

# Evaluation of Nonlinear Elements Effects on Aircraft Pilot Coupling for Highly Maneuverable Aircraft

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## Abstract

This paper proposes a practice that is performed to evaluate the effects of the nonlinear flight control system elements, with regard to Pilot-Induced-Oscillations. Flight control law design aspects for highly maneuverable aircrafts has been mentioned, an appropriate design has been presented for longitudinal and lateral-directional axes. OLOP criterion and well-studied linear PIO criteria have been applied. Impacts of the actuator parameters and pilot models have been studied. Linear and nonlinear simulations have been conducted to conclude the comments for the proposed methodology. Effect of aerodynamic loading on the actuators have been modeled for nonlinear simulations.

## 1. Introduction

The quantification of the interaction between the human pilot and aircraft dynamics is compulsory for the highly augmented, highly maneuverable aircrafts with digital fly-by-wire control systems. These interactions are known as Pilot-in-the-Loop Oscillations or Pilot Involved Oscillation (PIO) and mostly generated an adverse impact on aircraft control. In MIL-HDBK-1797A, PIO is defined as the sustained and uncontrollable oscillations resulting from efforts of the pilot to control the aircraft [1]. PIO has following categories due to its trigger and behavior [1], [2]:

Type 1: Linear behavior due to time delays and excessive phase loss resulting from the filters.

Type 2: Nonlinear behavior due to the actuator position and rate limits.

Type 3: Highly nonlinear behavior of highly augmented aircraft due to flight control mode changes.

In aviation history, there have been catastrophic events caused by Type-2 PIO related with highly augmented aircrafts [2]. As explicitly stated above, the nonlinear elements belong with the flight control system have to be taken into consideration for the PIO Type 2 susceptibility assessment.

It is aimed to present the necessary procedure to involve PIO analyses, especially for Type 2, in the early steps of the control algorithm design studies. As a flight control law designer, it is crucial to comment on the requirements about the critical parts of the FCS like the actuators. Actuators have their rate limits inherently in order to protect the system against high input rates [3]. The rate limiters create an inevitable nonlinearity in the FCS, however, most of the well-known flight control system analyses methods do not include them as an effector.

The Open-loop onset point (OLOP) criterion introduced a structured way to study the Type 2 PIO occurrences for the first time in 1990's [3]. OLOP criterion is a promising method to investigate stability issues related with the nonlinear elements in the fly-by-wire flight control systems. The methodology is based on the jump phenomenon which could be detected with the use of describing functions for the nonlinear elements.

Prior to the control algorithm design and analysis studies, the linear system has been constructed as follows. A high performance jet trainer aircraft database based on CFD analyses has been used to develop the 6-DOF mathematical model. The model also includes the engine model and atmospheric relations. Trim and linearization have been performed accordingly. Later, command and stability augmentation system has been designed to ensure stability and provide Level 1 handling qualities for all operational flight envelope without resulting an uncontrollable event.

The control algorithm and aircraft dynamics have been analyzed in detail. Besides OLOP criterion, widely practiced PIO Type 1 criteria such as Bandwidth-Phase Delay, Smith-Geddes and Average Phase Rate are also applied. Linear

and nonlinear simulations have been conducted. In nonlinear simulations, the effect of the hinge moments on the rate limiting element has been observed.

## 2. Nomenclature

$\alpha$	: Angle of attack
$\beta$	: Angle of sideslip
$V_N$	: True airspeed
$h$	: Pressure altitude
$\delta_{ail}$	: Aileron deflection
$\delta_e$	: Elevator deflection
$\delta_{stick}$	: Stick deflection
$\phi$	: Roll angle
$\theta$	: Pitch angle
$p, q, r$	: Body angular rates
$R, M, N$	: Body axis moments
$D$	: Drag
$L$	: Lift
$Y$	: Side force
$C_{X,Y,Z,l,m,n}$	: Aerodynamic coefficients
PIO	: Pilot Induced Oscillations
OLOP	: Open Loop Onset Point
CFD	: Computational Fluid Dynamics

## 3. Mathematical Models of System Elements

### 3.1. Air Vehicle Model

A high performance jet trainer aircraft is studied for this work. The aerodynamic database is based on the CFD analyses and the wind tunnel tests. In this study, the flight envelope is limited to sea level to 20 kft altitude and 0.2 Mach to 0.8 Mach speed flight conditions. The engine model is based on a generic engine database.

The MATLAB & Simulink environment has been used in order to simulate the nonlinear aircraft dynamics. A six degree-of-freedom mathematical model, which is defined on the Body-fixed reference frame, has been prepared. The reference frames is shown on the *Figure 1:Figure 1*.

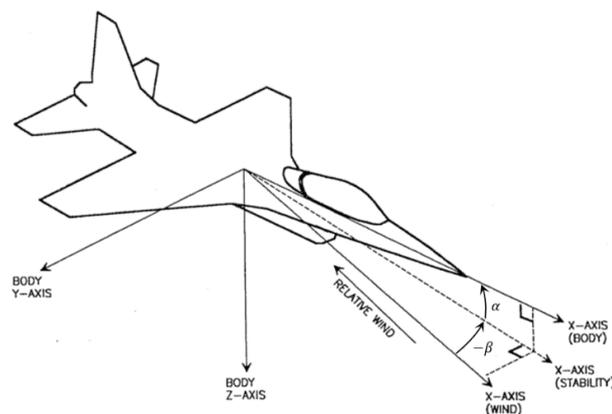


Figure 1: Aircraft Reference Frames

### 3.2. Control Law Design

The aircraft linear model has been inspected, it has decoupled longitudinal and lateral-directional dynamics in the envelope mentioned in introduction part. Thus, the design of the control laws has been accomplished using the appropriate techniques for those axes [4]. A detailed summary about the control laws could be found in following sections.

#### 3.2.1. Longitudinal Control Law

The basic mode of the aircraft longitudinal dynamics is the short period motion, which represents the short term pitch response. It is normally an oscillatory second order motion of dominantly including the variations of the pitch rate and angle-of-attack states. Linear system representation of this mode which is approximated by the small perturbation theory is given in Eq. 1.

$$\begin{bmatrix} \dot{q} \\ \Delta\dot{\alpha} \end{bmatrix} \approx \underbrace{\begin{bmatrix} M_q & M_\alpha \\ 1 - \frac{L_q}{V_N} & -\frac{L_\alpha}{V_N} \end{bmatrix}}_A \begin{bmatrix} q \\ \Delta\alpha \end{bmatrix} + \underbrace{\begin{bmatrix} M_{de} \\ -\frac{L_{de}}{V_N} \end{bmatrix}}_B \Delta\delta_e \quad (1)$$

Since the dynamic stability issues and the longitudinal flying and handling qualities requirements mostly address the short period motion, the command and stability augmentation system design has been done based on that.

Actuator and sensor dynamics have also their effects in the total aircraft and FCS response. In this study, the actuator dynamics are included in the linear system which would be used as the design basis. The sensor dynamics have been neglected in design part but they are included in the analyses. The overall linear system is given through Eq. 2 to 4.

$$\begin{bmatrix} \dot{q} \\ \Delta\dot{\alpha} \\ \Delta\dot{\delta}_e \\ \Delta\ddot{\delta}_e \\ \Delta\ddot{\delta}_e \end{bmatrix} \approx \underbrace{\begin{bmatrix} A & B * C_{act} \\ \text{zeros}(3,2) & A_{act} \end{bmatrix}}_{A_1} \begin{bmatrix} q \\ \Delta\alpha \\ \Delta\delta_e \\ \Delta\ddot{\delta}_e \end{bmatrix} + \underbrace{\begin{bmatrix} B * D_{act} \\ B_{act} \end{bmatrix}}_{B_1} \Delta\delta_e^c \quad (2)$$

Where;

$$\dot{x}_{act} = A_{act}x_{act} + B_{act}u_c, x_{act} = [\Delta\delta_e \quad \Delta\dot{\delta}_e \quad \Delta\ddot{\delta}_e]^T \quad (3)$$

$$y_{act} = C_{act}x_{act} + D_{act}u_c \quad (4)$$

The main strategy is not altering the original dynamic behavior much if the aircraft response is adequate in terms of performance and stability requirements. The stability augmentation has been performed using the washed out pitch rate and the AoA feedback signals. The control law architecture is given in the *Figure 2*.

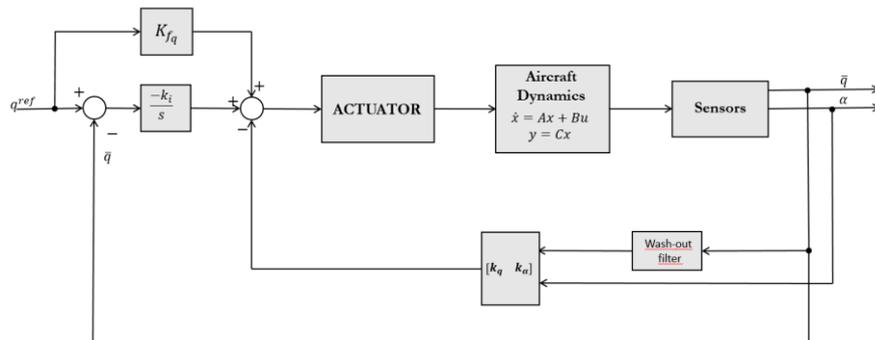


Figure 2: Longitudinal Control Law Architecture

The command path is designed as pitch rate command system. This type of control architecture is advantageous for the tracking performance while the load factor command systems are generally better in maneuvering performance. Especially for the high performance jet aircrafts, pitch rate command systems is widely used for longitudinal dynamics in Category A flight phases.

The longitudinal control laws use the elevator surface deflection as the only control effector. Control law parameters and objectives are assigned considering the time domain performance characteristics and pole zero distributions of all the elements of the control system. The closed loop linear system has been constructed. Then, the output feedback pole assignment problem has been defined and solved for the controllable pair of system and control matrices.

### 3.2.2. Lateral-Directional Control Law

The aircraft lateral-directional dynamics consist of spiral, dutch-roll and roll motions, all those dynamics have been considered in control law design. Linear system representation of the lateral-directional dynamics is given in Eq. 5-7.

$$\dot{x}_b = Ax_b + Bu \quad (5)$$

$$A = \begin{bmatrix} N_r & N_v & N_p & 0 \\ Y_r - u_N & Y_v & Y_p + w_N & g \cos \alpha_N \\ R_r & R_v & R_p & 0 \\ \tan \alpha_N & 0 & 1 & 0 \end{bmatrix}, B = \begin{bmatrix} N_{\delta_a} & N_{\delta_r} \\ Y_{\delta_a} & Y_{\delta_r} \\ R_{\delta_a} & R_{\delta_r} \\ 0 & 0 \end{bmatrix} \quad (6)$$

$$u = [\Delta\delta_a \quad \Delta\delta_r]^T, x_b = [r \quad v \quad p \quad \phi]^T \quad (7)$$

Stability issues are mostly related with the Dutch-roll and spiral motions, flying and handling qualities requirements address the roll and Dutch-roll dynamics. Thus, the command and stability augmentation system design has been done considering those requirements plus the performance requirements.

Similar to the longitudinal part, actuator and sensor dynamics have also their effects in the total aircraft and FCS response. The sensor dynamics have been neglected in design part but they are included in the analyses. The actuator system could be defined with the system elements in Eq.8.

$$A_{act} = \begin{bmatrix} A_{act\_a} & 0^{3*3} \\ 0^{3*3} & A_{act\_r} \end{bmatrix}, B_{act} = \begin{bmatrix} B_{act\_a} & 0^{3*1} \\ 0^{3*1} & B_{act\_r} \end{bmatrix}, C_{act} = \begin{bmatrix} C_{act\_a} & 0^{1*3} \\ 0^{1*3} & C_{act\_r} \end{bmatrix}, D_{act} = 0^{2*2} \quad (8)$$

The overall linear system which is used as the design basis is given in Eq. 9-11.

$$\dot{x}_1 = A_1x_1 + B_1u_c, y_1 = C_1x_1 \quad (9)$$

$$x_1 = [r \quad v \quad p \quad \phi \quad \Delta\delta_a \quad \Delta\dot{\delta}_a \quad \Delta\ddot{\delta}_a \quad \Delta\delta_r \quad \Delta\dot{\delta}_r \quad \Delta\ddot{\delta}_r]^T, u_c = [\Delta\delta_a^c \quad \Delta\delta_r^c]^T \quad (10)$$

$$A_1 = \begin{bmatrix} A & B * C_{act} \\ \text{zeros}(6,4) & A_{act} \end{bmatrix}, B_1 = \begin{bmatrix} 0 \\ B_{act} \end{bmatrix} \quad (11)$$

In order to include the sideslip angle as a system state instead of side velocity, transformation has been performed on the system defined above. The stability augmentation has been performed using the stability axis roll rate, lateral load factor, stability axis yaw rate and bank angle feedback signals. The control law architecture is given in the *Figure 3*.

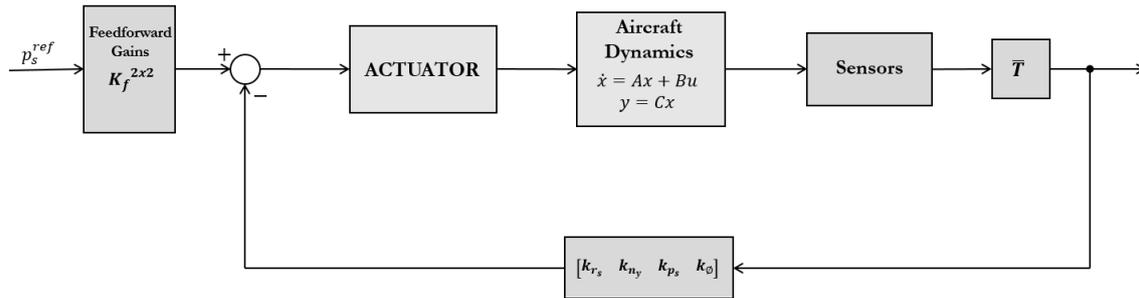


Figure 3: Lateral-Directional Control Law Architecture

The main strategy is resulting in level 1 flying and handling qualities over all the design envelope without disrupting the airplane like response. For high performance jet aircrafts, the eigenstructure assignment technique is an appropriate method in terms of performance issues.

### 3.3. Actuator Model

The actuators have been modelled by modifying the model given in Ref. [5]. Throughout the analyses, those models were also altered in terms of their parameters. The base actuator model 3<sup>rd</sup> order transfer function is given in the Eq. 12.

$$\frac{\text{Actuator Output}}{\text{Actuator Input}} = \frac{35}{s + 35} * \frac{71.4^2}{s^2 + 2 * 0.736 * 71.4 * s + 71.4^2} \quad (12)$$

The actuator transfer function was modelled as the continuous time linear Simulink model. The actuator model is shown in the Figure 4.

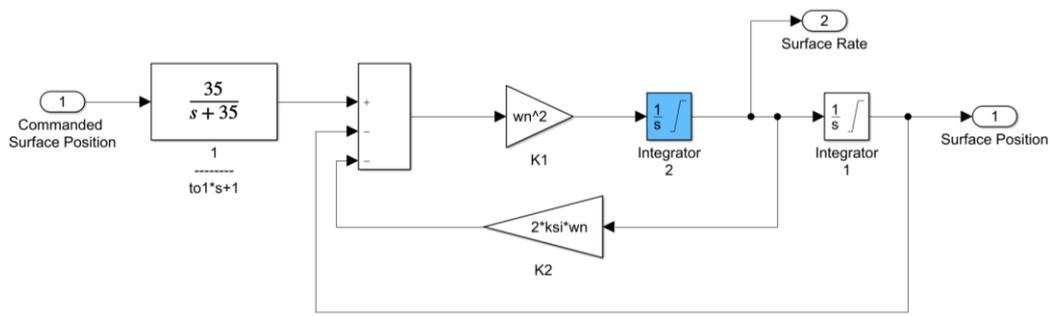


Figure 4: Actuator Simulink Diagram

The rate limit in the actuator is included in the integrator which is shown in a blue color and the position limit is included in the surface position integration element. The first order characteristic have been represented on the transfer function in the forward path, which is the dominant behavior. While constructing and manipulating the actuator models, the actuator performance parameters given in Table 1 were taken as the requirement.

Table 1: Actuator Parameters

Surface	Position Limit [°]	Rate Limit [°/sec]	Time Constant [sec]
$\delta_e$	$\pm 30$	60	0.0286
$\delta_a$	$\pm 30$	60	0.0286
$\delta_r$	$\pm 30$	120	0.0286

### 3.4. Pilot Model

Modelling the pilot dynamics is comprehensive issue by itself. In this paper the pilot model is needed for the Type 2 PIO analyses. The pilot behavior is defined as the “synchronous precognitive behavior” in the case of a developed nonlinear PIO. For that scenario, it is suggested to use a pure gain pilot model to represent the piloting action[6, 7, 8].

In this paper, three different pilot models have been used and integrated with the analyses. Two of them are the pure gain pilot models, as suggested, to represent the high gain and low gain pilot behavior. The pilot gain was assessed according to the method described in the [7] based on the researches held in DLR. Then, the gain was set in order to attain the crossover frequencies tabulated in *Table 2*.

Table 2: Pure Gain Pilot Model Parameters

	LOW Gain	HIGH Gain
Longitudinal Motion	$\phi_c = -130^\circ$	$\phi_c = -90^\circ$
Lateral Motion	$\phi_c = -160^\circ$	$\phi_c = -110^\circ$

Where  $\phi_c$  is the gain crossover frequency.

The third pilot model is the Neal-Smith model, which is widely used in the closed loop pilot aircraft analyses. [7, 9]. The simple mathematical representation of that model is given with the transfer function in Eq. 13.

$$\frac{\delta_{stick}(s)}{\theta(s)} = \frac{K_p(\tau_{p_1}s + 1)e^{-0.25s}}{(\tau_{p_2}s + 1)} \quad (13)$$

The model parameters are assigned considering the minimum bandwidth requirement for only Category A flight phases (high bandwidth tracking tasks) which is 3.5 r/s. The resulting Neal-Smith model parameters are given in *Table 3*.

Table 3: Neal-Smith Pilot Model Parameters

Parameter	$K_p$	$\tau_{p_1}$	$\tau_{p_2}$
Value	-0.125	0.060	0.010

## 4. “PIO” Criteria

Control law synthesis for highly augmented manned aircrafts have to be performed considering the PIO criteria besides the flying and handling qualities [1]. During control law synthesis, the PIO criteria are used as guidelines in order to prevent foreseeable means of PIO. The frequently applied ones are cited in the following sections.

### 4.1. Linear (Type-1)

There are numerous linear PIO detection criteria introduced in literature [10]. In this paper, few of them are selected according to their success rates and practicality [11]. The selected criteria are explained through the following paragraphs.

**Bandwidth and Phase Delay:** This criterion focuses on the aircraft stick force to attitude transfer function. The aircraft attitude bandwidth is a measure of the frequency range which does not require an excessive compensation from the pilot while the pilot is exerting good closed-loop control. The phase delay expresses the sensitivity to pilot gain of the augmented system closed loop characteristics when the system is operating at the highest gains. [10]

**Average Phase Rate + Gibson Gain Template:** In this criterion, the consideration is the phase rate which is defined as the gradient of the phase angle with respect to the frequency in the neutral stability region (180o phase delay). It is the direct measure of the high frequency phase roll-off. This criterion is important, since the related parameters are strongly effect the PIO generation process. [11]

**Smith-Geddes:** This criterion provides an estimate of the crossover frequency of the pilot and controlled vehicle system based on a linear formula developed from fixed base experiments. Then, the phase angle is obtained from that crossover frequency. That value gives the conclusion about PIO occurrence susceptibility of the system. [10]

**Gibson Dropback:** It is an open loop method which is derived from the attitude step response of the system. It is a measure of K/s characteristic of the system. Thus, this criterion is applicable on only the rate command systems.

Except the Gibson Dropback, all the criteria have been checked for all the design points of the longitudinal and lateral-directional augmented aircraft dynamics. Dropback criterion has been only considered for the pitch axis. Garteur's comprehensive study about the success rates for those criteria with flight tests and a lot of engineering applications. The outcomes of that study is given in *Table 4*.

Table 4: Success Rates of PIO Criteria

PIO Criterion	Global Success Rate [%]	Conservatism [%]	Safety [%]
Bandwidth & Phase Delay	81.6	89.5	77.3
Avg. Phase Rate & Gain Template	72.4	68.3	97.7
Smith – Geddes	80.3	82.2	84.1
Gibson Dropback	61.8	68.3	63.6

## 4.2. Nonlinear (Type-2): Open Loop Onset Point

The definition of the OLOP has given as the frequency response value of the open-loop aircraft-pilot system at the closed loop onset frequency [3, 8]. The onset frequency is defined as the frequency where the magnitude curve of the closed loop transfer function and the straight line which has the slope of 20 dB/decade. This straight line would have the gain crossover at the local onset frequency which is dependent on the rate limit. The determination of the parameters is exemplified in the related literature.

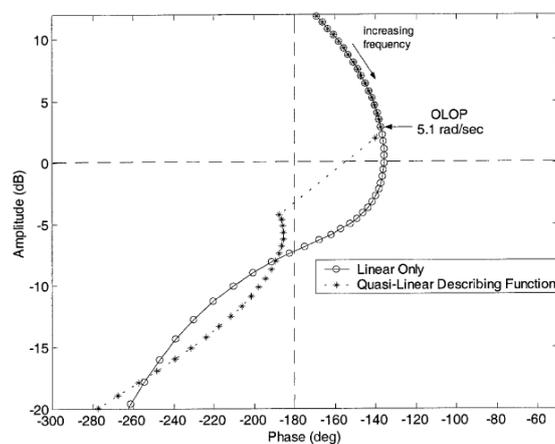


Figure 5: Nichols Plot of the Rate Limiter Describing Function

In *Figure 5*, it is shown that, at a specific frequency value, the nonlinear element describing function frequency response has a jump. The location where the jump occurs is important in terms of closed loop stability issues. The physical background of the OLOP boundaries could be explained in the lights of this information. In the *Figure 6*, the effect of the activation of a nonlinear element, or saturation of a rate limiter, is figured out. If the location of the OLOP in the Nichols chart is above the 0 dB line, then the jump would occur towards higher closed loop magnitude values. Depending on the location of the OLOP, the phase jump may even result in a closed loop instability.

The OLOP criteria is a well-documented method that could be implemented in the clearance process of the flight control law design. As the type-1 PIO criteria, the effects of the nonlinearities could be reported before the simulation studies. Therefore, the designer would have the answer of the question that “in what kind of a flight control system the designed control algorithm would be operating efficiently”.

The OLOP analyses are driven by using the linearized systems and predefined parameters. Thus, it is a linear analysis procedure.

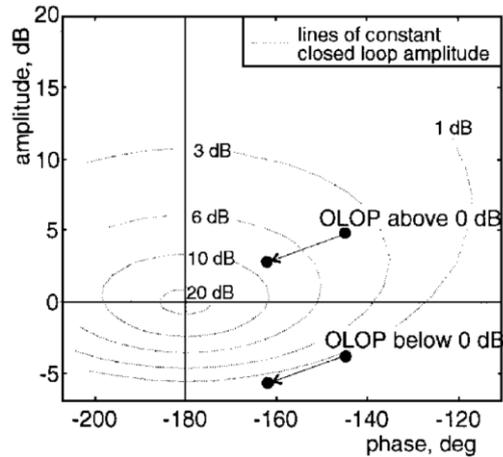


Figure 6: Phase Jump in Frequency Response

In this study, the stability boundaries which firstly documented by Duda is taken as the criteria to pass [3]. The Figure 7 shows the stability boundaries proposed by the theory for prediction of Type-2 PIO.

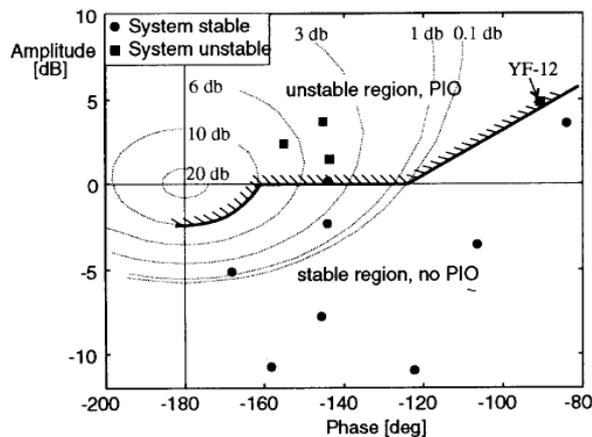


Figure 7: OLOP Stability Boundary

The nonlinear simulations have also been performed to observe the effects of the nonlinearities as they are. For the simulation input, an attitude tracking task is used. This step and ramp tracking task is taken from the MIL-HDBK-1797A [1]. Desired performance of this task requires no pilot induced oscillations. It is also considered as the PIO Type-2 criteria to pass.

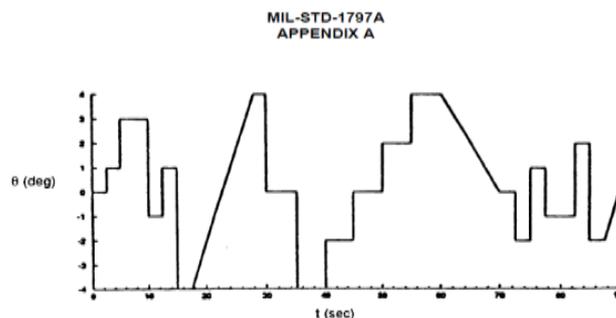


Figure 8: Pitch Attitude Tracking Task

## 5. Results

### 5.1. Linear Analyses' Results

The linear analyses have been conducted using the methodology explained in the sections 4.1 and 4.2. The control laws design points have been cleared according to the flying and handling qualities and the linear PIO criteria, in the first place. In this study, a limited envelope of flight conditions have been considered. The control law gains have been scheduled for this envelope. Design points of the control laws are tabulated in *Table 5*.

Table 5: Control Law Design and Analysis Grid

	Altitude [kft]	Mach
Longitudinal	[0 : 5 : 10]	[0.20 : 0.10 : 0.50]
Lateral-Directional	[0 : 10 : 20]	[0.30 : 0.10 : 0.80]

The clearance of the control laws could be summarized with Table 6 and Table 7.

For the pitch rate command system:

Table 6: Longitudinal Control Laws Analysis Summary

Mach	h (kft)	Gain Margin(de) [dB]	Phase Margin (de) [deg.]	Delay Margin (de) [s]	Stability Margin(de)	Bandwidth $w_{BW_0}$ [rad/s]	Phase Delay $\tau_{P_0}$ [s]	Average Phase Rate $\dot{\phi}_{w_{u_0}}$ [deg/Hz]	Freq. at 180 deg lag $w_{180_0}$ [Hz]	Dropback $\Delta G(q)$ [dB]	Smith-Geddes CHR	CAP
0.2	0	20.43	79.13	0.64	0.86	2.92	0.050	36.21	1.06	8.05	7.74	0.76
0.3	0	17.85	72.60	0.45	0.82	3.87	0.051	36.59	1.25	5.61	6.66	0.68
0.4	0	16.71	73.16	0.40	0.79	4.55	0.051	36.72	1.38	5.25	5.77	0.69
0.5	0	15.86	71.24	0.35	0.77	5.08	0.051	36.66	1.48	4.94	5.00	0.70
0.2	5	19.73	79.28	0.59	0.85	2.97	0.051	36.55	1.07	6.08	7.70	0.56
0.3	5	18.59	70.52	0.48	0.83	3.48	0.051	36.57	1.18	5.80	7.12	0.59
0.4	5	17.25	73.59	0.43	0.80	4.21	0.051	36.71	1.32	5.42	6.23	0.59
0.5	5	16.37	69.91	0.37	0.78	4.69	0.051	36.75	1.40	5.16	5.58	0.61
0.2	10	28.81	-163.17	2.41	0.58	1.64	0.048	34.86	0.77	9.03	1.05	0.67
0.3	10	19.55	68.84	0.53	0.84	3.08	0.050	36.05	1.10	5.91	7.59	0.51
0.4	10	17.70	61.62	0.39	0.80	3.72	0.051	36.45	1.22	5.70	6.88	0.52
0.5	10	17.08	69.20	0.40	0.80	4.25	0.051	36.63	1.32	5.30	6.18	0.52

For the roll rate command system:

Table 7: Lateral Control Laws Analysis Summary

Mach	h (kft)	Gain Margin(da) [dB]	Phase Margin (da) [deg.]	Delay Margin (da) [s]	Stability Margin(da)	Bandwidth $w_{BW_0}$ [rad/s]	Phase Delay $\tau_{P_0}$ [s]	Average Phase Rate $\dot{\phi}_{w_{u_0}}$ [deg/Hz]	Freq. at 180 deg lag $w_{180_0}$ [Hz]
0.3	0	Inf	Inf	Inf	0.982	2.32	0.055	39.925	0.98
0.4	0	Inf	Inf	Inf	0.992	2.73	0.057	41.119	1.095
0.5	0	Inf	Inf	Inf	0.999	3.06	0.058	41.901	1.183
0.6	0	Inf	Inf	Inf	0.96	3.09	0.057	40.831	1.198
0.7	0	Inf	Inf	Inf	0.864	3.06	0.056	40.002	1.198
0.8	0	Inf	Inf	Inf	0.787	3.01	0.055	39.464	1.189
0.3	10	31.272	Inf	Inf	0.95	1.86	0.056	40.287	0.875
0.4	10	Inf	Inf	Inf	0.994	2.11	0.057	41.239	0.941
0.5	10	Inf	Inf	Inf	0.995	2.48	0.058	41.958	1.035
0.6	10	Inf	Inf	Inf	0.995	2.81	0.058	42.084	1.117
0.7	10	Inf	Inf	Inf	0.995	3.04	0.059	42.261	1.175
0.8	10	Inf	Inf	Inf	1.008	3.07	0.058	41.618	1.186
0.3	20	29.622	117.053	0.685	0.92	1.85	0.057	40.936	0.891
0.4	20	36.253	Inf	Inf	0.968	1.81	0.058	41.784	0.88
0.5	20	Inf	Inf	Inf	0.997	1.86	0.059	42.27	0.883
0.6	20	Inf	Inf	Inf	0.997	2.15	0.059	42.27	0.953
0.7	20	Inf	Inf	Inf	0.996	2.39	0.059	42.576	1.009
0.8	20	Inf	Inf	Inf	0.994	2.59	0.059	42.762	1.057

In terms of flying and handling qualities and the Type 1 linear PIO criteria, the control laws are clear for all of the design points.

## 5.2. Application of the OLOP Criterion

In section 4.2, the proposed theory for this criterion has been presented. The procedure has applied for all the design points with the high gain pilot model (pure gain). Results are displayed for the nominal conditions (*Figure 9*). Additionally, effects of different actuator dynamics and different pilot models have been observed, those comparisons have been located in this section (*Figure 10 & Figure 11*).

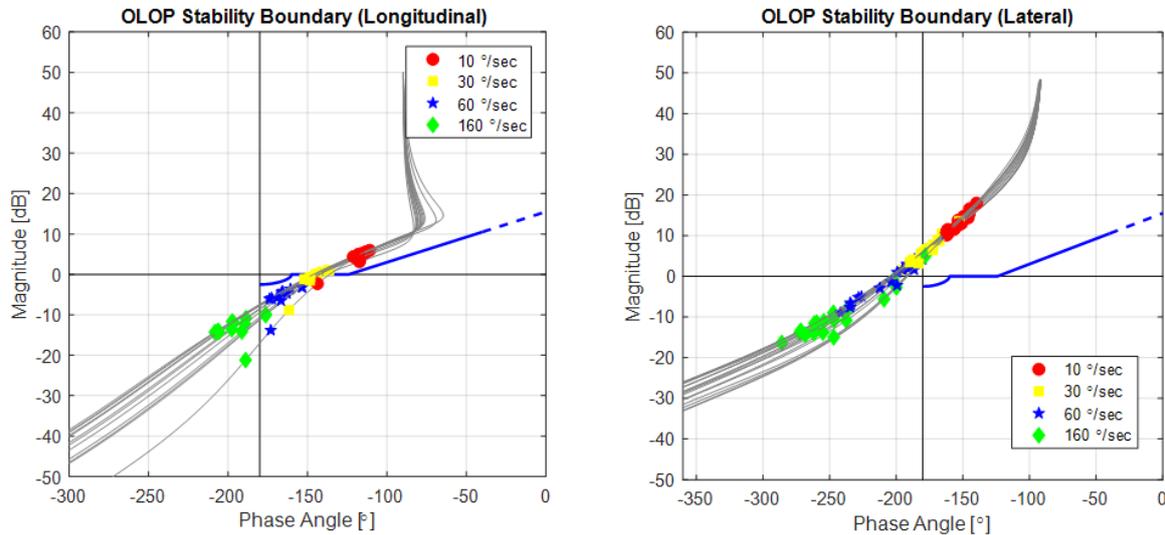


Figure 9: OLOP Analysis for Design Points

From *Figure 9* left part, most of the longitudinal design points are seem to become unstable below 30 deg/sec rate limiting in actuators. For the lateral-directional design points (*Figure 9* right part) the tolerance to decrease in rate limits is a bit smaller, the unstable behavior emerges for the rate limits below 60 deg/sec, generally.

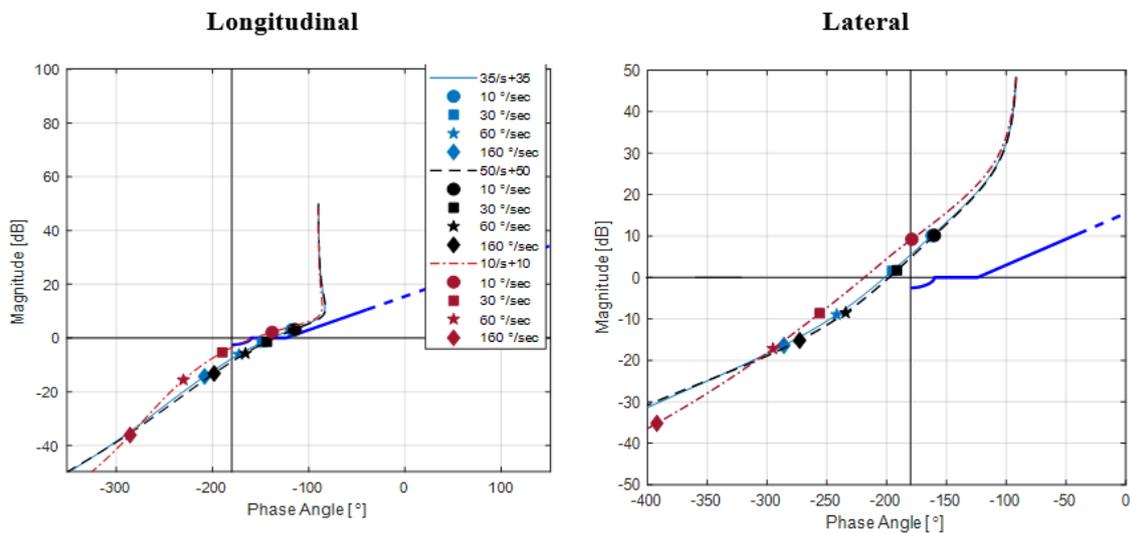


Figure 10: Effect of Actuator Time Constant on OLOP

The time constant of the actuator has been altered in design and the analysis tools in order to show the sensitivity of the OLOP to main actuator characteristic. It could be inferred that if the actuator time constant is small enough (blue and black lines in *Figure 10*), the shape of the frequency response would remain similar but the location of the OLOP may shift. However, for the longer time constant value, for instance 100 ms, the frequency response has shifted in phase. This shifted system is become more sensitive to the gain changes, and the instability is more probable for the longitudinal system.

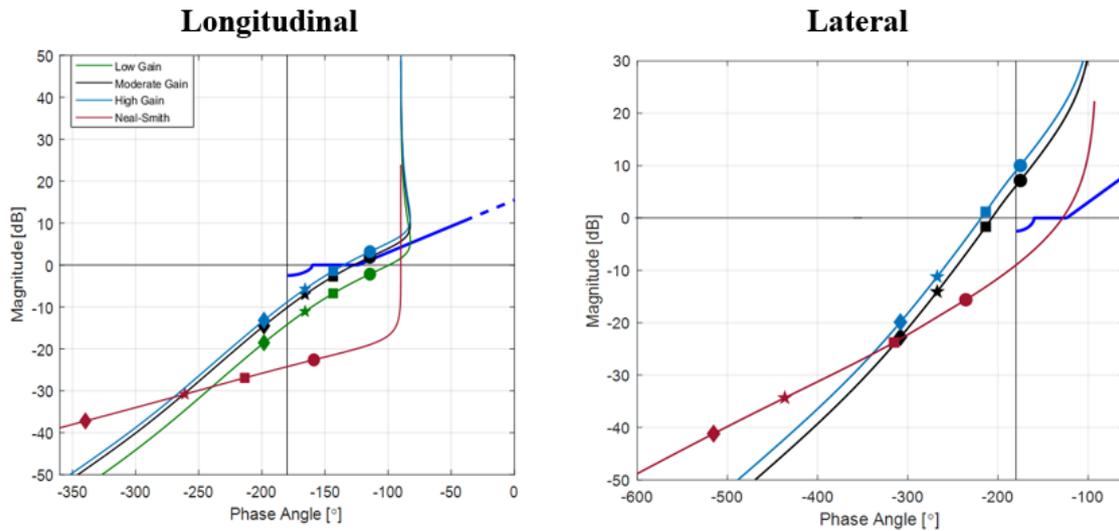


Figure 11: Effect of Pilot Model on OLOP

Effect of the pilot models is shown in *Figure 11* for longitudinal and lateral directional control systems. Pilot gain alters the location of OLOP significantly. Pure gain pilot model with the high gain parameters results in the most critical scenarios. Thus, for the continuation of this study, pure gain pilot model with high gain would be used.

### 5.3. Simulation Analyses

In order to support the meaning of the OLOP criterion with the nonlinear simulations, the attitude tracking task has been performed for both longitudinal and lateral motions.

#### 5.3.1. Linear Simulations

The linear simulations have been conducted for lateral and longitudinal motions, with the use of the linear decoupled aircraft dynamics and the appropriate controller architecture. Onto the augmented aircraft model, the pilot model and the command gradient have been added. *Figure 12* shows the architecture of the linear simulation model.

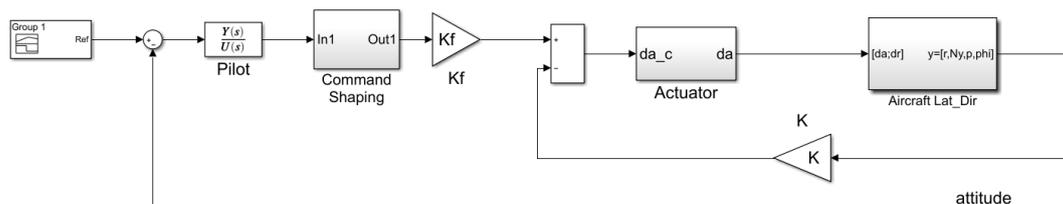


Figure 12: Linear Simulation Model

Selected flight condition is sea level altitude and 0.3 Mach speed for longitudinal, and 20 kft altitude and 0.5 Mach speed for lateral-directional analyses. The simulations have run starting from that condition. High gain pilot model is used. The results of the OLOP analysis for those specific conditions is shown in the *Figure 13*.

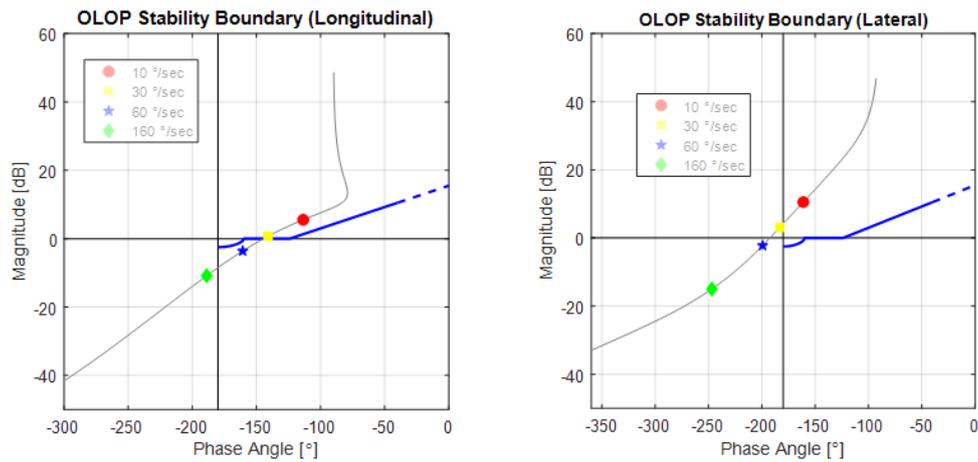


Figure 13: OLOP Analysis Results for Simulation Analysis Conditions

The time histories of the linear simulations is presented in Figure 14.

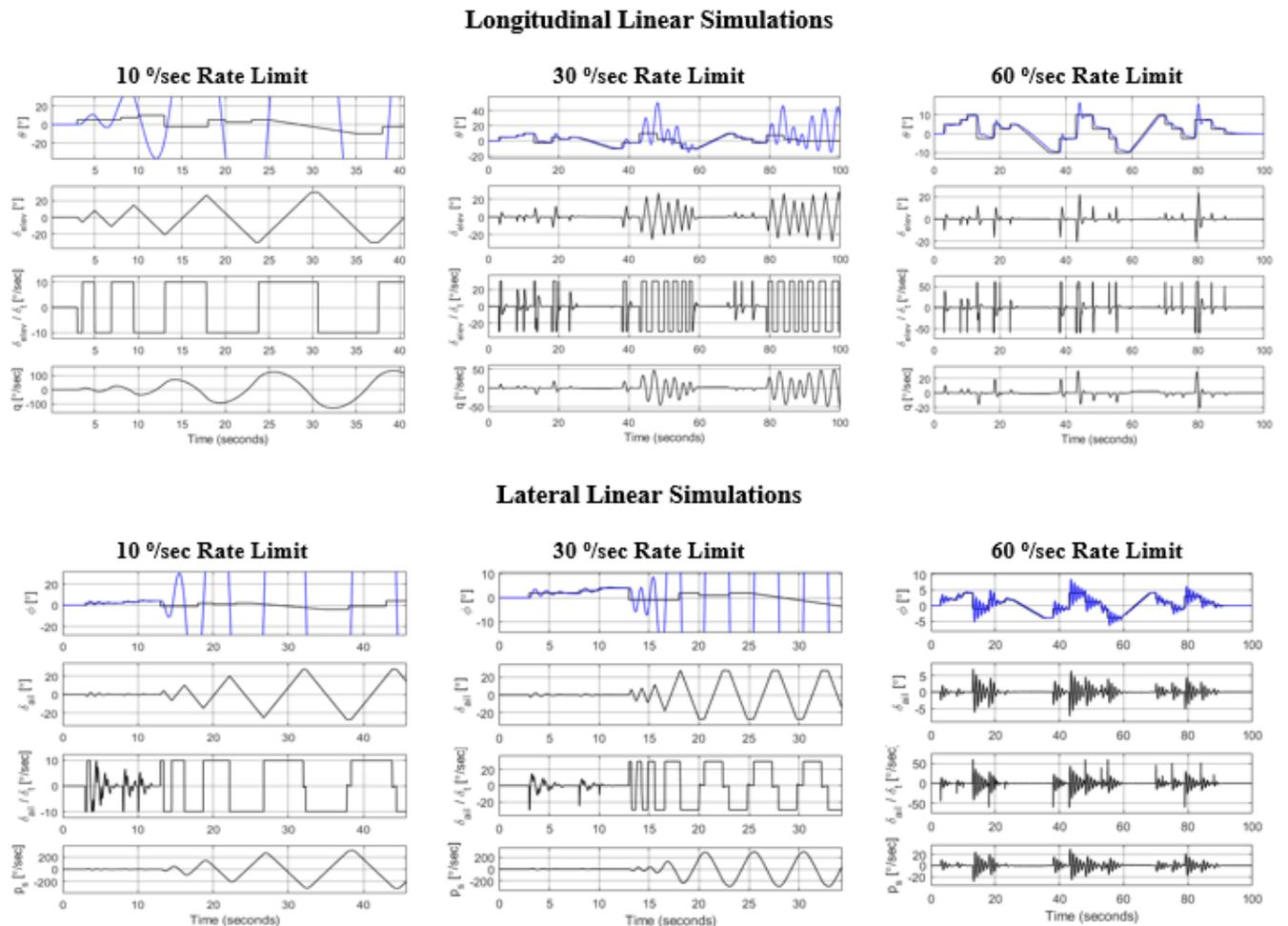


Figure 14: Linear Simulation Results

During the longitudinal simulations, the instability has almost started at the 30 °/sec rate limit case (Figure 14, upper part). By looking at the Figure 13, the yellow point stands for that value which is a marginal case very near to the limit. Similarly, the lateral directional system become unstable for the rate limits below 30 °/sec, this is also observed from

the OLOP analyses results from *Figure 13*, right part. Generally, linear simulations show that at which rate limit value the instability started to occur could be found with the use of the OLOP criterion.

### 5.3.2. Nonlinear Simulations

Nonlinear simulations have been conducted in order to verify the former results. Nonlinear simulations includes the coupled motion (lateral-longitudinal) different from the linear simulations.

In reality, characteristics of the rate limiter element has been changed with respect to hinge moments (aerodynamic loading). This behavior has been implemented to the simulations in order to increase the fidelity of the model.

The time histories of the longitudinal simulations are presented through the pieces of the *Figure 15*. Results of the OLOP analyses (*Figure 13*) and the linear simulations has shown that at the 30 deg/sec rate limit, instability has started to occur. In *Figure 15*, top right part, the nonlinear response shows marginal characteristics, yielding the same conclusion with the former analyses. During the longitudinal attitude tracking task, the loads on the control surfaces remain moderate. Thus, the effect of the changing rate limit value could not deduced seemingly.

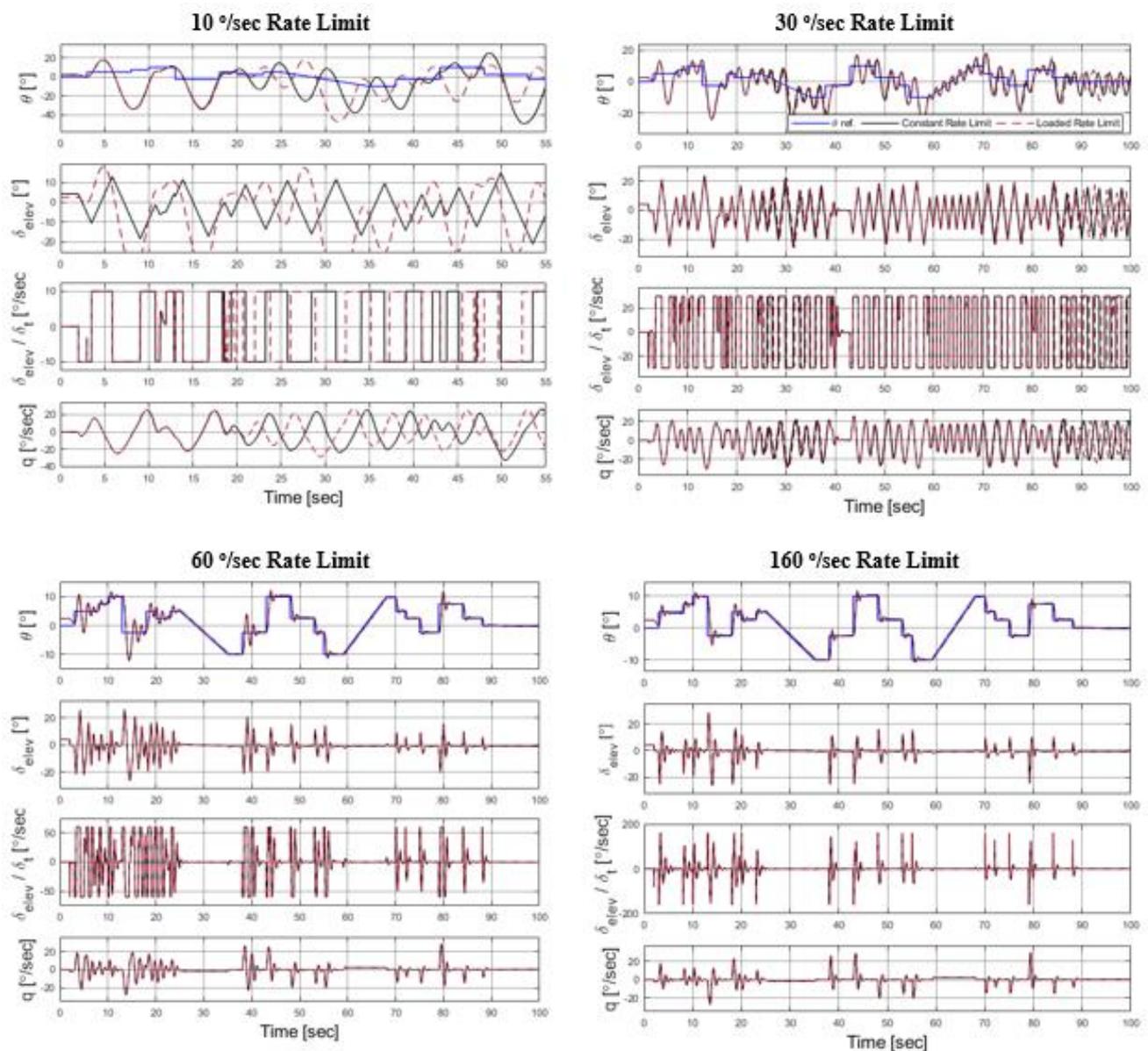


Figure 15: Nonlinear Simulation Results – Longitudinal

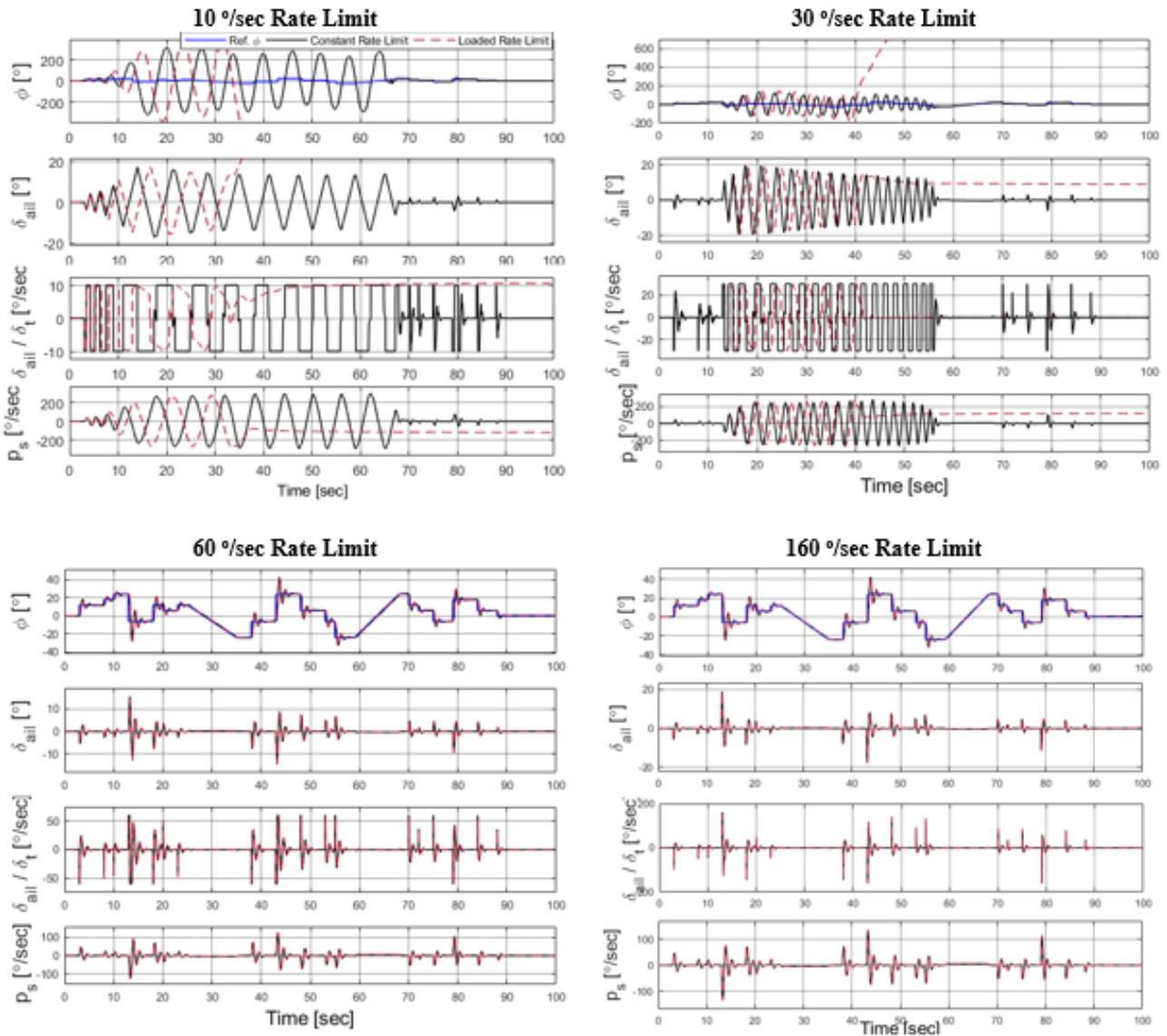


Figure 16: Nonlinear Simulation Results – Lateral

Lateral - directional attitude tracking tasks have conducted. The time histories of the simulations are presented through the parts of the *Figure 16*. OLOP criteria predicts the instability for the rate limit values below 30 deg/sec (*Figure 13*). Linear simulations has also the same conclusion with the OLOP analysis. However, in the nonlinear simulations, the response is marginal for the constant rate limit. With the higher fidelity in actuator model, nonlinear simulation results show instability (*Figure 16*, top right part). On the top left part, the degradation in the rate limit due to the hinge moments could be detected.

## 6. Conclusion

This study presents a structured way to integrate the Type-2 PIO analysis in the early phases of control law design procedure. Comparisons are held between different analysis methods in order to support the conclusions. Effect of the hinge moments on the actuator rate limiting also has been investigated. Important notices are listed in following paragraphs.

1. During the control laws design process, the OLOP criterion could be integrated as the other PIO criteria. The appropriate pilot model and command path design (stick gradients etc.) are prerequisites.
2. Actuator first order dynamics has an effect on the OLOP, however, if the time constant is small enough, the effect may be neglected.

3. Pilot model has a noticeable effect on the location of the OLOP. Pilot models should be considered separately for lateral directional and longitudinal control actions. The usage of high gain pilot models adds more conservatism to the analyses. The scope of the analysis should be considered while selecting the pilot model.
4. Linear simulations with the attitude tracking task lead almost the same conclusion with the OLOP criterion.
5. Nonlinear simulations also gave the similar outcomes. Especially for the lateral control characteristics, the OLOP criterion and the linear simulations are more conservative in terms of PIO prediction.
6. Effect of the loading on the actuator has been integrated into the nonlinear simulations. For the rate limit values smaller than 30 deg/sec, the contribution of the drop in the rate limit is significant. These values are considered as too slow for a high performance jet aircrafts. However, in case of a failure, this condition would be important.

In the operational conditions, with the nominal rate limiting values, the proposed control law design does not have any soft point. Thus, as the future work, the real failure scenarios could be tested. Furthermore, the piloted simulations would be beneficial to verify the pilot models are appropriate or not.

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