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Finding the Parameters of Surface Roughness in Rarefied Gas Flow Solving the Inverse Problem

Olga A. Aksenova and Iskander A. Khalidov***
**St.-Petersburg Naval Polytechnic University, Department of Mathematics,*
17/1 Ushakovskaya Quai, 197045, St.-Petersburg, Russia; olga.a.aksenova@gmail.com
***Peter the Great St.-Petersburg Polytechnic University, Department of Mathematics and Mechanics,*
195251, Polytechnicheskaya ul., 29, St.-Petersburg, Russia; iskander.khalidov@gmail.com.

Abstract
The new algorithm is proposed to determine the real values of the roughness parameter from aerodynamic measurements solving the inverse problem. The main parameter defining surface roughness in rarefied gas is different from technical parameter applied in standard procedures, so it is not measured in the experiments. Moreover, even if this parameter could be measured, it could not describe real rough surface, because the main input into the influence of surface roughness on aerodynamic values takes the roughness of smaller scale, usually negligible on profile diagrams. Therefore roughness parameter must be determined from the solution of the inverse problem, based on the simulation of surface roughness on micro-level by Gaussian or poly-Gaussian random field and on the representation of the scattering function on rough surface.

1. Introduction

Surface roughness is often underestimated in aerodynamic experiments and calculations. Especially important is the influence of surface roughness on aerodynamic macroscopic parameters of rarefied gas flows because of two reasons. First, without a boundary layer the velocities and other properties of gas molecules in the whole rarefied gas flow depend on the scattering kernel on the surface. Second, the parameters of surface roughness are obtained usually from profile diagram measurements: it seems to be most simple and precise method to find the characteristics of the roughness. However, this approach does not permit determining exact values of roughness parameters in rarefied gas, because profile diagrams show only the roughness of the largest scale. So the most part of the effect of surface roughness produced by the roughness of smaller scale remains unnoticed.
The main purpose of the present paper is to develop solution algorithm for the inverse problem of determining the main surface roughness parameter from aerodynamic experimental data.
Surface roughness is usually characterized in basic technical applications (like determining surface friction or destruction) either by the main parameter – root-mean-square deviation of a rough surface $\sigma = \sqrt{Mz^2}$ or by the average distance R_z between the highest peak and the lowest valley (or similar peak-to-peak distance). Here $z(x,y)$ is a deviation of random field simulating rough surface from its average level. This approach is indeed most effective in different technical applications including the interaction of rough surface with continuum flow. Nevertheless more relevant to the rarefied gas dynamics is the parameter $\sigma_1 = \sqrt{Mz_x^2} = \sqrt{Mz_y^2}$ determined not by the height z, but by its partial derivatives $z_x = \dfrac{\partial z}{\partial x}$ and $z_y = \dfrac{\partial z}{\partial y}$ in directions x and y of a random field [1]–[6].
The parameter σ_1 is independent of the absolute values of the roughness amplitude and describes root-mean-square values of the tangent of the angle θ_n between the local $\mathbf{n}$ and the global $\mathbf{N}$ normal vectors on rough surface. The parameter σ_1 is obtained usually from statistical processing of profile diagram measurements: it seems to be most exact method to find σ_1. However, the precision of determining the parameter σ_1 from experimental profile diagram measurements could be unsatisfactory, because profile diagrams show only the roughness of the largest scale. And the main input into the influence of surface roughness on aerodynamic values in rarefied gas is provided by the roughness of smaller scale, usually negligible on the profile diagram [1]–[4]. Moreover, the parameter σ_1 is not

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included in technical standard of surface quality. Hence it is difficult to find the values of σ_1 in the literature. Therefore the only precise algorithm to determine the real values of the roughness parameter σ_1 is extracting it from aerodynamic measurements solving the inverse problem. The solution of this problem is based on analytical and numerical evaluation of the influence of surface roughness on scattering function and on momentum and energy exchange coefficients in rarefied gas flow.

2. Simulation of gas – rough surface interaction

The expansion of scattering function V on a rough surface is derived from its analytical representation in the form $V = \hat{S}V_0$, where the roughness operator $\hat{S}$ depends only on the geometrical shape of a roughness and on the trajectory of a reflected gas atom [5], [6]. The operator $\hat{S}$ does not depend on physical and chemical parameters of the gas and of the surface. These parameters influence only the local scattering function V_0 (scattering function on smooth surface without roughness). Gas particles are called atoms because the structure of the molecule is also accounted by the local scattering function V_0, so it does not affect the roughness operator $\hat{S}$.

The separation of roughness operator $\hat{S}$ and of local scattering function V_0 is based on the assumption that the number of rough irregularities in a small surface area dS is large, so that the scale of these irregularities is smaller than the aerodynamic scale of the flow [6]. Consequently, the characteristic scale of the roughness is smaller than the characteristic scale of the flow.

The new approach for solving the problem of simulating the gas atoms scattering from a rough surface numerically has been developed in our previous papers [1]–[4]. This approach is based on the expansion of the roughness operator considered for Gaussian [5] or poly-Gaussian [1]–[4] random field. This expansion has good agreement with the DSMC methods. We have used the approximations of momentum and energy exchange coefficients on the rough surface to solve the problem [1]–[5].

The simulation of gas-surface interaction applying fractal and deterministic models of a roughness (especially constructed of flat elements [7] or sine waves) seem to be simpler [8]–[10]. However, the simplicity of rough surface simulation does not affect the computation time appreciably [3] and gives no advantages. Most difficult part of the calculation is not the simulation of roughness, but the computation of gas atom scattering from rough surface. Moreover, the computation time is even reduced in our method in comparison to the methods based on simple roughness models. In fact, the roughness operator $\hat{S}$ contains the whole information about the roughness geometry. If we obtain the correct representation of $\hat{S}$ and corresponding numerical algorithm, then there is no need to simulate the shape of the rough surface in the DSMC calculations.

Poly-Gaussian model of the roughness is studied [1]–[4], which is confirmed on the base of experimental data [11]. This model is more general, than Gaussian homogeneous isotropic random fields studied in St.-Petersburg State University [5], [6], and it has several advantages in comparison with other different more simple models of surface roughness applied in practice, including polygonal-line roughness [7], fractal [5], [12], conical-hole based model of the roughness [8], and the surface constructed of flat elements [9]–[10]. In particular, these simple models take into account only the input of large scale roughness into momentum and energy exchange coefficients on the surface [5]. So simple models have no possibility to take into account the micro-roughness (the roughness of small scale) – it is the advantage of poly-Gaussian model.

Other basic advantages of Gaussian and poly-Gaussian models are following.

1. Poly-Gaussian and Gaussian random fields are differentiable, so that the local normal exists everywhere on the surface under simple conditions [4], [5].
2. The simulation of different real surfaces processed in various ways is easy by simple variation of the parameters (scale coefficients, correlation function and weighted function).
3. Analytic representation of stochastic characteristics of the number of the crossings of inclined trajectory of a gas atom by the random field (such as the probability of the absence of the crossings) simplifies numerical Monte Carlo calculation of gas-surface interaction.

The reason to prefer poly-Gaussian model of roughness (to Gaussian one) is explained in [11] and in [4]: “Non-Gaussian statistics have rough surfaces of mixed structure, modeled as a result of several stages of processing (by pressing, extrusion, grinding, honing etc.) of wearing surfaces after being operated, maintained and so on. Other reason for non-Gaussian relief is the deterministic component, which is typical for the processing by whetting, milling, polishing etc.”

The possibility of precise approximation of real micro-reliefs is confirmed in different technology processes. In particular, the model is applied to simulate rough surfaces produced as by ionic bombardment of the steel using nitrogen ions, as well as by chemical etching of the steel by alcohol solution of nitric acid. In both cases a very good

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coincidence is confirmed. If we compare with the profiles of real surfaces applied for space vehicles (for instance, metallic or synthetic), then we conclude that Gaussian and poly-Gaussian models are closer to reality.

The scattering function $V(\mathbf{v},\mathbf{v}') = \hat{S}_1 V_0(\mathbf{v},\mathbf{v}',\mathbf{n})$ on the rough surface (taking into account only the first reflection from the surface, if there are multiple collisions [5]) can be rewritten as

$$V(\mathbf{v},\mathbf{v}') = \frac{1}{M_1} \iint\limits_{z_y<\cot\theta} dz_x\, dz_y\; V_0(\mathbf{v},\mathbf{v}',\mathbf{n})\, \frac{(\mathbf{n}\cdot\mathbf{v}')}{(\mathbf{n}\cdot\mathbf{N})(\mathbf{N}\cdot\mathbf{v}')} \int\limits_{-\infty}^{\infty} p(z,z_x,z_y)\,\Pi^{(1)}(\theta,\sigma_1,z,z_x,z_y)\,dz. \qquad \textbf{(1)}$$

Here $\mathbf{v}$ and $\mathbf{v}'$ are the velocities of incident and reflected gas atoms, $\mathbf{v} = (0, -\sin\theta, -\cos\theta)$, $V_0(\mathbf{v},\mathbf{v}',\mathbf{n})$ is the local scattering function on a smooth area on the rough surface, M_1 is the normalizing factor [3], $(\mathbf{n}\cdot\mathbf{v}')$ designates a scalar product of the vectors **n** and $\mathbf{v}'$, **n** and **N** are local and global normal unit vectors to the rough surface (**n** – to the smaller area on a rough surface), **N** and *z*-axis are directed perpendicular to the mean level of the roughness.

The roughness parameters in (1) are the square mean derivative σ_1 and a functional roughness parameter – the normalized correlation function ρ of *the* random field $z(x,y)$. All statistical parameters of the roughness (σ, R_z, skewness coefficient etc.) can be expressed in terms of the most important for gas interaction with rough surface parameters σ_1 and ρ.

The joined poly-Gaussian probability density *p* of the field $z(x,y)$ and of its derivatives $z_x = \frac{\partial z}{\partial x}$ and $z_y = \frac{\partial z}{\partial y}$ in directions *x* and *y* is defined by the formula

$$p(z,z_x,z_y) = \int\limits_0^{\infty} g^{v^2 r}_{x_0,y_0}(z,z_x,z_y)\,dF(v), \qquad \textbf{(2)}$$

where $g^{v^2 r}_{x_0,y_0}(z,z_x,z_y)$ is probability density of Gaussian process with correlation function $v^2 r(h)$ and its derivatives in the point t_0 of a gas atom interaction with the surface, $F(v)$ is a non-decreasing on the interval $[0;\infty)$ weighted function, and the correlation function $r(h)$ must satisfy the restriction: its spectrum must contain continuous component [2].

3. Aerodynamic shadowing effect on rough surface

The main difficulty in the calculation of scattering function (1) on rough surface and of aerodynamic macro-parameters of the flow is the computation of the conditional probability $\Pi^{(1)}$ of the absence of the level-crossings between the random field $z(x,y)$ and the trajectory of the gas atom (assuming the values of z, z_x, z_y to be given) [1], [5], [6]

$$\Pi^{(1)}(\theta_1,\sigma_1,z,z_x,z_y) = P\{z(0,y) \le z + y\cdot\cot\theta \mid z(0,0) = z,\ z_x = z'_x(0,0),\ z_y = z'_y(0,0)\}. \qquad \textbf{(3)}$$

This probability $\Pi^{(1)}$ allows us taking into account aerodynamic shadowing effect, which consists in the shadowing of a noticeable part of the area on a rough surface by the dimples of the relief for incident or reflected rarefied gas atoms, consequently, this area does not take part in the interaction of these gas atoms with the surface. The calculation of scattering function (1) becomes much more simple without the factor (3), because $\Pi^{(1)}$ depends on the values of a random field along the trajectory of a gas atom. So (3) is the integral in functional space (continuum integral), and it must be approximated by the integrals of high dimension (in practice, of dimension not less than 200 [5])

$$\Pi^{(1)}(\theta_1,\sigma_1,z,z_x,z_y) = \lim_{\substack{n\to\infty,\\ h\to 0}} \int\limits_{-\infty}^{u_1} d\eta_1 \int\limits_{-\infty}^{u_2} d\eta_2 \ldots \int\limits_{-\infty}^{u_n} p(0,h,2h,\ldots,nh,z,\eta_1,\eta_2,\ldots,\eta_n,z_x,z_y)\; d\eta_n, \qquad \textbf{(4)}$$

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where p is joined poly-Gaussian probability density of the field $z(x,y)$ and its derivatives $\eta_1, \eta_2, \ldots, \eta_n$ in n points u_1, ..., u_n, corresponding to the arguments $0, h, 2h, \ldots, nh$, according to the equation of the trajectory of a gas atom near the point (0, 0, z) of the first collision with the surface (here h is a pitch of movement along the y–axis in the computation).

The expression (4) requires too much computing time, and becomes useless for DSMC procedure. The difficulty of taking into account aerodynamic shadowing effect applying continuum integral computation is the reason why this approach is developed only in St.-Petersburg and is ignored in most calculations of rarefied gas flow interacting with rough surface (for example, in [7], [8], [9]).

However, the same problem remains if other random models of the roughness are applied (not Gaussian or poly-Gaussian). Even rough surface constructed of flat elements, if they are distributed randomly, requires calculating of continuum integrals (4) to take into account aerodynamic shadowing effect of the roughness (p becomes joined probability density of the field $z(x,y)$ with corresponding distribution). Only most simple deterministic models of the roughness like sine waves or other regular periodic functions permit computing of rarefied gas-surface interaction without calculating continuum integrals. But deterministic models do not take into account the micro-roughness, therefore this approach reduces substantially the influence of the roughness on scattering function (about 2–3 times in practical calculations).

The conditional probability $\Pi^{(1)}$ of absence of its level-crossings with the trajectory of a gas atom (3) (or (4)) depends only on the values of the random field under the trajectory of a gas atom, so we can consider only the profile of the surface in the plane of two vectors: of the local normal **n** and of the velocity **v** (or $\mathbf{v}'$) of gas particle. Thus the probability $\Pi^{(1)}$ for a random process can be studied instead of more complicated probability $\Pi^{(1)}$ for a random field. Therefore we consider the properties of poly-Gaussian random processes.

The probability density of poly-Gaussian random process in n points x_1, ..., x_n, corresponding to the arguments t_1, ..., t_n, is the mixture of Gaussian densities defined by the formula

$$p_{t_1,\ldots,t_n}(x_1,\ldots,x_n) = \frac{1}{(2\pi)^{n/2}} \int_0^\infty \frac{1}{\sqrt{|R_n(v)|}} \exp\left\{ -\frac{1}{2} (\mathbf{x} - \sigma(v)) R_n^{-1}(v) (\mathbf{x} - \sigma(v))^T \right\} dF(v), \qquad \textbf{(5)}$$

where $\mathbf{x} = (x_1, \ldots, x_n)$, $\sigma(v)$ and $R_n(v)$ are the vector of a mean value and the correlation matrix of the values of random process in n points, $|R_n(v)|$ is the determinant of the correlation matrix, $R_n^{-1}(v)$ – its inverse, the upper index T denotes transposition, $F(v)$ is a non-decreasing on the interval [0;∞) weighted function. The function $F(v)$ can be as continuous, as well discrete (stepped) – only the convergence of the integral is necessary.

Poly-Gaussian random process has two known advantages. First, is that it can approximate an arbitrary random process with any desired accuracy [13]. Second, the sample relief of poly-Gaussian random process can be simulated numerically by the simple algorithm based on the transformation of Gaussian distributions [11]. This algorithm is developed for many applications like light scattering on rough surface, growing of thin films for the micro-electronics, surface diagnostics by electronic spectroscopy, a friction in machinery etc. [11]. The possibility of precise approximation of real micro-reliefs is confirmed also in different technology processes. In particular, rough surfaces are simulated which have been modeled as well by ionic bombardment of the steel using nitrogen ions, as by chemical etching of the steel by alcohol solution of nitric acid. In both cases very good agreement with experimental measurements is confirmed.

4. Analytical and numerical solution of the inverse problem of determining the parameters of Gaussian or poly-Gaussian surface roughness

The techniques of similar type are applied to model the surfaces of flying vehicles moving in rarefied gas. To calculate the conditional probability $\Pi^{(1)}$ we apply suggested by Miroshin [6] Rice expansion in terms of the factorial moments N_m of m-th order of the number of the intersections of the random field with the trajectories of gas atoms

$$\Pi^{(1)} = \sum_{m=0}^{\infty} \frac{(-1)^m}{m!} N_m . \qquad \textbf{(6)}$$

The values N_m can be expressed as the $2m$-fold integrals of the joint probability density of $z(x,y)$ and its derivatives η_i (i=1,2,…m). From the generalized expression (2) of the probability density of poly-Gaussian random field and of

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its derivatives we have derived the representation of the factorial moments N_m of m-th order of the number $A_u[0; T]$ of the excursions of the random field above the level u of a trajectory of a gas atom

$$N_m(u) = \int_0^\infty N_m^G\left(\frac{u}{v}\right) dF(v), \tag{7}$$

where N_m^G are the factorial moments of m-th order of the number of the excursions of corresponding Gaussian random field above the level $\frac{u}{v}$ on the same interval [0; T].

Asymptotic evaluation of the factorial moments $N_m(u)$ for high level u is well-known for Gaussian processes [6], and for poly-Gaussian (spherically-invariant [14]) processes it takes the form

$$\lim_{u\to\infty} N_k(T,u) = \frac{\lambda_2^k}{c_1^{k-1}}, \quad k = 1, 2, \ldots, \tag{8}$$

where the constant value λ_2 is defined by the asymptotic expansion $r(h) = 1 - \frac{1}{2}\lambda_2 h^2 + O\left(|h|^{2+a}\right)$ of correlation function for $h \to 0$, and the weighted function $F(v)$ has a step on the right side of the interval, i. e. for some v_0 the function $F(v) = 1$ for all $v > v_0$, and the left-sided limit at this point v_0 is less than 1: $\lim_{v\to v_0-0} F(v) < 1$. This inequality means that the probability mixture determining poly-Gaussian field has Gaussian component with the maximal value of the variable v. Thus the asymptotic evaluation of the number of the excursions of poly-Gaussian process above the high level is similar to corresponding Gaussian asymptotic evaluation only if the weighed function $F(v)$ defines Gaussian component with the maximal value of the variable v_0. In all other cases asymptotically all the moments (8) are substantially different, i. e. all aerodynamic characteristics of the surface simulated by poly-Gaussian random field can be noticeably different from the characteristics of the surface simulated by Gaussian random field. Typical poly-Gaussian profiles and probability densities (5) are clearly different from Gaussian profiles and probability densities as presented in [11].

Numerically the factorial moments $N_m(u)$ (7) can be computed applying the expression of Gaussian random process as an integral of a Wiener process [5], [15].

There are two possibilities to apply obtained results to the problem of studying the influence of surface roughness on aerodynamic characteristics in rarefied gas flow.

On one hand, it is possible to derive asymptotic evaluations describing studied influence on smooth surface. However, in this case we need to take into account that similar evaluations in Gaussian case are very complicated and they are containing integrals in a functional space – continuum integrals. The computation of these integrals requires their approximation by the integrals of high dimensions (4). Besides, real surfaces always contain micro-roughness (even by the best processing), which could not be considered as smooth surface. Therefore asymptotic evaluations on smooth surface are not widely applied in practice. The main practical conclusion that could be made from asymptotic investigations is that the influence of surface roughness for poly-Gaussian model of roughness may be more noticeable, than for Gaussian model (because each next factorial moment N_{m+1} turns to infinity sooner, than each previous one).

On the other hand, poly-Gaussian model could be applied in numerical calculations, first of all in DSMC. Obtained results allow us selecting appropriate random field on preliminary stage of the computation so that it could satisfy two conditions. First, the random field must approximate well the roughness profiles of real surfaces applied in practice. Second, it must have necessary properties for optimizing numerical procedures (depending on physical and chemical characteristics of the gas and of the surface) for the best simulating rarefied gas atoms scattering from the surface.

The most general expansion of the scattering function V on a rough surface

$$V(\mathbf{v},\mathbf{v}') = \sum_{k=0}^{\infty} b_k(\mathbf{v},\mathbf{v}') K_k(\mathbf{v},\mathbf{v}'), \tag{9}$$

could be obtained from the expansion of the local scattering function $V_0(\mathbf{v},\mathbf{v}',\mathbf{n})$ in a series in terms of the orthogonal functions $\varsigma_k(\mathbf{n})$

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$$V_0(\mathbf{v},\mathbf{v}',\mathbf{n}) = \sum_{k=0}^{\infty} b_k(\mathbf{v},\mathbf{v}')\varsigma_k(\mathbf{n}), \qquad \textbf{(10)}$$

where $b_k(\mathbf{v},\mathbf{v}')$ are the coefficients in the expansion (9) or (10), and $K_k(\mathbf{v},\mathbf{v}') = \hat{S}_1\varsigma_k(\mathbf{n})$ are continuum integrals depending only on the parameters of the random field simulating the rough surface

$$K_k(\mathbf{v},\mathbf{v}') = \frac{1}{M_1}\iint\limits_{z_y<\cot\theta} dz_x\, dz_y\; \varsigma_k(\mathbf{n})\,\frac{(\mathbf{n}\cdot\mathbf{v}')}{(\mathbf{n}\cdot\mathbf{N})(\mathbf{N}\cdot\mathbf{v}')}\int\limits_{-\infty}^{\infty} g(z,z_x,z_y)\,\Pi^{(1)}(\theta_1,\sigma_1,z,z_x,z_y)\,dz\,. \qquad \textbf{(11)}$$

The values $K_k(\mathbf{v},\mathbf{v}')$ are completely defined by gas atom trajectory and by roughness parameters: σ_1 and $\rho(r)$. Thus it is possible to compute these integrals preliminary, before the main DSMC computation of a rarefied gas flow. This proposition is valid in our approach for an arbitrary local scattering function $V_0(\mathbf{v},\mathbf{v}',\mathbf{n})$ with only one restriction. The scattering function $V_0(\mathbf{v},\mathbf{v}',\mathbf{n})$ must have an approximation in the form (10) with only a few parameters to describe the physical and the chemical features of the gas and of the surface. Applied in practice scattering functions including diffuse, specular, Nochilla, Cercignani–Lampis and others satisfy this condition.

Knowing the analytical scattering model for the smooth surface without a roughness, the estimated parameters $b_k(\mathbf{v},\mathbf{v}')$ in (9) can be transformed according to the roughness operator in (11). The integrals $K_k(\mathbf{v},\mathbf{v}')$ are computed at preliminary step. From the equations (9)–(11), it follows that the parameters for the rough surface are linear combinations of the parameters on the smooth surface; the coefficients of the linear expansion depend on $K_k(\mathbf{v},\mathbf{v}')$. Simulating the calculated distribution, we obtain the velocities $\mathbf{v}'$ of the gas atoms that are scattered by the rough surface. In terms of the computational speed, our algorithm has an advantage over the methods that use simple geometrical models to simulate the shape of the rough surface [7], [8], [9]. This advantage is achieved by eliminating the need for the geometrical-shape simulation in DSMC; all the shape information is accounted for at the preliminary steps.

Applying all considered methods of the calculation of surface roughness effect in rarefied gas, we propose following solution algorithm for the inverse problem.

1. Determining the set of the orthogonal functions $\varsigma_k(\mathbf{n})$ (we have applied trigonometric approximations used in the local interaction theory).
2. Expanding the local scattering function $V_0(\mathbf{v},\mathbf{v}',\mathbf{n})$ in a series in terms of the system $\varsigma_k(\mathbf{n})$; hence the coefficients $b_k(\mathbf{v},\mathbf{v}')$ are calculated.
3. Computing the integrals $K_k(\mathbf{v},\mathbf{v}')$ in function space for different angles of incidence and reflection and for different values of roughness parameter σ_1 and the number *k*; obtained integrals $K_k(\mathbf{v},\mathbf{v}')$ in function space could be expanded in a series in terms of the system $\varsigma_k(\mathbf{N})$ for different values of σ_1 and k (where **N** is the normal vector to the average level of rough surface).
4. Expanding the scattering function $V(\mathbf{v},\mathbf{v}')$ on rough surface (or the momentum and energy exchange coefficients obtained from the experimental data) in a series in terms of the system $\varsigma_k(\mathbf{N})$.
5. Calculating the values of $K_k(\mathbf{v},\mathbf{v}')$ from two expansions of $V(\mathbf{v},\mathbf{v}')$ in a series in terms of orthogonal functions $\varsigma_k(\mathbf{N})$ with the coefficients $b_k(\mathbf{v},\mathbf{v}')$.
6. Finding the roughness parameter σ_1 and the correlation function from calculated $K_k(\mathbf{v},\mathbf{v}')$.

5. DSMC computation of rarefied gas atoms scattering from rough surface

According to the proposed algorithm, to solve the inverse problem we need original experimental measurements of aerodynamic values (scattering function, momentum or energy exchange coefficients) on the same surface with different roughness parameters. Unfortunately, only few experiments in rarefied gas are known satisfying these conditions. Experimental data from [9] and [10] we have applied testing solution algorithm of the inverse problem.

The comparison of computational results for two different models of a surface roughness with experimental data obtained in TSAGI [9]–[10] is presented for argon atoms scattering from Kapton surface for incidence angles $\theta = 60°$ and $\theta = 60°$ (Fig.1).

Dotted line presents the scattering function (indicatrix) on smooth surface without roughness; it is closer to diffuse scattering, than to specular or ray reflection, and coincides well with Nochilla and Cercignani–Lampis models. Dash-

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dotted line shows the numerical results obtained by Erofeev, Friedlander et.al. [9]. Solid lines present our results on poly-Gaussian surface with the same roughness parameters. And different signs present experimental results [9] for smooth (white, 1) and rough (black, 2) Kapton surface.

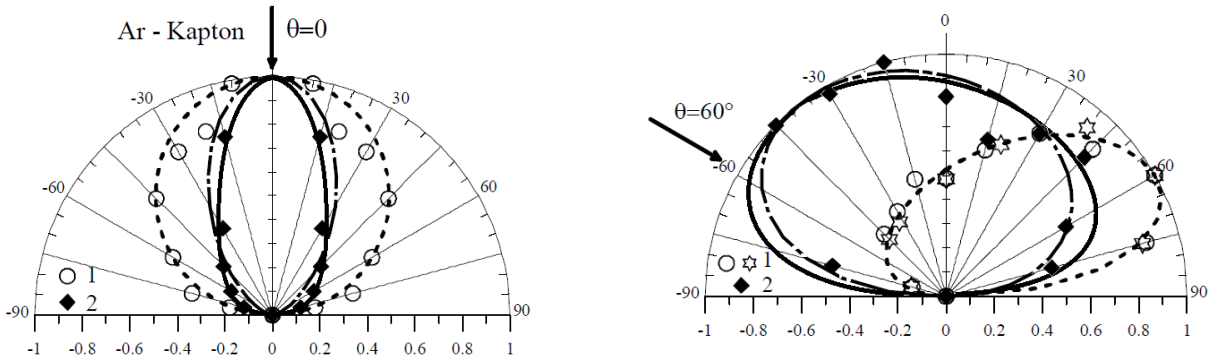

Figure 1: The argon scattering indicatrix on smooth and rough Kapton surfaces for incidence angles θ=0°; θ=60°.

Comparing numerical results of direct simulation with the experiment and analyzing the graphs we can conclude that aerodynamic shadowing effect either has not been taken into account in [9], or considered partially: aerodynamic shadowing effect gives a wider indicatrix graph as well onwards, as in backwards direction. The results depend on the local scattering kernel, and numerical DSMC computations for some local scattering functions show that the influence of surface roughness on momentum and energy exchange coefficients increases noticeably for poly-Gaussian model compared to Gaussian or to simple deterministic one.

The result of the solution of inverse problem shows that the value of σ_1 obtained by proposed algorithm from experimental data is 10%–30% higher than the value of σ_1 from the profile diagram measurements (or from direct measurements of geometrical parameters of artificial roughness). The difference is the input of the roughness of smaller scale not visible on profile diagrams.

Similar results are obtained from another experimental measurements in [9] and [10], for example from the normal momentum exchange coefficients of reflected gas flow (Ar and He) for different molecular energy E of incident molecular beam and different (Kapton or aluminium) surface roughness. In all computations the calculated value of σ_1 is 5%–35% higher than in profile diagram measurements.

6. Conclusion

Considered Gaussian and poly-Gaussian models of surface roughness allow us analytical and numerical solution of the inverse problem of determining roughness parameter σ_1 in the interaction of rarefied gas flows with rough surface. Obtained results lead us to following conclusion.

1. The solution of inverse problem is the best way to determine surface roughness parameters, because the roughness parameter σ_1 is not measured in the experiments: it is not included in technical standard of surface quality. The precision of measuring σ_1 from the profile diagram could be low, because profile diagrams show only the roughness of the largest scale. The computational error increases with σ_1.
2. The most effective solution of inverse problem is based on Poly-Gaussian model of roughness which is applied to take into account aerodynamic shadowing effect on rough surface: it means to calculate the probability of absence of level-crossings and related function space integrals in scattering function. This model is verified by experiment and its statistical parameters have better coincidence with the parameters of real surfaces applied in practice.
3. To reduce computing time in solving inverse problem the random field (modeling rough surface) must be simulated preliminary, before the main DSMC computation of rarefied gas flow.
4. The results for different models of roughness (including more simple) are compared and the effect of aerodynamic shadowing effect on rough surface is studied for real surfaces (like Kapton and Al) and gases (like Ar and He).
5. The value of σ_1 obtained from the solution of inverse problem is substantially higher (up to 30–35%) than the value of σ_1 from the profile diagram measurements. So this example confirms the advantage of our method of determining roughness parameter σ_1.

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