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Reliability-Based Design Optimisation of Composite Structure using Multi-fidelity Modelling

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Abstract
In this work, multi-fidelity modelling is studied to conduct the Reliability-Based Design Optimisation (RBDO) of a steel drain cover and a mono-stringer composite panel. Design of Experiments (DOE) creates sampling points from the desired design space. Artificial Neural Networks (ANNs) generate surrogate models using input and output pairs calculated by finite element analyses. Two optimisation process were multi-objective problems that find the optimal designs which satisfy the given constraints. Each random design variable was assigned an expected level of uncertainty for these reliability analyses. Results of the surrogate model are compared with those of the finite element model.

1. Introduction

The conventional design approach of composite materials not considering design uncertainties might bring about the over-designed structure [1]. There are many research papers which cover the structural analysis and optimal design concerning mechanical loading or thermal loading. Yoo and Kim presented an analysis on the thermo-mechanical buckling and vibration of a sandwich composite panel using the finite element method (FEM). Genetic algorithm (GA) found the best stacking sequence to prevent thermal buckling and thermal vibration by finding maximum eigenvalues [2]. NASA summarised the lessons learned of composite mechanics from micromechanical behaviour to macro-mechanical behaviours, such as multi-scaling modelling, design and fabrication, etc. They proposed the method of low-cost composite manufacturing and the design approach of more analysis - fewer tests as future challenges [3].
To enhance the structural reliability, various fields of structural optimisation have been interested in the development and application of the Reliability-Based Design Optimisation (RBDO) that minimise the probability of failure. RBDO was conducted to design a composite C-beam having design uncertainties. The purpose of this work is to not only maximise the reaction force but also minimise the twist angle. Deterministic optimisation (DO) was also performed to compare to the results of RBDO. Structural analysis and optimisation were conducted using Simulia Abaqus and Simulia Isight, respectively, to find the best design values [4]. Lopez et al. applied RBDO and DO to composite stiffened panels. In particular, these optimisation methods found that the stacking sequence of these composite panels to maximise the ultimate load with respect to post-buckling regime including progressive failure analysis. Using the initial configuration by DO, RBDO was performed to obtain the orientation of layers with respect to each target reliability index. The values of the objective function calculated by RBDO were low compared to those from DO because RBDO considers the uncertainties of design variables [5].
In general, the optimisation process like RBDO requires expensive computational efforts to find the optimal solutions which are satisfied with their objective functions and constraints. To overcome these efforts, a surrogate model has been studied and applied to the area of structural optimisation. The variety of surrogate models was defined by three components that are the statistical model and assumptions, the basic functions and the loss functions. The cross-validation method, a type of error analysis, was introduced because it reduces the time to create surrogate models. The comparison between single instances of different metamodel techniques and multiple instances of different metamodel techniques was introduced. This comparison demonstrated that multiple surrogate models produce more robust optimisation results despite a longer optimisation time than single surrogate models [6]. Bacarreza et al. used the radial basis function (RBF) that is a type of the artificial neural networks (ANN) to create the surrogate models of composite stiffened panels. The output of this method is a linear combination of input and output of the original model, so the multivariable function can be approximated by the combination of a simple function. This surrogate model was validated by the leave-one-out cross-validation method [7].

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The concept of the multi-fidelity model has been introduced in the area of optimisation since 2000. Through the use of high-fidelity models and low-fidelity models, the multi-fidelity model shows good results as accurate as of the high-fidelity models whereas the computational time is much less than that of the high-fidelity models. Morse et al. applied a multi-fidelity modelling approach to the reliability analysis of a rectangular plate with a circular hole. Response Surface Method (RSM) produced surrogate models that can approximate the high-fidelity and the low-fidelity models. The correction response surfaces between those two models were calculated to obtain the multi-fidelity models. Through various parameter studies, the best multi-fidelity model was determined by evaluations for accuracy and computer working time [8].
In this paper, the methods of how to generate multi-fidelity models are introduced and the surrogate model produced by ANN is applied to the RBDO of a steel drain cover and a composite panel having design uncertainties. DOE and ANN are also presented which are necessary to create a good quality of the surrogate model to represent the finite element model. Both the drain cover and composite panel were optimised using NSGA-II which is good for a multi-objective optimisation problem. The result shows that the surrogate model provides not only reasonable accuracy to the finite element model but also large computation time savings.

2. Multi-Fidelity Model

2.1 Design of Experiments

Sampling techniques are very important to generate a surrogate model that represents the original model efficiently and accurately. If the sampling techniques select appropriate points in the given design space, the accuracy of the generated surrogate model is improved and it represents the broad range of characteristics of the original model. Design of Experiments (DOE) is one of the methods to decide the location of sampling points in the design area. This DOE is a process having the general target of maximizing the amount of information obtained from a limited number of sampling points. The sampling points using such DOE estimate performance variability led by uncertain design parameters. When DOE produces a design matrix in the given condition, the design matrix indicates the values of uncertain design parameters. DOE is recommended when the distribution of random variables is not seen and when uncertain parameters follow a uniform distribution [9]. In this work, Optimal Latin Hypercube Sampling (OLHS) was used to generate the surrogate model. In OLHS, the design space is divided by the uniform interval of probability and these sampling points in each interval are combined randomly. Then, an optimisation process is adapted to the initial design matrix. Through swapping the order of two-factor levels in a column of the matrix, a new design matrix is created, and this matrix shows evenly distributed sample points in the whole design space. The purpose of this optimisation is to generate a matrix where the sample points are scattered as evenly as possible within the design area designated by the lower and upper ranges.

2.2 Surrogate Model

A surrogate model or a metamodel is an approximation method that can resolve high computational cost in the field of structural analysis and optimisation. This model provides an explicit form about given design variables based on sampling points created by DOE. There are a number of approximation methods that can generate surrogate models, such as the response surface method, the kriging method, artificial neural networks, support vector machine, etc. In this work, RBFs that is a type of ANN were used to generate the surrogate model [10]. The surrogate model is validated because it should provide not only less calculation time but also accurate solutions. There are two methods to evaluate this surrogate model, separation and cross-validation. As can be seen from Figure 1, the separation method is that the original dataset is divided by the training and testing datasets, which are determined by sampling methods like the random sampling or DOE matrix. The training set is used to generate the surrogate model using global approximation techniques. Once the surrogate model is produced, the testing dataset validates this surrogate model by comparing output values between the original model and the surrogate model. The testing dataset is a hidden dataset used only once and this dataset is not opened until the surrogate model is generated. This testing dataset is not involved in the data training job to create the surrogate model. The errors between the two models are calculated using the testing dataset and the created surrogate model can be validated. On the other hand, the cross-validation error analysis involves partitioning a sample of data into the training set and the testing set. This error analysis is also called the leave-one-out cross-validation because one point is used as the testing set during each step of this analysis. The validating point can increase from one to all sampling points depending on the designer's decision. ANN consists of three components such as neurons, weight, activation function and learning rules. RBF network of ANN uses the radial basis function as its activation function. In the basic form, all inputs are connected to each hidden layer as shown in Figure 2. As shown in the above figure, input x is used as an input to RBF and the output y of this network is a linear combination of the outcomes of RBF. Mathematically, the basic RBF model is expressed as

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$$h(x) = \sum_{n=1}^{N} w_n exp(-\gamma \|x - x_n\|^2)$$

where N is the number of neurons in the hidden layer. $h(x)$ and w_n are hypothesis and the weight of neuron i , respectively. $\|x - x_n\|$ is the radial distance between the input and centre point of neuron i, the norm is typically calculated by the Euclidean distance.

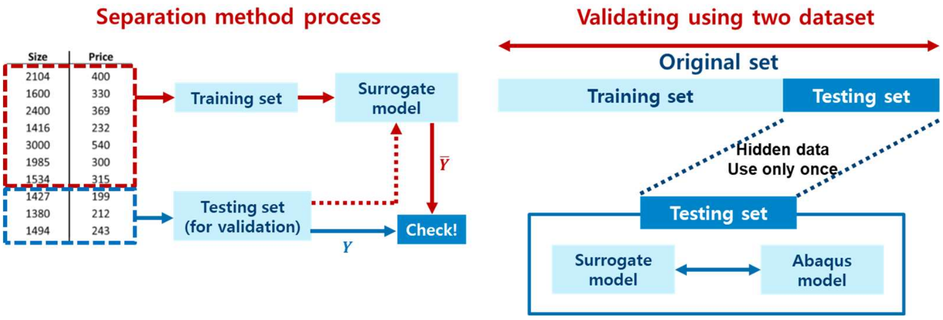

Figure 1: Separation method

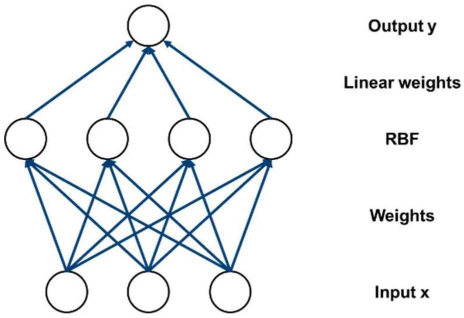

Figure 2: Basic form of RBF

3. Numerical Example

Two numerical examples of RBDO were conducted for a drain cover and a composite panel using surrogate models. The reliability analyses in the optimisation process were carried out using the First-Order Reliability Method. Non-dominated Sorting Genetic Algorithm-II (NSGA-II) was applied to the RBDO problem. The results were compared to the results of the finite element model. The aim of these two examples was to find the optimal design of structures considering their design uncertainties. These optimisation problems were multi-objective optimisation problems since the structures are optimised using different objectives which have to be minimised or maximised. The objectives of the drain cover are to minimise both the vertical displacement and the cost, whereas those of the composite panel are to maximise the first buckling load and to minimise the mass.

As shown in Figure 3, the drain cover was modelled in Abaqus Simulia and the optimal design solutions were calculated using Isight Simulia. The input data including material properties, dimension and loading condition for this work are described in Table 1. For the reliability analysis, FORM was applied. To calculate the gradient of von Mises stress, the finite difference method was adopted by the step size of 0.01 and the convergence criteria for the most probable failure point was set to 0.001. The mean values of drain height and shell thickness were 180.0 and 5.0, respectively. The standard deviation of drain height was 54.0 and that of shell thickness was 0.5. The constraint was determined by half of the yield stress, 160.0. Finally, the two objectives of minimum cost and minimum vertical displacement were found by NSGA-II. The results from this method were compared with the outcomes using surrogate models.

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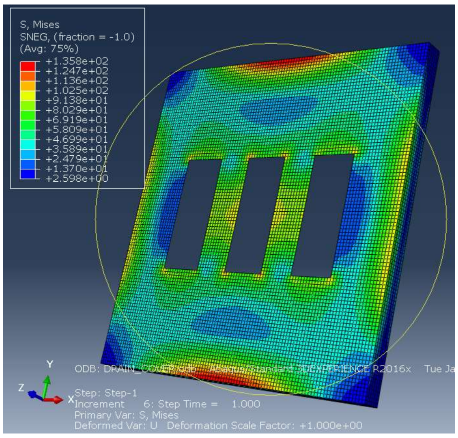

Figure 3: Model of drain cover for RBDO

Table 1: Input data

Parameter	Value
Young's Modulus	138 GPa
Poisson's Ratio	0.3
Outer Dimension	500 mm × 500 mm
Inner Dimension	440 mm × 440 mm
Drain Height	180 mm
Drain Width	70 mm
Edge Height	30 mm
Shell Thickness	5 mm
Load(vertical pressure)	10 kN

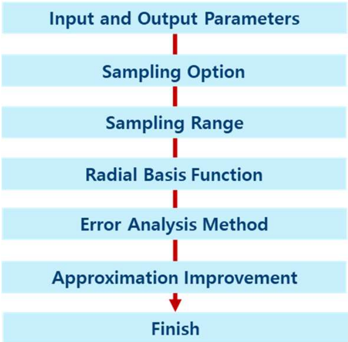

Figure 4: Process to generate a surrogate model

FORM and optimisation algorithm with the identical condition applied to the FEM model was carried out to find the optimal design parameters using the validated surrogate model. The optimal solutions using the FEM model and the surrogate model were obtained and compared in Table 2. The cost and vertical displacement showed nearly similar results between the two models. The other output, von Mises stress and shell thickness were also almost the same values with less than 1% of error. The reliability index and the probability of failure also had very similar results. Most notably, the computational time decreased dramatically by using the surrogate model. The optimisation was conducted in a PC of Intel Core i7-7700 CPU 3.40GHz and RAM 16GB. The time using FEM was over 28 hours, whereas that of the surrogate model was less than an hour. This confirms that surrogate models can dramatically reduce computational time while providing nearly identical levels of accuracy. There was a magnificent difference in the final value of drain height and it was caused that the surface of its solution is mostly flat, in contrast, the solution surface of shell thickness is very steep.

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Table 2: RBDO result comparison between FEM model and surrogate model

	FORM + NSGAII		Error
	FEM	Surrogate	
Drain height	175.60	204.63	
Shell thickness	5.36	5.49	
von Mises stress	121.03	119.52	1%
Cost	122.86	123.27	0%
Reliability index	2.06	1.88	
Probability of failure	0.04	0.06	
Vertical displacement	3.03	2.99	
Computation time	1day 04:44:08	00:40:41	

Figure 5 shows the composite panel having mono-stringer stiffened considered in this work. The geometry of this panel is made up of X1, X2, X3 and X4 which are stringer foot length, stringer height, horizontal distance between top and foot, and stringer top length, respectively. The material properties and dimension of this composite panel are described in Table 3.

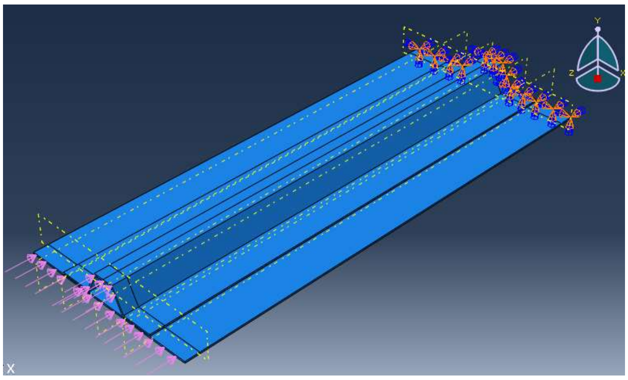

Figure 5: Mono-stringer stiffened composite panel

Table 3: Material properties and dimension

Parameter		Value
Longitudinal modulus of elasticity	E_{11}	139.0 GPa
Transversal modulus of elasticity	E_{22}=E_{33}	8.1 GPa
Poisson's ratio	ν	0.33
Out-of-plane shear modulus	G_{12}=G_{13}	3.1 GPa
In-plane shear modulus	G_{23}	4.8 GPa
Skin and stringer thickness		2.208 mm
Skin and stringer layup		$[45/-45/0/0/90/0]_s$
Panel length, *L*		600.0 mm
Panel width, *W*		250.0 mm

In Figure 6, the optimisation results using surrogate models are compared. As can be seen this figure, Pareto Fronts show the non-dominated sorting designs that are satisfied with the desired objectives and constraints. The optimal structure mass is over around 0.94kg. Table 4 shows the comparison of chosen geometry between high-fidelity models and low-fidelity models when the mass of the structure is 0.94 kg. The figure and table show that the optimisation results using the low-fidelity models are different from those of the high-fidelity models, which provides the most accurate solution. Computation time savings, as well as reasonably accurate solutions, are the main goals in this work. To compare the computation time between the high-fidelity models and the low-fidelity models, the simulation time using the low-fidelity models are normalised by that using the high-fidelity models. As can be seen in Table 4, the surrogate model of the low fidelity models requires around 18% of the simulation time using the surrogate model of

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the high-fidelity models, even though the accuracy of the low-fidelity models are not as good as that of high-fidelity models.

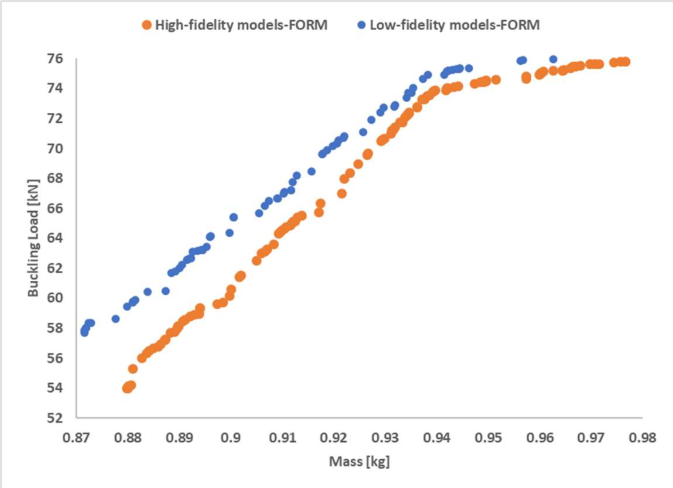

Figure 6: Comparison to RBDO results using FORM (High-fidelity vs Low-fidelity)

Table 4: Chosen geometry of composite panel and computation time

Model	**X1**	**X2**	**X3**	**X4**	**MEAN**	**STD**	**Mass**	**Time**
	[mm]	**[mm]**	**[mm]**	**[mm]**	**[kN]**	**[-]**	**[kg]**	**[-]**
High-fidelity	51.6	24.3	18.0	30.0	73.86	0.40	0.94	1
Low-fidelity	44.5	32.7	17.8	30.0	77.46	0.34	0.94	0.18

4. Conclusion

In this work, a surrogate model which is necessary to create multi-fidelity models is demonstrated by application to the RBDO of a drain cover and composite panel having design uncertainties. The surrogate models were generated by ANN that uses sampling points in the design space. The reliability analyses considering design uncertainties were performed by FORM that estimates the reliability index considering the limit-state function. The numerical examples of the drain cover and the panel show that the results of the created surrogate model provide good agreement with those of the finite element model. In particular, the computation time of the reliability optimisation using the surrogate model is a lot more economical than that of the finite element model. The optimisation problem for the composite panel highlights the comparison with surrogate models between high-fidelity models and low-fidelity models. These two examples show that multi-fidelity models using surrogate models could provide a certain level of accurate solution and considerable computational time savings when the multi-fidelity models are applied to the design of a composite structure.

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