ is searched. To this end, a thrust-constraint-free optimal control problem (denoted as *Prob3*) is used. In *Prob3*, the performance index is chosen as

$$J_3 = \frac{1}{2} \int_0^{t_f} \boldsymbol{a}^T \boldsymbol{a} dt \tag{19}$$

Eq. (19) approximates to Eq. (16) if λ_0 is chosen as $T_{\max}{}^2 / (2m_0{}^2)$. And the approximate solution for the initial costates is as follows:

DOI: 10.13009/EUCASS2019-404

$$\begin{cases} \boldsymbol{\lambda}_{r0} = \dfrac{6\left(t_f \boldsymbol{v}_0 + t_f \boldsymbol{v}_f + 2\boldsymbol{r}_0 - 2\boldsymbol{r}_f\right)}{t_f^{\ 3}} \\ \boldsymbol{\lambda}_{v0} = \dfrac{2\left(2t_f \boldsymbol{v}_0 + t_f \boldsymbol{v}_f + 3\boldsymbol{r}_0 - 3\boldsymbol{r}_f\right)}{t_f^{\ 2}} \\ \lambda_{m0} = \dfrac{1}{m_0}\left(\dfrac{1}{3}\boldsymbol{\lambda}_{r0} \cdot \boldsymbol{\lambda}_{r0} t_f^{\ 3} - \boldsymbol{\lambda}_{r0} \cdot \boldsymbol{\lambda}_{v0} t_f^{\ 2} + \boldsymbol{\lambda}_{v0} \cdot \boldsymbol{\lambda}_{v0} t_f\right) \end{cases} \tag{20}$$

In the fast homotopy method, *Prob2* with $\varepsilon_2 = 0$ is solved using the solution of *Prob3*. Then, a homotopy process is conducted to solve *Prob2* with $\varepsilon_2 = 1$ which is equivalent to *Prob1* with $\varepsilon_1 = 1$. Thereafter, *Prob1* with $\varepsilon_1 = 0$, which is *Prob0*, is solved by a homotopy proces. Last, the solution of *Prob0* is used to output the minimum-fuel descent trajectory.

4. Numerical simulation

In this section, the fast homotopy method will be applied to solving minimum-fuel descent trajectory in a binary asteroid system. The code is written in FORTRAN language and run on the laptop in the Release mode.

4.1 Simulation parameters

In the simulation, a simplified sphere-sphere model for 1999 KW4 is used. The parameters for the spacecraft and the 1999 KW4 are listed in Table 1. In this table, the physical parameters of 1999 KW4 is obtained from [5] [7].
The initial position and velocity are chosen as $[0.0, -1.6585, 0.0]^T$ km and $[0.01, 0.01, 0.01]^T$ m/s, respectively. The landing site is chosen as $[-0.1382, -0.6585, 0.0]^T$ km and the landing velocity is set to zero. The flight time is set to 2000 s.

Table 1: Simulation parameters

Item	Value
M (kg)	2.488×10^{12}
d (m)	2548
μ	0.9458
T_{max} (N)	2
m_0 (kg)	1000
I_{sp} (s)	225

4.2 Results

The result of the optimal control for different optimal control problems are shown in Figure 2. According to this figure, the optimal control of *Prob2* ($\varepsilon_2 = 0$) is almost the same as that of *Prob3*, whichs supports the correctness of approximating *Prob2* ($\varepsilon_2 = 0$) by *Prob3*. It can also been seen that the optimal control of *Prob0* (i.e. minimum-fuel control problem) is a bang-bang control.

DOI: 10.13009/EUCASS2019-404

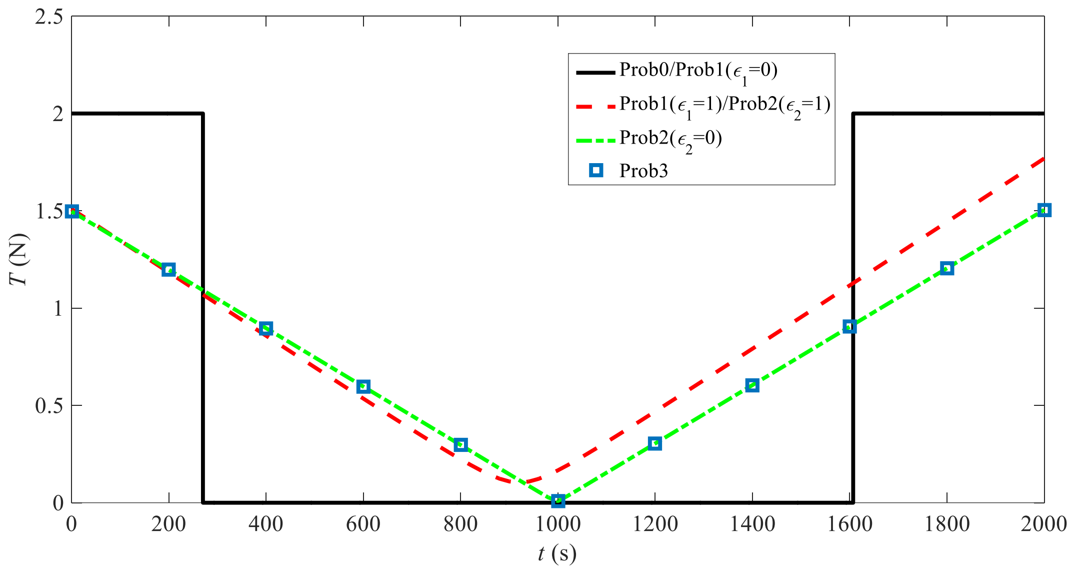

Figure 2: Optimal control for different optimal control problems

The mass history for different optimal control problems is shown in Figure 3. According to this figure, the minimum-fuel control costs the least.

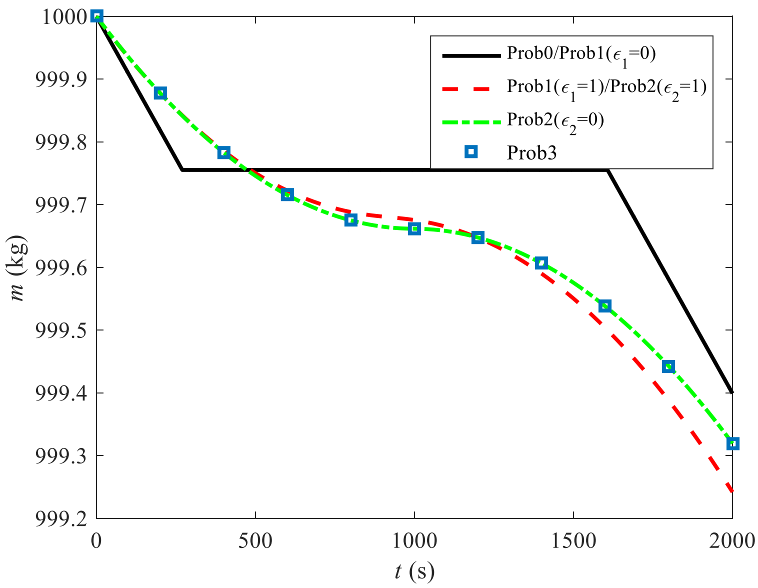

Figure 3: Mass history for different optimal control problems

Figures 4 - 6 give the three dimensional descent trajectory, position history and velocity history. According to these figures, the spacecraft lands at the desired site with zero relative velocity. It validates the fast homotopy method can be applied for binary asteroid landing trajectory optimization.

Moreover, the recorded CPU for solving the minimum-fuel control problem is only 1.56×10^{-2} s, which indicates the homotopy method is high efficient.

5. Conclusion

This study extends the previously developed fast homotopy method to binary asteroid systems, through replacing the dynamical model of single asteroid systems by the dynamical model of binary asteroid systems. The simulation results show that the fast homotopy method can be applied to the trajectory optimization problem of landing on 1999 KW4 with a simplified sphere-sphere model. Optimal bang-bang control is obtained and the final position and

DOI: 10.13009/EUCASS2019-404

velocity satisfy the boundary constraints. The reported CPU time for the test case is less than 0.02 s which reflects the high solution efficiency of the fast homotopy method.

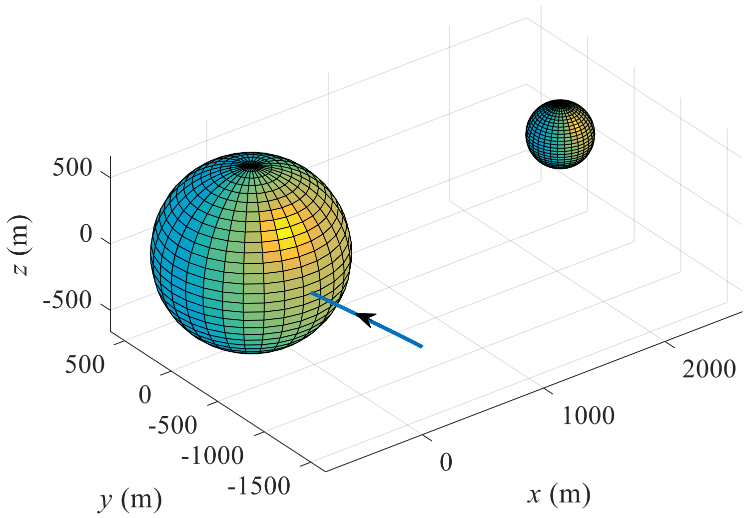

Figure 4: Minimum-fuel descent trajectory

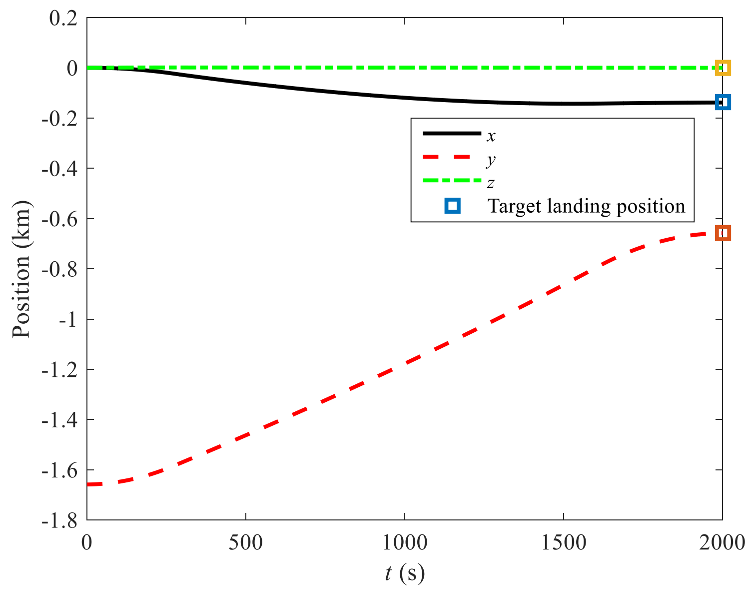

Figure 5: Position history with minimum-fuel control

DOI: 10.13009/EUCASS2019-404

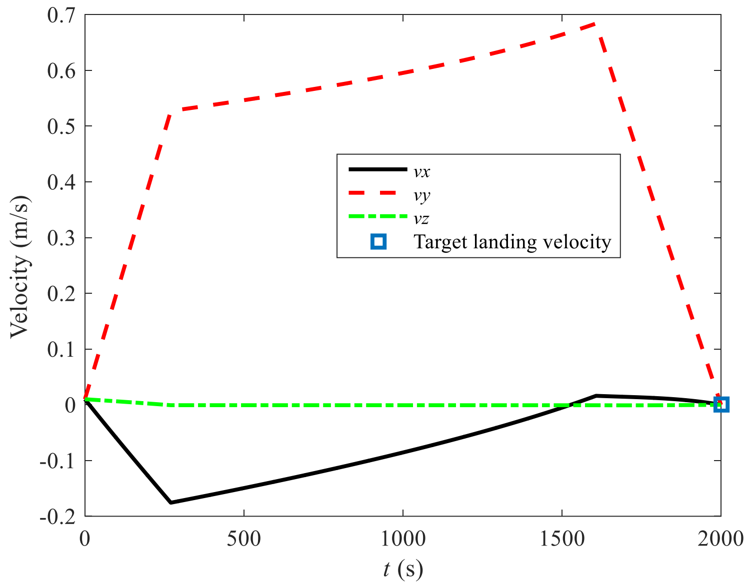

Figure 6: Velocity history with minimum-fuel control

Acknowledgments

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