DOI: 10.13009/EUCASS2019-577

Output feedback control for optimal propellantless formation keeping using Electrostatic forces

Yehia Abdel-Aziz[1], S. K. Tealib[1], M. E. Awad[2,3], Kh. I. Khalil[1], and M. Radwan[2]
[1]National Research Institute of Astronomy and Geophysics (NRIAG), 11421-Helwan, Cairo, Egypt
[2]Faculty of Science, Cairo University
[3]Faculty of Aeronautic and Space Science, Benisef University, Egypt
yehia@nriag.sci.eg

Abstract The concept of Lorentz force can provide a propellantless electrostatic propulsion, through the interaction between an electrostatically charged satellite and the Earth's magnetic field to provide a useful thrust, which can be used for control both orbital and attitude motion. This idea needs to install an ion collector on the satellite to increase the level of charging as artificial charging. In this work the Lorentz force has been developed for two terms, a) first term, which was experienced with the magnetic field in the case of absolute charging of the spacecraft; b) the second term, which was experienced with the electric field in the case of differential charging of the spacecraft. We developed a mathematical model of fuel-optimal satellite formation keeping using a low thrust propulsion system using Electrostatic forces The linear time-varying relative dynamics which describe the relative motion in the presence of second zonal harmonic perturbation and total Lorentz force have developed. The proposed controller includes a feedback control using a real-time fuel-optimal control approach are considered. Application of a Legendre pseudospectral method is presented using quadratic programming for the fuel-optimal control problem. Feedforward control (iterative learning control) is used to improve formation keeping accuracy by eliminating the effects of periodic perturbations due to second zonal harmonics. The simulation results confirm the capability of using Lorentz force to provide an optimal propellantless to control the of formation keeping.

Keywords: Lorentz-force; a Legendre pseudospectral; nonlinear relative motion;

Introduction

According to the fundamental physical principle, a moving charged particle experiences the Lorentz force in a magnetic field. It is deduced that a charged spacecraft could actively generate the Lorentz force by modulating its surface charge when it moves through the Earth's magnetic field. Therefore, the Lorentz force is a possible good means to control the spacecraft without the fuel consumptions. However, due to the limitations that the directions of Lorentz force are determined by the local magnetic field and the velocity of the spacecraft with respect to the local magnetic field. As a result of this constraint, the Lorentz force cannot completely replace the traditional propulsion technologies. Kumar and Eyer 2012 developed linearized state of controlling a spacecraft formation, under J_2 effect. HarijonoDjojodihardjo 2014 developed Linearized Hill Clohessy-Wiltshire equations in modified form under the effect of J2 on spacecraft of formation flying. Bakhtiari et al. 2017 used Lagrangian mechanics to develop a model of formation flying considering J_2 perturbation. The Natural spacecraft charging level may reach about 10^{-8} C/kg (Vokrouhlicky 1989) and the induced Lorentz force with such charging level is insufficient to perturb the orbit and attitude of satellite significantly Peck M 2005 has proposed a new concept of active application of charge a spacecraft is introduced for artificial charging which is referred to Lorentz spacecraft. Abdel-Aziz 2007 developed, the variation in orbital elements of the satellite motion, under the effect of Lorentz force of a charged satellite in Earth's magnetic field. Pollock et al. 2011 studied the relative motion of a charged spacecraft subject to perturbations from the Lorentz force due to interactions with the planetary magnetosphere. Tsujii et al. 2012 derived a mathematical model of a charged satellite, taken the effect of the Lorentz force. Huang et al. 2014 studied the analytical expressions for the orbital motion of Lorentz spacecraft with respect to inclined low Earth orbit. Abdel-Aziz & Khalil 2014 studied the effects of a Lorentz force on the orbital motion in Low Earth Orbit (LEO) and developed a model for the effects of electromagnetic forces (Lorentz force) to modify or perturb the spacecraft orbits. Abdel-Aziz & Shoaib 2015 studied the attitude Dynamics of spacecraft, they studied the stability of the attitude orientation. Peng & Gao 2017 investigated periodic orbits under inter-satellite Lorentz force. Inalhan et al. 2002 developed linear programming to determine fuel-optimal control for formation-keeping. Wu, B, et al 2009 &2016 developed control system including feedback control and feed-forward control for satellite formation keeping.

Most of the previous studies interested in the study the effect of the magnetic field alone. The main idea in this work is a development the effect of Lorentz force for both magnetic and electric fields to increase the level of charging on the spacecraft surface, using a small device (Ion collector) to correct the drift in relative position considering second zonal harmonics perturbation. We focus on the design of fuel-optimal satellite formation keeping strategy using a low thrust propulsion system. The proposed controller includes feedback control and feed forward control. For feedback control, a real-time fuel-optimal control approach is proposed

1. The dynamic nonlinear model of Relative Motion

The full nonlinear equations of relative motion are given by the following (Alfriend et al. 2010).

$$\begin{aligned}
\ddot{x} &= 2\dot{\theta}_z\dot{y} + \ddot{\theta}_z y + \dot{\theta}_z^2 x + \frac{\mu}{r^2} - \frac{\mu}{r_d^{\,3}}(x+r) \\
\ddot{y} &= -2\dot{\theta}_z\dot{x} - \ddot{\theta}_z x + \dot{\theta}_z^2 y - \frac{\mu}{r_d^{\,3}}y \\
\ddot{z} &= -\frac{\mu}{r_d^{\,3}}z
\end{aligned} \tag{1}$$

where r_d & r are the position for the deputy and chief satellites, μ is the gravitational parameter, $\dot{\theta}_Z = \frac{h}{r^2}$ is the angular rotational velocity in the orbital rate direction of the LVLH frame and $\ddot{\theta}_z$ is the angular acceleration.

3. Nonlinear Relative J_2 Perturbations

The gravitational potential energy of chief satellite can be written as (Ginn 2007)

$$\mathbf{U} = -\frac{\mu}{r} - \frac{3\mu J_2 R_e^2}{2r^3}\left(\frac{1}{3} - \sin^2\Phi\right) = -\frac{\mu}{r} - \frac{3\mu J_2 R_e^2}{2r^3}\left(\frac{1}{3} - Z^2/r^2\right) \tag{2}$$

The gradient of U in Eq. (2) is computed in the LVLH frame to be (Wang et al. 2016)

$$\nabla U = \frac{\mu}{r^2}\hat{x} + \frac{3\mu J_2 R_e^2}{2r^4}\begin{Bmatrix} 1-3\sin^2 i \sin^2\theta \\ 2\sin^2 i \sin 2\theta \\ \sin 2i \sin\theta \end{Bmatrix}\begin{bmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \end{bmatrix} \tag{3}$$

The relative dynamics of the chief satellite can be written by the Lagrangian formulation:

$$\frac{d}{dt}\left(\frac{\partial \mathbf{K}}{\partial \dot{\mathbf{q}}}\right) = \frac{\partial \mathbf{K}}{\partial \mathbf{q}} - \frac{\partial \mathbf{U}}{\partial \mathbf{q}} \tag{4}$$

where U and K are the potential and kinetic energies of the satellite and $\mathbf{q} = [x \;\; y \;\; z]^T$ is the configurations of a satellite in LVLH coordinate.

The kinetic energy per unit mass of chief satellite is computed as

$$\mathbf{K} = \frac{1}{2}\left(\dot{x} + \dot{r} - \dot{\theta}_z y\right)^2 + \frac{1}{2}\left(\dot{y} + \dot{\theta}_z (r + x) + \dot{\theta}_x z\right)^2 + \frac{1}{2}\left(\dot{z} + \dot{\theta}_x y\right)^2 \tag{5}$$

The steering rate of the orbital plane can be written as:

$$\dot{\theta}_x = -\frac{3\mu J_2 R_e^2}{2hr^3}\sin 2i \sin\theta, \qquad \ddot{\theta}_z = -\frac{2h\dot{r}}{r^3} - \frac{3\mu J_2 R_e^2}{r^5}\sin^2 i \sin 2\theta$$

$$\ddot{\theta}_x = -\frac{3\mu J_2 R_e^2}{2r^5}\sin 2i \cos\theta + \frac{9\mu J_2 R_e^2}{2r^4 h}\sin 2i \cos\theta - \frac{36\mu^2 J_2^2 R_e^4}{r^6 h^2}\sin^2 i \cos i \sin^2\theta \cos\theta \tag{6}$$

By substituting potential energy in Eq. (2) and kinetic energy in Eq. (5), into a Lagrangian formulation of Eq. (3), the nonlinear dynamic equations of the satellite relative motion can be presented

$$\begin{aligned}
\ddot{x} &= 2\dot{\theta}_z \dot{y} + \ddot{\theta}_z y + \dot{\theta}_z^{\,2} x - k_d x - \dot{\theta}_x \dot{\theta}_z z - (\zeta_d - \zeta)\sin i \sin\theta - r(\xi_d - \xi) + \frac{\mu}{r_d^3}(x + r) \\
\ddot{y} &= -2\dot{\theta}_z \dot{x} + 2\dot{\theta}_z \dot{z} - \ddot{\theta}_z x + (\xi_d - \dot{\theta}_z^{\,2} - \dot{\theta}_x^{\,2})y + \ddot{\theta}_x z - (\zeta_d - \zeta)\sin i \cos\theta + \frac{\mu}{r_d^3} y \\
\ddot{z} &= -2\dot{\theta}_x \dot{y} - \dot{\theta}_x \dot{\theta}_z x - \ddot{\theta}_x y - (\xi_d - \dot{\theta}_x^{\,2})z - (\zeta_d - \zeta)\cos i + \frac{\mu}{r_d^3} z
\end{aligned} \tag{7}$$

where $\zeta = \frac{3\mu J_2 R_e^2}{r^4}\sin i \sin\theta, \quad \xi = \frac{\mu}{r^3} + \frac{3\mu J_2 R_e^2}{2r^5} - \frac{15\mu J_2 R_e^2}{2r^5}\sin^2 i \sin^2\theta$,

$\zeta_d = \frac{3\mu J_2 R_e^2}{r_d^5}\mathbf{r}_{dz}, \quad \xi_d = \frac{\mu}{r_d^3} + \frac{3\mu J_2 R_e^2}{2r_d^5} - \frac{15\mu J_2 R_e^2}{2r_d^7}\mathbf{r}_{dz}, \quad \mathbf{r}_{dz} = (r + x)\sin i \sin\theta + y \sin i \cos\theta + z \cos i$

4. Electromagnetic force (Lorentz force)

The total Lorentz force consisted of two compounds, 1) magnetic force ($\boldsymbol{F}_m$) is always perpendicular to the magnetic field, and experienced by a particle of charge q (Coulombs) moving through a magnetic field **B**, 2) the electric force ($\boldsymbol{F}_e$) always in the direction of the electric field, acts on a charged particle whether or not it is moving. (Ulaby & Ravaioli 2014 and Abdel-Aziz & Khalil 2014)

$$\mathbf{F}_L = \mathbf{F}_m + \mathbf{F}_e = q(\mathbf{v_r} \times \mathbf{B}) + q\mathbf{E} = q[\mathbf{v_r} \times \mathbf{B} + \mathbf{E}] \tag{8}$$

where $\mathbf{v}_r$ is the velocity vector with respect to the magnetic field, and **E** is an electric vector field.

4.1. Lorentz Magnetic force

In this section, the acceleration vector due to the Lorentz magnetic force is developing in case of absolute charging. The Lorentz magnetic force is defined by the size and polarity of the charge (q) on the satellite, ($\mathbf{v}_r$) is the velocity of the charged particle relative to the magnetic field, and the strength and direction of the magnetic field **(B)** The Lorentz force on a charged particle giving

$$\mathbf{F}_m = q(\mathbf{v_r} \times \mathbf{B}) \tag{9}$$

The acceleration vector is given by:

$$\mathbf{a}_m = \frac{\mathbf{F}_m}{m} = \frac{q}{m}(\mathbf{v_r} \times \mathbf{B}) \tag{10}$$

where $\frac{q}{m}$ is the charge-to-mass ratio of the satellite in Coulombs per kilogram.

The velocity of the charged and magnetic field can be written as:

$$\mathbf{v_r} = \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} = \begin{bmatrix} \dot{r} + \dot{x} - y\left(\dot{\theta}_z - \omega_E \cos i\right) - z\,\omega_E \cos\theta \sin i \\ \dot{y} - (r+x)\left(\dot{\theta}_z - \omega_E \cos i\right) + z\,\omega_E \sin\theta \sin i \\ \dot{z} - (r+x)\omega_E \cos\theta \sin i + y\,\omega_E \sin\theta \sin i \end{bmatrix} \tag{11}$$

Where θ & i are the true anomaly and inclination of chef satellite, and ω_E is Earth's rotation rate.

the magnetic field (B) has the form

$$\mathbf{B} = \begin{bmatrix} B_x & B_y & B_z \end{bmatrix} = \frac{B_0}{r_d^3}\left[3(\hat{\mathbf{n}}.\hat{\mathbf{r}}_d)\hat{\mathbf{r}}_d - \hat{\mathbf{n}}\right] \tag{12}$$

where B_0 is the magnetic dipole moment of Earth (8×10^{15} T m^3), $\hat{\mathbf{n}}$ The unit vector in the direction of the magnetic dipole moment, and $\hat{\mathbf{r}}_d = \frac{1}{r_d}[r+x \quad y \quad z]^T$ The expression of the magnetic dipole unit vector ($\hat{\mathbf{n}}$) in the LVLH frame is : (Huang et al. 2015)

$$\hat{\mathbf{n}} = \begin{bmatrix} n_x & n_y & n_z \end{bmatrix}^T = \begin{bmatrix} -(\cos\varepsilon\cos\theta + \sin\varepsilon\cos i\,\sin i)\sin\alpha - \sin i\,\sin\theta\cos\alpha \\ (\cos\varepsilon\sin\theta - \sin\varepsilon\cos i\,\cos\theta)\sin\alpha - \sin i\,\cos\theta\cos\alpha \\ \sin\varepsilon\sin i\,\sin\alpha - \cos i\,\cos\alpha \end{bmatrix} \tag{13}$$

where α is the tilt of the dipole angle between $\hat{Z}$ and $\hat{\mathbf{n}}$. The angle $\varepsilon = \Omega_m - \Omega$, with $\Omega_m = \omega_E\, t + \Omega_0$ is the inertial rotational angle of the magnetic dipole, with Ω being the right ascension of the ascending node of the chief, and Ω_0 is the initial rotation angle of the dipole.

The expressions of Lorentz magnetic acceleration in relative motion in case of a magnetic dipole can be derived by substituting Eq. (11) and Eq. (12) into Eq. (10) yields

$$\mathbf{a}_m = \frac{q}{m}\begin{bmatrix} a_{mx} \\ a_{my} \\ a_{mz} \end{bmatrix} = \frac{q}{m}\begin{bmatrix} v_x B_z - v_z B_y \\ v_z B_x - v_x B_z \\ v_x B_y - v_y B_x \end{bmatrix} \tag{14}$$

4.4. Equations of Motion of Electric Field

The charge distribution in a material is discrete, meaning that charge exists only where electrons and nuclei are and nowhere else (Ulaby & Ravaioli 2015). In case of differential charging ions on satellite surface suppose two point charges of equal magnitude but opposite polarity, separated by a distance d, the electric dipole is consisted, so to determine the electric potential V_e at any point P by applying equation:

$$V_e = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r_1} + \frac{-q}{r_2}\right) = \frac{q}{4\pi\varepsilon_0}\left(\frac{r_2 - r_1}{r_1 r_2}\right) \tag{15}$$

where $\varepsilon_0 = 8.85\times10^{-12} C^2/\left(N - m^2\right)$ is the permittivity of free space and $|r - r_i|$ is the distance between the observation point and the location of the charge q (Ulaby, 2005).

$$V_e = \frac{P.\hat{\mathbf{r}}}{4\pi\varepsilon_0 r^2}, \quad P = qd \tag{16}$$

where P is the electric dipole moment, d is the distance vector from the charge (+ q & - q)

The final expressions of Lorentz acceleration Expand by the electric field in Cartesian coordinates can be derived as:

$$\mathbf{F}_e = \frac{qd}{4\pi\varepsilon_0}\left(\frac{3xz}{r^2}\hat{\mathbf{X}} + \frac{3yz}{r^2}\hat{\mathbf{Y}} + \left(\frac{3z^2}{r^2} - 1\right)\hat{\mathbf{Z}}\right)$$

(17)

$$\mathbf{a}_e = \begin{bmatrix} a_{ex} & a_{ey} & a_{ez} \end{bmatrix}^T = \frac{\mathbf{F}_e}{m} = \frac{q}{m}\frac{d}{4\pi\varepsilon_0 r^5}\begin{pmatrix} 3xz \\ 3yz \\ 2z^2 - x^2 - y^2 \end{pmatrix} \tag{18}$$

5.1 Real-Time Optimal Control Law Design

Satellite formation will drift apart slowly because of various environmental perturbations after the desired formation is established. Thus, the formation keeping strategy is required. In this section, a real-time fuel-optimal control approach was subsequently proposed for satellite formation keeping in eccentric orbits, based on the developed nonlinear J_2 dynamic model. The control input constraints were included in the optimal control problem formulation to avoid control saturation of low-thrust propulsion system. The constraints fuel-optimal control for satellite formation keeping. (Pencil et al. 2004).

1- Relative dynamics constraints
   Using relative motion equations (1) considering J_2 perturbation equations (7) and total Lorenz (magnetic and electric) forces (14) & (18) to make the real-time fuel-optimal control approach practical. The following relative state error dynamics (Inalhan et al. 2002)

DOI: 10.13009/EUCASS2019-577

$$\delta \dot{\mathbf{x}}(t) = A(t)\delta \mathbf{x}(t) + B(t)\mathbf{u}(t) \tag{19}$$

where $\delta \mathbf{x} = \mathbf{x} - \mathbf{x}_d$ denotes the formation keeping error. $\mathbf{x} \in \mathbb{R}^6$, $\mathbf{x}_d \in \mathbb{R}^6$ denote, respectively, the measured, reference relative state of the follower with respect to the leader. $\mathbf{u}$(t) is the control acceleration vector.

2- Control acceleration constraints. (Pencil et al. 2004)

$$-\mathbf{u}_{\max} \le \mathbf{u}_{\max}(t) \le \mathbf{u}_{\max} \tag{20}$$

3- Initial condition constraints

$$\delta \mathbf{x}(t_0) = \mathbf{x}_d(t_0) - \mathbf{x}(t_0) \tag{21}$$

4- Final condition constraints

$$\delta \mathbf{x}(t_f) = 0 \tag{22}$$

The above optimal control problem can be summarized. The problem is to determine the control acceleration $\mathbf{u}(t)$ and the corresponding state trajectory $\delta \mathbf{x}(t)$ to minimize the following cost function

$$J = \frac{1}{2}\int_{t_0}^{t_f} \mathbf{u}^T(t)\, \mathbf{u}(t)dt \tag{23}$$

Subject to

$$\begin{cases} \delta \dot{\mathbf{x}}(t) = A(t)\delta \mathbf{x}(t) + B(t)\mathbf{u}(t) \\ -\mathbf{u}_{\max} \le \mathbf{u}_{\max}(t) \le \mathbf{u}_{\max}, \quad \forall t \in [t_0, t_f] \\ \delta \mathbf{x}(t_0) = \mathbf{x}_d(t_0) - \mathbf{x}(t_0) \\ \delta \mathbf{x}(t_f) = 0 \end{cases} \tag{24}$$

5.2 Legendre Pseudospectral Method

Numerical methods for solving the previous optimal control problem [Eqs. (23)–(24)] can be grouped into two major categories: indirect methods and direct methods. Direct methods are actively investigated by many researchers. There are two primary reasons for the widespread use of direct methods. First, they can be applied without explicitly deriving the necessary optimality conditions. Second, direct methods do not require a prior specification of the arc sequence for problems with path inequalities. In particular, a direct method that has shown tremendous promise is Legendre pseudospectral method (Elnagar et al. 1995; Qi et al. 2006). The Legendre pseudospectral method (Qi et al. 2006) makes real-time optimal control possible. A detailed description of the Legendre pseudospectral method for solving optimal control problems is provided in (Elnagar et al. 1995). The pseudospectral differentiation matrix transforms the differential equation of relative motion into a set of algebraic equations. Because the above optimal control problem is formulated over the time interval $[t_0, t_f]$, and the LGL points lie in the interval [−1, 1], the following transformation is used to express the problem for $\tau \in [\tau_0, \tau_N] = [-1,1]$, It follows that (23) and (24) can be replaced by

DOI: 10.13009/EUCASS2019-577

$$\begin{cases} J = \dfrac{t_f - t_0}{4} \displaystyle\int_{-1}^{1} \mathbf{u}(\tau)^T \mathbf{u}(\tau) d\tau \\ \delta\dot{\mathbf{x}}(\tau) = \dfrac{t_f - t_0}{4} \left[A(\tau)\delta\mathbf{x}(\tau) + B(\tau)\mathbf{u}(\tau) \right] \\ \delta\mathbf{x}(-1) = \mathbf{x}(t_0) - \mathbf{x}_d(t_0) \\ \delta\mathbf{x}(1) = 0, \quad -\mathbf{u}_{\max} \le \mathbf{u}(\tau) \le \mathbf{u}_{\max}, \end{cases} \tag{25}$$

5. NUMERICAL RESULTS

In this section, we discuss the numerical simulations for verification the effect of different acceleration on the relative position between two satellites due to J_2 Equation (7), Lorentz magnetic field Equations (14), and Lorentz force electric field Equation (18). We can apply those equations to get the perturbation due to second zonal harmonic J_2 then, using the separate magnetic and electric components of the Lorentz force to obtain the required values of q/m to maintain the relative position between two satellites. We can apply those equations to get the perturbation in the separate magnetic and electric components of the Lorentz force. These numerical simulations were performed used MATLAB©. The nonlinear differential equations of motion were solved using the 8th order Runge-Kutta method. The fuel-optimal control problem is transcribed into a quadratic programming problem by using Legendre pseudospectral method. The quadratic programming problem is solved by the command “quadprog” in MATLAB. Assuming initial values of position and velocity chief satellite are

r = [0 -7163.61171 0] km & v= [2.053396 0 -7.170630] km/s.

Deputy satellite r_d = [-0 -7163.51 0] km & v_d= [2.053596 -0.000000 -7.170668] km/s.

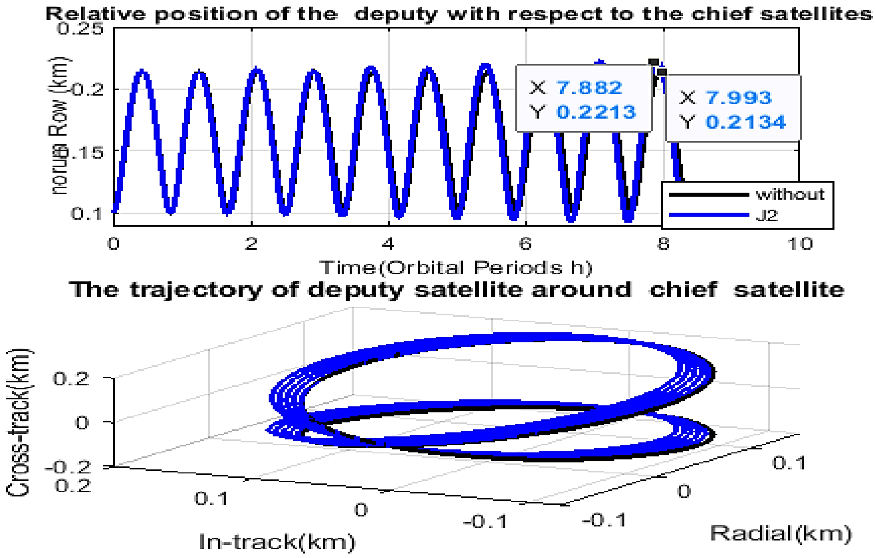


Fig. 1(a) Error in relative position for formation flying Satellite due to J2

DOI: 10.13009/EUCASS2019-577

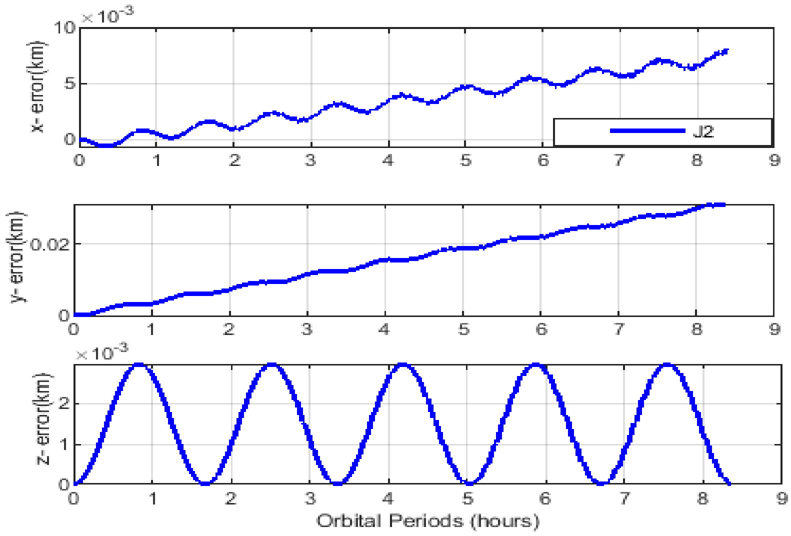
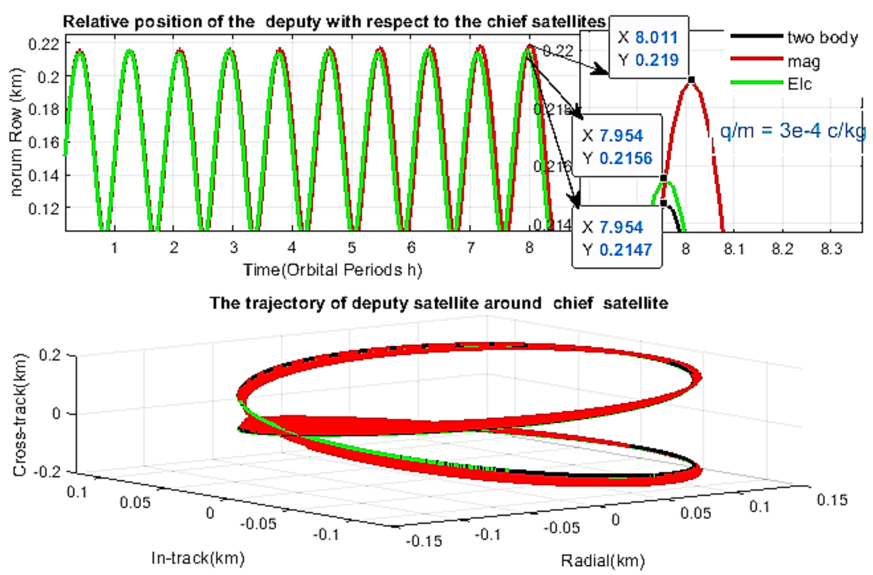


Fig. 1(b) Error in relative position for formation Flying Satellite due to J2



DOI: 10.13009/EUCASS2019-577

Fig2. Comparison the effect of the magnetic and electric fields at q/m = 3 e-4 c/kg on relative position and trajectory.

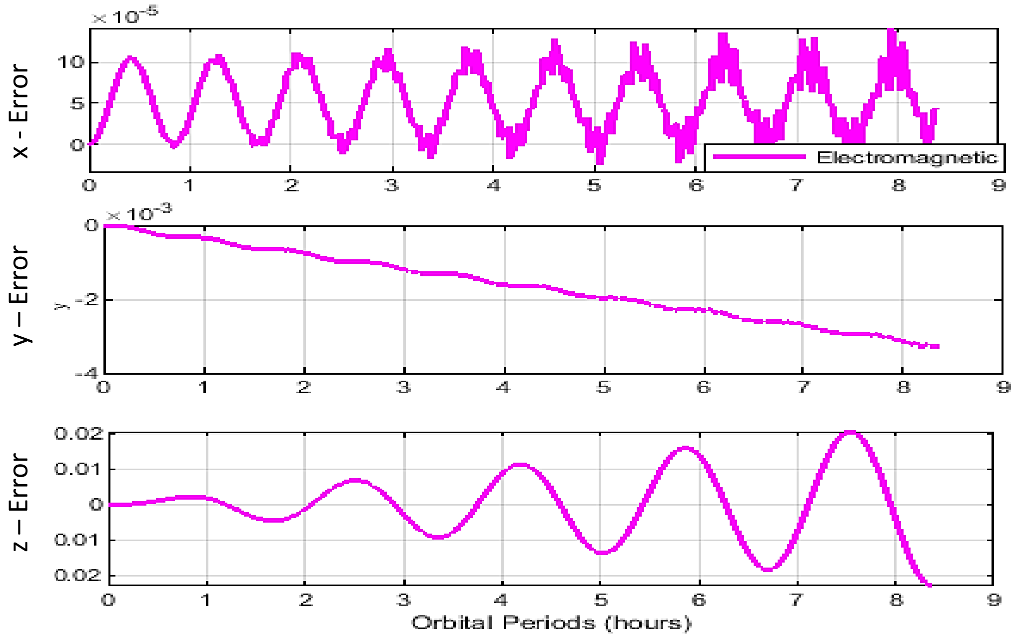


Fig 3. Error in relative position for electromagnetic force Satellite at q/m = 3e-4 electromagnetic

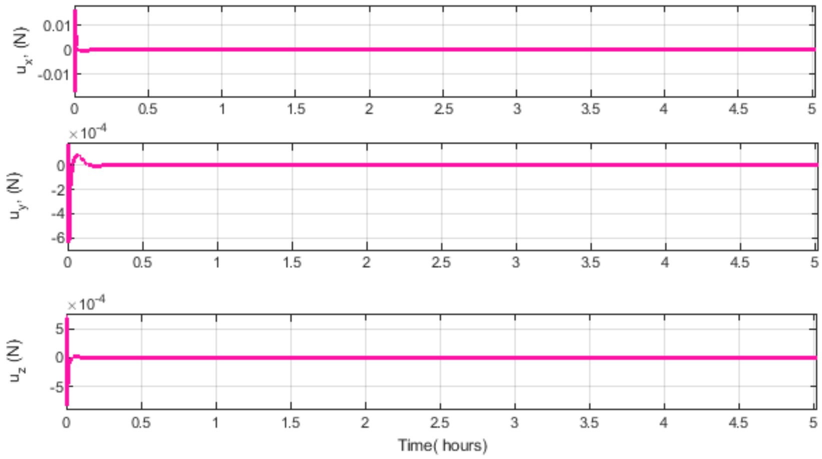


Fig (3.a) Control input by using real-time optimal control at m= 100 kg

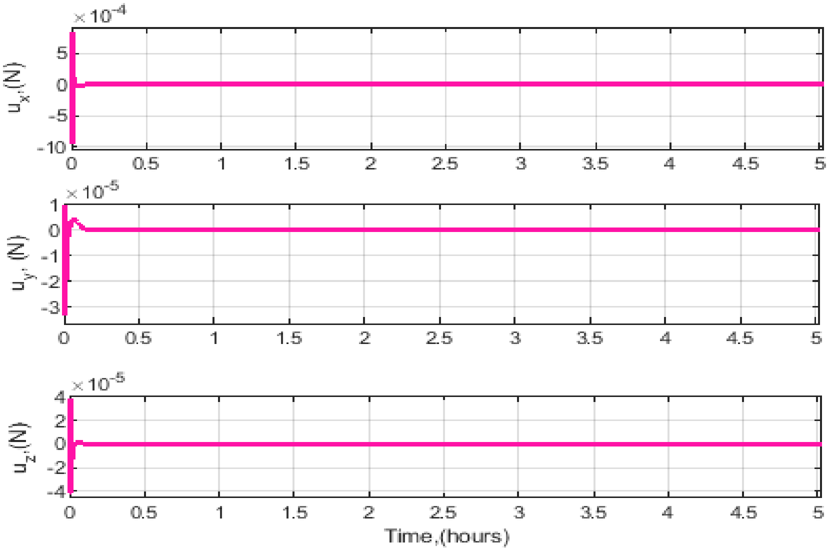


Fig (3. b) Control input by using real-time optimal control and total Lorentz force, m= 100 kg, q/m = 3*10-4 C/kg and time 3 periods

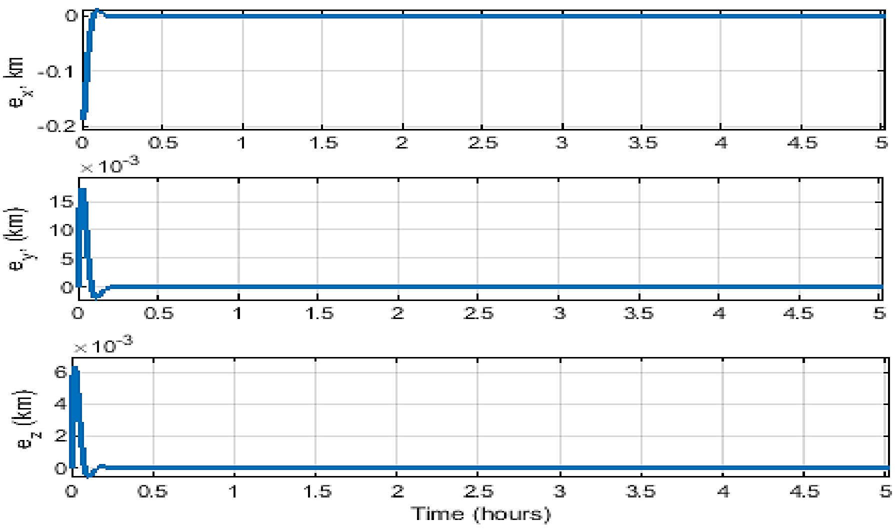


Fig.4 position errors by using the real-time fuel-optimal control

DOI: 10.13009/EUCASS2019-577

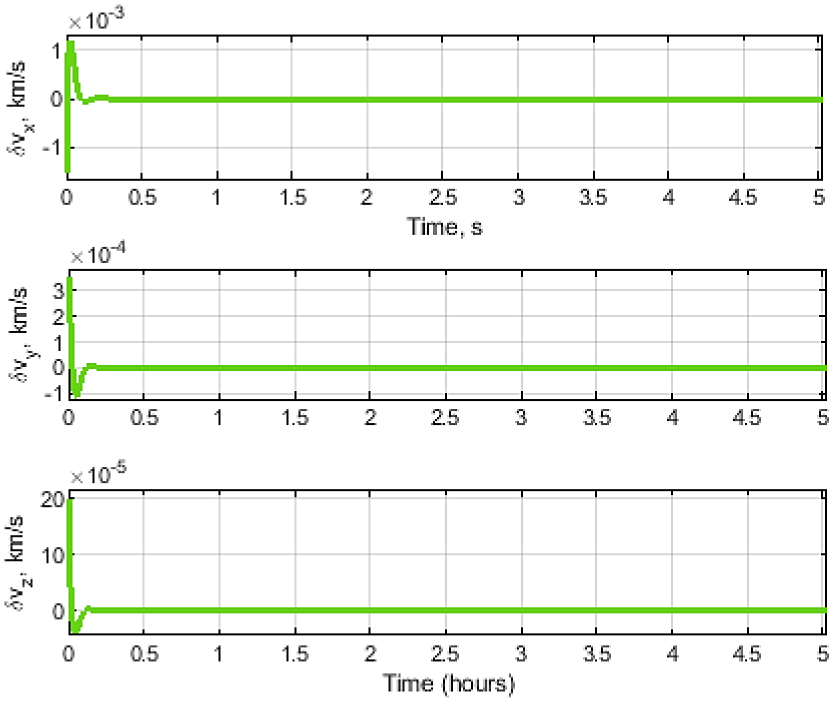


Fig.5 velocity errors by using the real-time fuel-optimal control

6. Results and discuss

We applied our model to study relative motion considering J_2 perturbation and we are assuming increase the level of charging in the spacecraft surface by small Ion collector to determine the optimal value of the charge to mass ratio which will be required to correct the drift in relative position due to the effects of J_2 .The model errors of the exact J_2 nonlinear perturbation (Eq.7) on relative position of as shown in Figures (1. a &1. b) where fig (1.a) is referred to as the variation in relative position and trajectory under the effect of J_2 after 5 periods. The results for Fig. (1.b) show that the model error of J_2 nonlinear relative dynamics was small because this model takes into account nonlinearity and J_2 perturbation. The primary error of the model was the drift in the in-track direction (y) about 20 m per 5 orbits.

Fig 2. It is shown the first main target in this study by developing the equation of motion by adding the electric field. We assume that the level of charge in the surface of the spacecraft is increased by a small ion collector up to 3e-4 c / kg. Where the black curve shows the norm relative and trajectory in the case of gravitational, while the red curve shows the effect of the magnetic field and the green curve shows the effect of the electric field, where the rate of change on relative position about 4 meters after 5 periods when we take the magnetic force only and about ± 1m Due to the effect of electric field under the same conditions.

Fig. 3 Shown the error in relative position, x-axis, y-axis z-axis, under the effect total Lorentz force (magnetic and electric) after 5 periods at a charge to mass ratio 3e-4 C/kg. We can conclude that, the magnitude of the charge to mass ratio 3e-4 can b sufficient to correct the drift in the relative position of formation flying due to the effect of second zonal harmonic J_2

In this subsection, we applied the proposed real-time fuel-optimal formation control method to study Formation Keeping. The initial position and velocity of the deputy and chief satellites were the same as those in the previous subsection. A full nonlinear propagator with second zonal harmonic perturbations, and total Lorentz forces are considered. The charge-to-mass ratio of the chief satellite was assumed $3*10^{-4}$ C/kg. Figs. (3.a & 3.b) shown Control by using real-time optimal control the mass of the deputy satellite was assumed to be 100 kg and Control by using real-time optimal with total Lorentz forces at q/m = $3*10^{-4}$ C/kg. Fig. (4 &5) shown correct position and velocity errors by using the real-time fuel-optimal control approach.

7. **Conclusion**

In this paper, we developed a new approach for formation flying satellites considering Electromagnetic force (Lorentz force). The Lorentz acceleration has been developed for two terms, a) the first term, which experienced with the magnetic field in the case of absolute charging of the spacecraft, including the effect of Earths tilted magnetic dipole; b) the second term, which is experienced with the electric field in the case of differential charging of the spacecraft. The main idea was to install a small device (Ion collector) to increase the level of charging in the spacecraft surface to obtain an order of magnitude for the charge to the mass ratio which can be valid for orbital control. We have investigated the different value of charge to the mass ratio in case of a magnetic part or electric part of Lorentz force which can be useful for control and correct the drift in relative position. The numerical results have shown that the value of the charge to the mass ratio in case of total Lorentz forces (magnetic + electric) is about ± 3e-4 C/kg can be valid to correct drift in relative position after 5 periods. We have applied our model to test the validly of Lorentz acceleration for formation flying control, and to correct the drift in relative position due to second zonal harmonic (J_2). Then, a real-time fuel-optimal continuous low-thrust control approach was proposed to keep the formation against various orbital perturbations. Finally a real-time fuel-optimal continuous with total Lorentz forces are considered with charge to mass ratio about $3*10^{\wedge -4}$ C/kg to correct the drift in the relative position of formation flying due to the effect of Second zonal harmonic.

6. References

Abdel-Aziz, Y.A., 2007, February. Lorentz force effects on the orbit of a charged artificial satellite: a new approach. In AIP Conference Proceedings (Vol. 888, No. 1, pp. 385-391). AIP.

Abdel-Aziz, Y.A., and Khalil, K.I., 2014. Electromagnetic effects on the orbital motion of a charged spacecraft. Research in Astronomy and Astrophysics, 14(5), p.589.

Abdel-Aziz, Y.A., and Shoaib, M., 2015. Attitude dynamics and control of spacecraft using geomagnetic Lorentz force. Research in Astronomy and Astrophysics, 15(1), p.127.

Alfriend, K. A., Vadali, S. R., Gurfil, P., How, J., & Breger, L. S. 2010, Spacecraft Formation Flying: Dynamics, Control and Navigation, 1st edn.

Bakhtiari, M., Daneshjou, K. and Fakoor, M., 2017. Long-term effects of main body's obliquity on satellite formation perturbed by third-body gravity in elliptical and inclined orbit. Research in Astronomy and Astrophysics, 17(4), p.039.

Kumar, B.S. and Eyer, J.K., 2012. Averaged relative states for spacecraft formation control in the presence of J2 disturbances. *Acta Astronautica*, *77*, pp.61-67.

DOI: 10.13009/EUCASS2019-577

Djojodihardjo, H., 2014. Influence of the earth's dominant oblateness parameter on the low formation orbits of micro-satellites. International Journal of Automotive and Mechanical Engineering, 9, p.1802.

Elnagar, G., Kazemi, M.A. and Razzaghi, M., 1995. The pseudospectral Legendre method for discretizing optimal control problems. *IEEE transactions on Automatic Control*, *40*(10), pp.1793-1796. Ginn, J.S., 2007. Spacecraft Formation Flight: Analysis Of The Perturbed J2-modified Hill-Clohessy-Wiltshire Equations

Huang, X., Yan, Y., Zhou, Y. and Yi, T., 2014. Improved analytical solutions for relative motion of Lorentz spacecraft with application to relative navigation in low Earth orbit. Proceedings of the Institution of Mechanical Engineers, Part G: Journal of Aerospace Engineering, 228(11), pp.2138-2154.

Huang, X., Yan, Y. and Zhou, Y., 2015. Optimal Lorentz-augmented spacecraft formation flying in elliptic orbits. Acta Astronautica, 111, pp.37-47.

Peck, M., 2005, August. Prospects and challenges for Lorentz-augmented orbits. In AIAA guidance, navigation, and control conference and exhibit (p. 5995).

Pencil, E., Kamhawi, H. and Arrington, L., 2004, September. Overview of NASA's Pulsed Plasma Thruster Development Program. In 40th AIAA/ASME/SAE/ASEE Joint Propulsion Conference and Exhibit (p. 3455).

Peng, C. and Gao, Y., 2017. Formation-Flying Planar Periodic Orbits in the Presence of Intersatellite Lorentz Force. IEEE Transactions on Aerospace and Electronic Systems, 53(3), pp.1412-1430.

Pollock, G.E., Gangestad, J.W. and Longuski, J.M., 2011. Analytical solutions for the relative motion of spacecraft subject to Lorentz-force perturbations. Acta Astronautica, 68(1-2), pp.204-217.

Ulaby, F.T., 2005. Electromagnetics for engineers (pp. 127-134). Pearson/Prentice Hall.

Ulaby, F.T., Ravaioli, U. and Michielssen, E., 2014. Fundamentals of applied electromagnetics. 7 e. Prentice Hall.

Vokrouhlicky, D., 1989. The geomagnetic effects on the motion of an electrically charged artificial satellite. Celestial Mechanics and Dynamical Astronomy, 46(1), pp.85-104.

Qi, G., Kang, W., and Ross, I. M. (2006). “A pseudospectral method for the optimal control of constrained feedback linearizable system.” IEEE Trans. Autom. Control, 51(7), 1115–1129.

Tsujii, S., Bando, M. and Yamakawa, H., 2012. Spacecraft formation flying dynamics and control using the geomagnetic Lorentz force. Journal of Guidance, Control, and Dynamics, 36(1), pp.136-148.

Inalhan, G., Tillerson, M. and How, J.P., 2002. Relative dynamics and control of spacecraft formations in eccentric orbits. Journal of guidance, control, and dynamics, 25(1), pp.48-59.

DOI: 10.13009/EUCASS2019-577

Wang, D., Wu, B. and Poh, E.K., 2016. Satellite Formation Flying: Relative Dynamics, Formation Design, Fuel Optimal Maneuvers and Formation Maintenance (Vol. 87). Springer.

Wu, B., Poh, E.K., Wang, D. and Xu, G., 2009, March. Satellite formation keeping via real-time optimal control and iterative learning control. In 2009 IEEE Aerospace conference(pp. 1-8). IEEE.

Wu, B., Xu, G. and Cao, X., 2016. Relative dynamics and control for satellite formation: accommodating J 2 perturbation. Journal of Aerospace Engineering, 29(4), p.04016011.