

# All-source navigation algorithm for Hypersonic Vehicle based on ERAIM

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## Abstract

The fault detection algorithm is crucial to the all-source navigation system of a hypersonic vehicle. Integrity is an important indicator for evaluating the reliability of the hypersonic vehicle's navigation system. Therefore, an extended receiver autonomous integrity monitoring (ERAIM) algorithm is used to detect the faults of an all-source navigation system. The simulation results show that the all-source navigation algorithm developed in this paper can effectively detect the faults. If an individual navigation system has failure, its errors can well meet the requirements for the fully autonomous, highly precise and long-time navigation of a hypersonic vehicle.

## 1. Introduction

A hypersonic vehicle refers to a vehicle that cruises for a long time at the speed exceeding 5 Ma in near space<sup>[1]</sup>. Wei Huang et al. proposed the key technology of a near-space hypersonic vehicle that uses as much anti-interference information as possible and at the same time enhances fault tolerance and information fusion to improve its navigation, guidance and control technology<sup>[2]</sup>. At present, most of hypersonic vehicles use the SINS/GPS integrated navigation system. Yang M et al. proposed the GFSINS/GPS/CNS integrated navigation system for a hypersonic vehicle<sup>[3]</sup>. This paper designed an all-source navigation system based on strapdown inertial navigation system (SINS), global navigation satellite system (GNSS), celestial navigation system (CNS) and terrain referenced navigation (TRN) system.

Fault detection is key to a navigation system. Brown R G introduced three RIAM methods for GPS integrity detection<sup>[4]</sup>. Dejie Jiang referred to the RIAM concept to the GNSS/SINS integrated navigation system, namely using the multi-level separation-based fault detection algorithm<sup>[5]</sup>. Steve Hewitson et al. used the ERAIM algorithm extended from the RAIM algorithm for a satellite's fault detection of the GNSS/INS integrated navigation system<sup>[6]</sup>. Dongliang Shu adopted the  $\chi^2$  test method as the fault detection algorithm for an all-source navigation system<sup>[7]</sup>. Because the RIAM algorithm is used for satellite fault detection, the multi-level separation-based fault detection algorithm has delay in detecting step faults. When detecting ramp faults, the ERAIM algorithm has advantages over the  $\chi^2$  test method to overcome delay. The all-source navigation system designed in this paper uses the ERAIM algorithm as the fault detection algorithm for various navigation systems before integration.

The all-source navigation system of a hypersonic vehicle includes a variety of information sources. When dealing with nonlinear problems existing in an information fusion algorithm for integrated navigation system, federal filtering can reduce the estimation error of its various navigation parameters, improve its positioning accuracy, stability and real-time<sup>[8]</sup>. Peng Rong introduced and analyzed the four structures and characteristics of a federal filtering system<sup>[9]</sup>. Since the sub-filter of the non-reset federal filtering system works independently, which is helpful for fault detection, this paper adopts the Kalman filtering as the local filtering algorithm and the no-reset federal filtering algorithm as the global filtering algorithm. The simulation results show that the all-source navigation algorithm developed in the paper can effectively detect the faults of the all-source navigation system in time and that the federal filtering algorithm can isolate the faults through system reconstruction. The navigation errors after information fusion can meet the requirements for the navigation of a hypersonic vehicle.

## 2. The architecture of an all-source navigation system

The all-source navigation system of a hypersonic vehicle designed in this paper supports various kinds of sub-navigation systems and adopts the plug-and-play architecture. Fig. 1 shows the diagram for the architecture of the all-source navigation system.

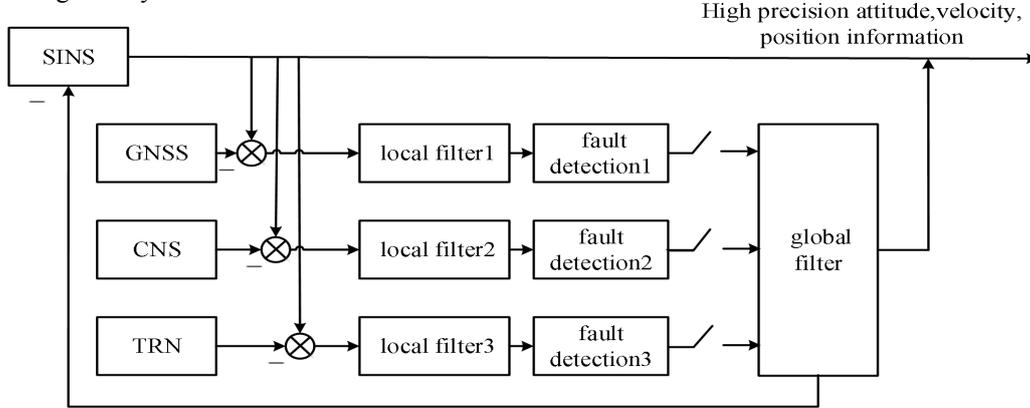


Figure 1: The diagram for the architecture of the all-source navigation system

The diagram shows that the all-source navigation system designed in this paper uses the SINS as the common reference system and other navigation systems as the individual navigation systems. Velocity information and position information are provided by GNSS and TRN, while attitude information is provided by the CNS. The all-source navigation system has such a high fault tolerance that one SINS is combined with another navigation system to form several local filters. Once it is determined that an individual navigation system has failure, the state estimation is not introduced into the global filter. When a fault detection module confirms that the faulty individual navigation system is back to normal for fault detection, the output of the local filter is introduced again into the global filter. The Kalman filter is used for implementing the fusion algorithm of the local filter, and the ERAIM algorithm is used for fault detection. The non-reset federal filtering system is used for implementing the fusion algorithm of the global filter.

The paper selects the local-level frame as the navigation reference frame and designs the Kalman filter for the all-source navigation system. The state vector is chosen as follows:

$$\mathbf{X} = [\delta\boldsymbol{\varphi} \quad \delta\mathbf{v} \quad \delta\mathbf{p} \quad \boldsymbol{\varepsilon}_b \quad \nabla_b]^T \quad (1)$$

Where  $\delta\boldsymbol{\varphi} = [\delta\varphi_E \quad \delta\varphi_N \quad \delta\varphi_U]^T$  represents the attitude error of a hypersonic vehicle;  $\delta\mathbf{v} = [\delta v_E \quad \delta v_N \quad \delta v_U]^T$  represents its Earth-referenced velocity error vector;  $\delta\mathbf{p} = [\delta L \quad \delta\lambda \quad \delta h]^T$  represents its position error vector;  $\boldsymbol{\varepsilon}_b = [\varepsilon_{bx} \quad \varepsilon_{by} \quad \varepsilon_{bz}]^T$  represents its gyroscope bias error vector;  $\nabla_b = [\nabla_{bx} \quad \nabla_{by} \quad \nabla_{bz}]^T$  represents its accelerometer drift error vector.

Then the matrix form of the state equation for the errors of the hypersonic vehicle is given as follows:

$$\dot{\mathbf{X}} = \mathbf{F}\mathbf{X} + \mathbf{G}\mathbf{W} \quad (2)$$

Where  $\mathbf{F}$  is the state transition matrix;  $\mathbf{G}$  is the noise distribution matrix;  $\mathbf{W}$  is the process vector.

Because the subsystems of the all-source navigation system can provide attitude, velocity and position information, it is necessary to select attitude error, speed error and position error as measures to correct the errors of the all-source navigation system.

The high-precision attitude information of the hypersonic vehicle is provided by the CNS navigation system in the all-source navigation system designed in this paper.

$$\mathbf{Z}_{dcm} = \hat{\mathbf{C}}_i^b (\hat{\mathbf{C}}_i^s)^T = \mathbf{I} + [\boldsymbol{\phi}_m \times] \quad (3)$$

Where  $\hat{\mathbf{C}}_i^b$  is the transformation matrix calculated by the SINS navigation system;  $\hat{\mathbf{C}}_i^s$  is the transformation matrix of the actual output from the CNS navigation system;  $\boldsymbol{\phi}_m$  is the small misalignment angle between the star sensor ( $s$ ) frame and the body ( $b$ ) frame.

The attitude measurement equation is:

$$\mathbf{Z}_\phi = \mathbf{H}_\phi \mathbf{X} + \mathbf{V}_\phi \quad (4)$$

Where  $\mathbf{H}_\phi$  is the transformation matrix for attitude measurement;  $\mathbf{V}_\phi$  is the attitude measurement noise, and  $\mathbf{Z}_\phi$  can be calculated with the following:

$$\mathbf{Z}_\phi = \frac{1}{2} \begin{bmatrix} \mathbf{Z}_{dcm}(3,2) - \mathbf{Z}_{dcm}(2,3) \\ \mathbf{Z}_{dcm}(1,3) - \mathbf{Z}_{dcm}(3,1) \\ \mathbf{Z}_{dcm}(2,1) - \mathbf{Z}_{dcm}(1,2) \end{bmatrix} \quad (5)$$

The position and velocity measurement  $\mathbf{Z}_{VP}$  consists of the difference between positions  $[L_l \ \lambda_l \ h_l]^T$  and velocities  $[v_{EI} \ v_{NI} \ v_{UI}]^T$  predicted by the SINS and their corresponding values  $[L \ \lambda \ h]^T$ ,  $[v_E \ v_N \ v_U]^T$  measured by the GNSS and TRN. The position and velocity measurement equation is:

$$\mathbf{Z}_{VP} = \mathbf{H}_{VP} \mathbf{X} + \mathbf{V}_{VP} \quad (6)$$

Where  $\mathbf{H}_{VP}$  is the transformation matrix for position and velocity measurement;  $\mathbf{V}_{VP}$  is the position and velocity noise, and  $\mathbf{Z}_{VP}$  can be calculated with the following:

$$\mathbf{Z}_{VP} = [v_{EI} - v_E \quad v_{NI} - v_N \quad v_{UI} - v_U \quad L_l - L \quad \lambda_l - \lambda \quad h_l - h]^T \quad (7)$$

These form the architecture of the all-source navigation system for a hypersonic vehicle.

### 3. The Fault detection algorithm based on the ERAIM algorithm

Traditional innovative RAIM algorithms for fault detection based on the Kalman filter innovations rely on the model of the Kalman filter. If the existing dynamics model is not accurate, the false alarm rate of a traditional RAIM algorithm is high. To solve this problem, this paper introduces the dynamics model of the Kalman filter into the traditional RAIM algorithm and proposes the extended RAIM (ERAIM) algorithm. The ERAIM algorithm comprehensively considers the measurement model and the dynamics model of the Kalman filter for state prediction. On this basis, it completes the fault detection of the all-source navigation system. In the following, the paper presents the derivation of the ERAIM algorithm.

With the least-squares principle, the  $\mathbf{X}_k$  can be estimated by integrating the measurement state  $\mathbf{Z}_k$  with the predicted state  $\hat{\mathbf{X}}_{k/k-1}$  of the Kalman filter. The corresponding measurement model is:

$$\mathbf{L}_k = \mathbf{A}_k \mathbf{X}_k + \mathbf{V}_k \quad (8)$$

Where  $\mathbf{L}_k = [\mathbf{Z}_k; \hat{\mathbf{X}}_{k/k-1}]$  represents the measurement vector of the least-squares principle;  $\mathbf{A}_k = [\mathbf{H}_k; \mathbf{E}]$  represents the new coefficient matrix made up of  $\mathbf{H}_k$  and the identity matrix  $\mathbf{E}$ ;  $\mathbf{V}_k = [\mathbf{V}_{Zk}; \mathbf{V}_{\hat{\mathbf{X}}_{k/k-1}}]$ , in which  $\mathbf{V}_{Zk}$  represents the residual vector of the measurement  $\mathbf{Z}_k$ , and  $\mathbf{V}_{\hat{\mathbf{X}}_{k/k-1}}$  represents the residual vector of state prediction.

The covariance of the stochastic model corresponding to the measurement model  $\mathbf{L}_k$  contains the measurement noise covariance matrix  $\mathbf{R}_k$  and the predicted error covariance matrix  $\mathbf{P}_{k/k-1}$  of the Kalman filter, as described in the following:

$$\mathbf{C}_{Lk} = [\mathbf{R}_k \quad \mathbf{0} \quad ; \quad \mathbf{0} \quad \mathbf{P}_{k/k-1}] \quad (9)$$

Based on the least-squares principle, the optimal estimation of the state vector  $\hat{\mathbf{X}}_k$  and its corresponding error covariance matrix  $\mathbf{Q}_{\hat{\mathbf{X}}_k}$  can be calculated with the following:

$$\begin{aligned} \hat{\mathbf{X}}_k &= (\mathbf{A}_k^T \mathbf{C}_{Lk}^{-1} \mathbf{A}_k)^{-1} \mathbf{A}_k^T \mathbf{C}_{Lk}^{-1} \mathbf{L}_k \\ \mathbf{Q}_{\hat{\mathbf{X}}_k} &= (\mathbf{A}_k^T \mathbf{C}_{Lk}^{-1} \mathbf{A}_k)^{-1} \end{aligned} \quad (10)$$

Furthermore, the residual vector  $\mathbf{V}_k$  of the least-squares method and its corresponding covariance matrix  $\mathbf{Q}_{\mathbf{V}_k}$  can be calculated with the following:

$$\begin{aligned} \mathbf{V}_k &= \mathbf{A}_k \hat{\mathbf{X}}_k - \mathbf{L}_k \\ \mathbf{Q}_{\mathbf{V}_k} &= \mathbf{C}_{Lk} - \mathbf{A}_k \mathbf{Q}_{\hat{\mathbf{X}}_k} \mathbf{A}_k^T \end{aligned} \quad (11)$$

Where  $V_k$  represents the white noise vector of zero mean normal distribution when the all-source navigation system is fault-free. Once some failure occurs,  $V_k$  is no longer zero mean white noise. So the fault can be detected with the covariance factor method, assuming the test statistics is as follows:

$$s_k = V_k^T C_{Lk}^{-1} V_k \quad (12)$$

When the all-source navigation system is fault-free,  $s_k$  represents the central  $\chi^2$  distribution variable with the degree of freedom that equals to  $n - m$ , where  $n$  represents the dimension of the residual vector  $V_k$ ;  $m$  represents the number of unknown parameters. Otherwise  $s_k$  represents the non-central  $\chi^2$  distribution variable.

The following discusses how to select test statistic thresholds, assuming that the false alarm rate is  $P_{FA}$ . The critical value  $T_{Di}$  corresponding to the  $\chi^2$  distribution variable is given in the  $\chi^2$  distribution function table.  $T_{Di}$  is the threshold value for detecting statistics. The judging criteria for ERAIM-based fault detection algorithm are:

If  $s_k \leq T_D$ , the all-source navigation system works normally

If  $s_k > T_D$ , then the all-source navigation system has faults.

#### 4. The federal filtering information fusion algorithm

This paper uses the Kalman filter for information fusion of a local filter, and the non-reset federal filter is used for information fusion of the global filter. There are four procedural steps for calculating federal filtering: information distribution, time update, measurement update, and estimation fusion.

Information distribution: According to the principle of information distribution, assign the process information of the all-source navigation system to the global filter and the local filter respectively. The formulas for calculating information distribution are as follows:

$$\begin{aligned} P_{i,k-1} &= \beta_i^{-1} P_{k-1} \\ Q_{i,k-1} &= \beta_i^{-1} Q_{k-1} \\ X_{i,k-1} &= X_{k-1} \end{aligned} \quad (13)$$

Where  $\sum_{i=0}^n \beta_i = 1$ . In the all-source navigation system designed in this paper, if the ERAIM algorithm detects that a local filter is faulty, the faulty local filter corresponds to  $\beta_i = 0$  and the remaining local filters correspond to  $\beta_i = 1/n$ , where  $n$  is the number of non-faulty local filters. In this case, the error-free information is introduced into the global filter.

Time update: The state and estimated error covariance of the local filters and the global filter are transferred according to their transfer matrixes. The local filters and the global filter operate independently of each other. The formulas for calculating time update are as follows:

$$\begin{aligned} \hat{X}_{i,k/k-1} &= \Phi_{i,k/k-1} \hat{X}_{i,k-1} \\ P_{i,k/k-1} &= \Phi_{i,k/k-1} P_{i,k-1} \Phi_{i,k/k-1}^T + \Gamma_{i,k/k-1} Q_{i,k-1} \Gamma_{i,k/k-1}^T \end{aligned} \quad (14)$$

Measurement update: The state and estimated error covariance are updated according to the latest measurement information. Since the global filter has no measurement information, the measurement update is performed independently only by the individual local filter. The formulas for calculating measurement update are as follows:

$$\begin{aligned} K_{i,k} &= P_{i,k/k-1} H_{i,k}^T R_{i,k}^{-1} \\ \hat{X}_{i,k} &= \hat{X}_{i,k/k-1} + K_{i,k} (Z_{i,k} - H_{i,k} \hat{X}_{i,k/k-1}) \\ P_{i,k} &= (I - K_{i,k} H_{i,k}) P_{i,k/k-1} \end{aligned} \quad (15)$$

Estimation fusion: The local optimal estimation of each local filter is optimally fused. The formulas for calculating estimation fusion are as follows:

$$\begin{aligned} \hat{X}_g &= P_g \sum P_{i,k}^{-1} \hat{X}_{i,k} \\ P_g &= \left( \sum P_{i,k}^{-1} \right)^{-1} \end{aligned} \quad (16)$$

## 5. Simulation and analysis

In order to validate the all-source navigation algorithm developed in this paper, a 2000 s trajectory of a near-space hypersonic vehicle is designed, as shown in Fig. 2. The initial information of the trajectory is as follows: both three velocities are 0 m/s, the latitude  $L_0 = 34^\circ 16'$ , the longitude  $\lambda_0 = 108^\circ 54'$ , the height  $h_0 = 0\text{m}$ , the heading angle  $\psi_0 = 120^\circ$ , the pitch angle  $\varphi_0 = 60^\circ$ , and the roll angle  $\gamma_0 = 0^\circ$ .

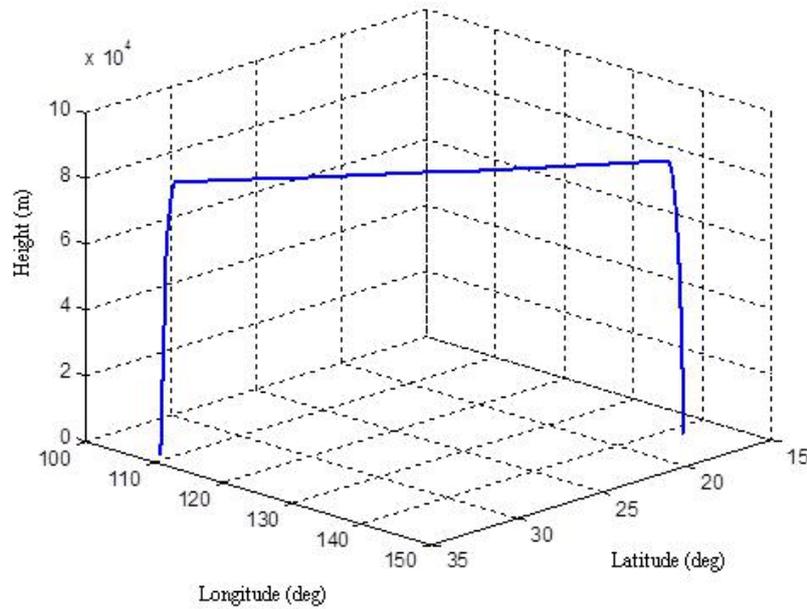


Figure 2: The hypersonic vehicle's trajectory

Table 1 gives the simulation parameters of the all-source navigation system; Table 2 gives the fault forms and sizes of various navigation systems of the all-source navigation system. Fig. 3 to Fig. 9 show the simulation results of the all-source navigation system.

Table 1: Simulation parameters

Simulation parameter	value	Simulation parameter	value
Gyroscope constant bias	0.01 °/h	SINS solution period	10 ms
Gyroscope random error	$0.001^\circ / \sqrt{h}$	Initial roll angle error	10 "
Accelerometer constant drift	$5 \times 10^{-5} g_0$	Initial heading angle error	120 "
Accelerometer random error	$5 \times 10^{-6} g_0$	Initial pitch angle error	10 "
Gyro scale factor error	20 ppm	Initial velocity error	0.01 m/s
Accelerometer scale factor error	20 ppm	Initial position error	5 m
Gyro installation error	15 "	GNSS positioning accuracy	20 m
Accelerometer installation error	15 "	GNSS velocity accuracy	0.3 m/s
Inertial device sampling period	5 ms	Star sensor attitude accuracy	(5 " , 5 " , 15 " )
Star sensitive sampling period	1 s	TRN positioning accuracy	30 m

Integrated navigation period	1 s	Simulation time	2000 s
False alarm rate	0.00001		

Table 2: The fault of various navigation systems

	Step fault	Ramp fault
GNSS	200 m (100s)	4 m/s (100 s)
TRN	200 m (100s)	4 m/s (100 s)
CNS	50 " (100s)	4 " /s (100 s)

### 5.1 Fault detection results

The fault detection of various navigation systems before integration and that of the all-source navigation system is performed by comparing the test statistic figures of the ERAIM algorithm with threshold values, and the fault detection results are shown in Fig. 3 to Fig. 6.

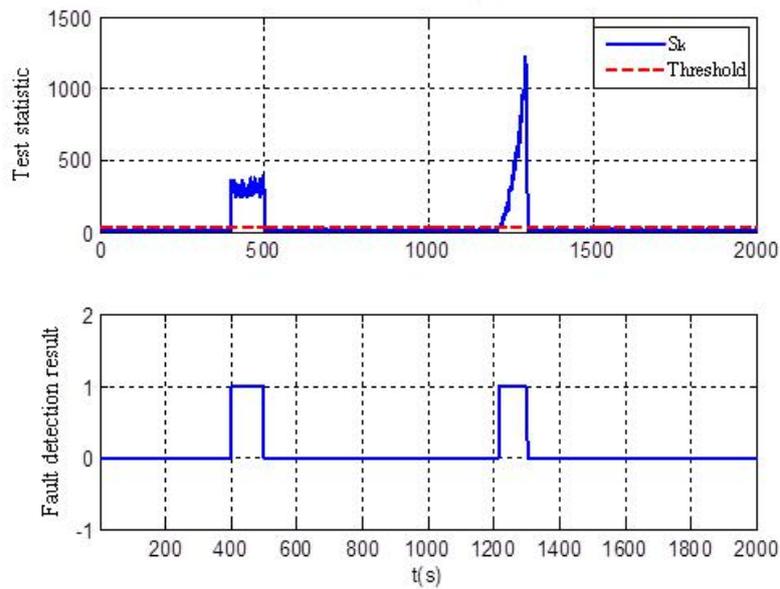


Figure 3: Fault detection results and statistics on the SINS/GNSS navigation system

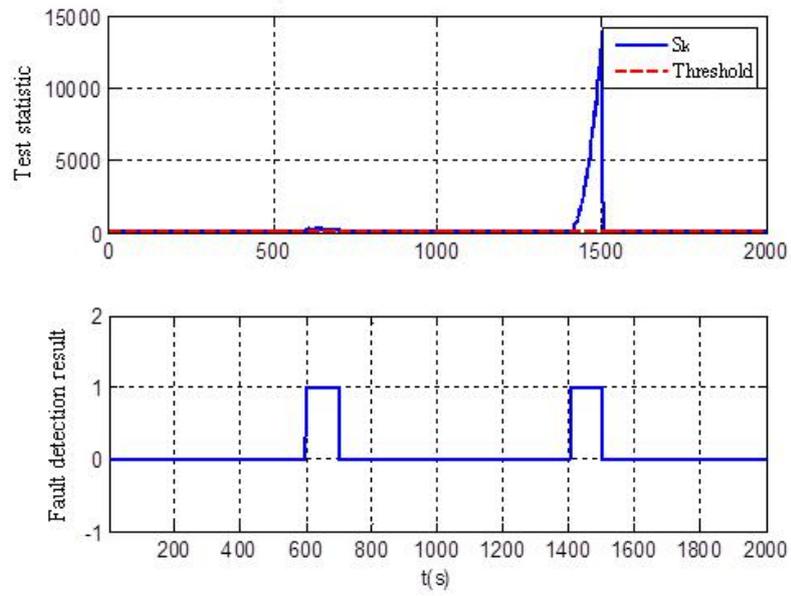


Figure 4: Fault detection results and statistics on the SINS/CNS navigation system

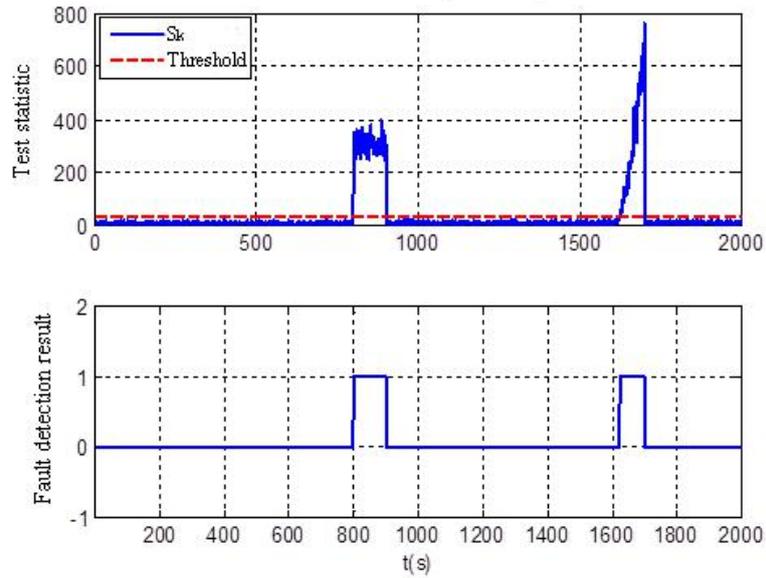


Figure 5: Fault detection results and statistics on the SINS/TRN navigation

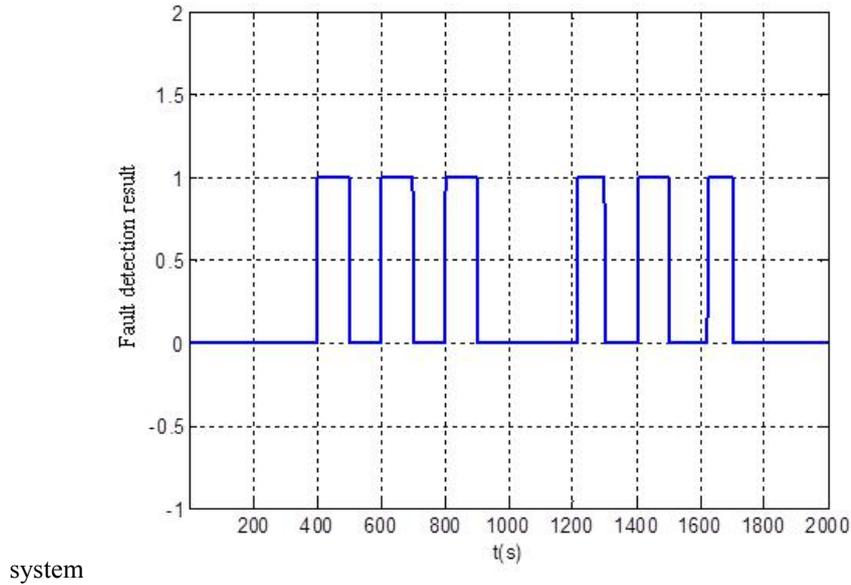


Figure 6: Fault detection results on the all-source navigation system

Figure 6 shows that the all-source navigation system designed in this paper can detect the faults of GNSS, CNS and TRN navigation systems. Fig.3 to Fig.3 show that there is no delay in detecting the step faults of GNSS, CNS and TRN navigation systems, the delay in detecting the ramp faults of the GNSS navigation system is 17 s, the delay in detecting the ramp faults of the CNS navigation system is 4 s, and the delay in detecting the ramp faults of the TRN navigation system is 23 s. The analysis of the fault detection results of the three navigation systems show that the ERAIM-based fault detection algorithm proposed in this paper can detect not only step faults quickly but also ramp faults, although there is a certain delay.

## 5.2 Error analysis of the all-source navigation system

With the global filter, the information distribution coefficient of the navigation systems detected to be faulty is  $\beta_i = 0$ , that is, the faults thus detected are isolated from the global filter. When the faults return to normal, the distribution coefficient are re-allocated to  $\beta_i = 1/n$ , so as to achieve the plug-and-play effect. The errors of the all-source navigation system after information fusion are shown in Fig. 7 to Fig. 9.

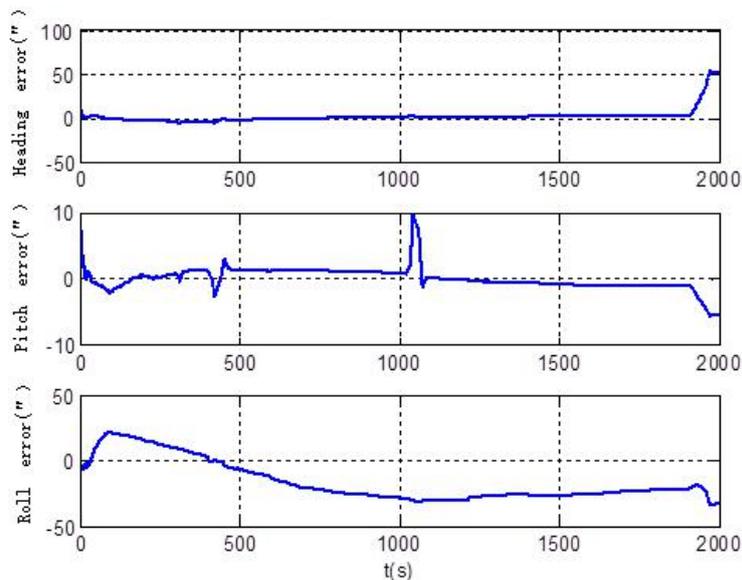


Figure 7: Attitude errors of the all-source navigation system

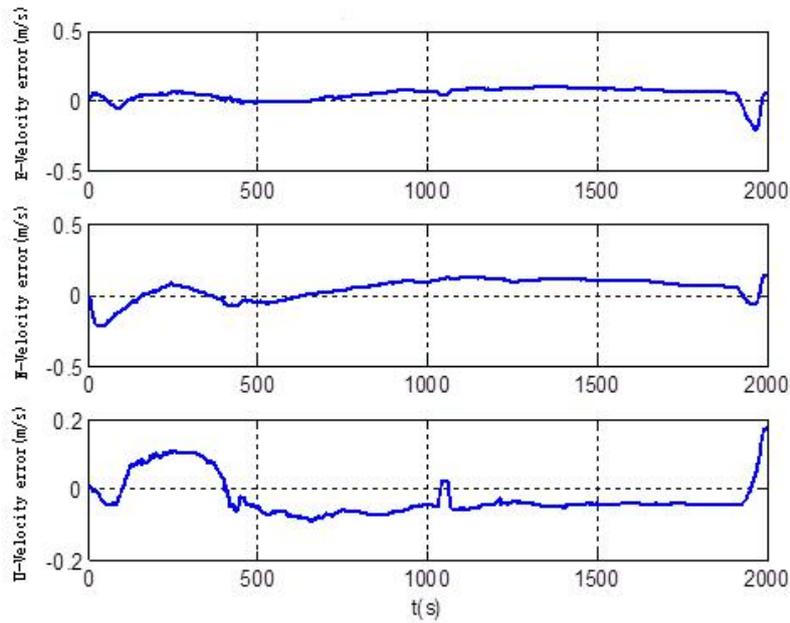


Figure 8: Velocity errors of the all-source navigation system

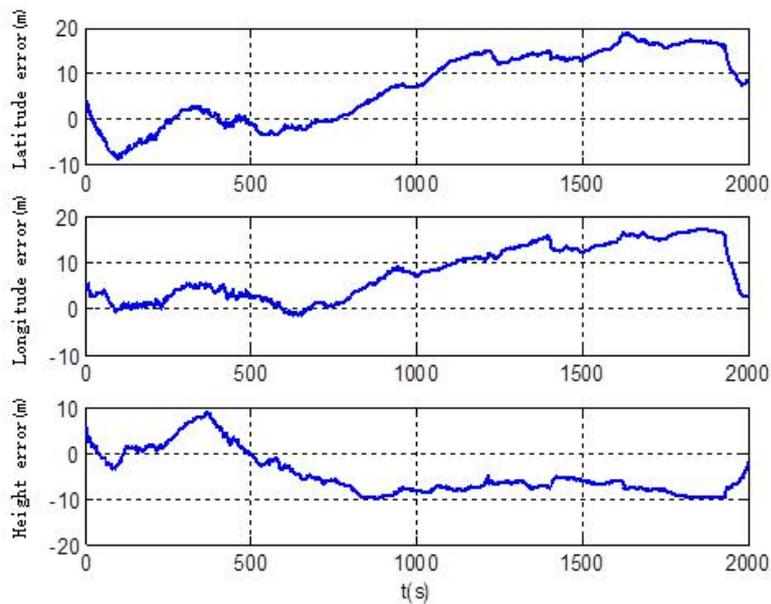


Figure 9: Position errors of the all-source navigation system

Figure 7 shows that the heading angle error of the all-source navigation system is less than  $60''$ , the pitch angle error is less than  $10''$ , and the roll angle error is less than  $40''$ . Figure 8 shows that the east-velocity error, the north-velocity error and the up-velocity error of the all source navigation system are all less than  $0.2$  m/s. Figure 9 shows that both the latitude error and longitude error of the all-source navigation system are less than  $20$  m and that the height error is less than  $10$  m.

## 6. Conclusion

This paper proposes the all-source navigation system suitable for a near-space hypersonic vehicle and uses navigation information to design the all-source navigation algorithm that has the plug-and-play effect. The ERAIM algorithm for the SINS/GNSS integrated navigation system is used to solve the problems existing in the fault detection by the all-source navigation system. The simulation results show that the all-source navigation system designed in this paper can navigate a spacecraft for 2000 s and quickly detect its faults when an individual navigation system has failure. In this case, it isolates the faulty individual navigation system and correct its attitude error, speed error and position error. If an individual navigation system has failure, its position error is within 20 m, the speed error is within 0.2m/s, and the attitude error is within 60", well meeting the needs of the fully autonomous, highly precise and long-time navigation of a hypersonic vehicle.

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