INVESTIGATION OF VORTEX FLOW WITH SHOCK WAVE

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Interaction of a streamwise vortex with a shock wave is a complex three-dimensional, most often unsteady phenomenon encountered both in the problems of external and internal flows (inlets, nozzles, combustion chambers). This is a classical problem in theoretical gas dynamics and is very important for analyzing an external flow around supersonic aircraft with wings of large sweep and small aspect ratio. A specific features of such flows is the generation of a vortex sheet near the leading edge of the wing and its subsequent rolling into two isolated streamwise vortices that interact with shock waves formed on the aircraft surface elements. Origination of vortices is also observed in a supersonic flow around a body under angle of attack. In the case of airplane configuration, such vortex structures can enter the engine air inlet. Depending on the interaction mode of a streamwise vortex with a shock wave always formed ahead of a supersonic inlet, the flow rate, drag, and other parameters of this propulsion element can be significantly changed. It was shown in the experimental works [1, 3, 4, 6 - 9] that at the action of a strong external disturbance on the streamwise vortex propagating in the flow results in a so called vortex explosion or vortex breakdown. This phenomenon took place both in incompressible subsonic flows containing vortices and supersonic compressible subsonic and supersonic ones. The term "vortex explosion" means the formation of a point (or a surface) of a total flow stagnation in the region of interaction of the vortex and the strong disturbance and the formation of a reverse flow region (which is sometimes rather extended) near the vortex centerline. In the range of supersonic velocities the first results on the vortex explosion were obtained in the experiments in [5], in which one put forward an assumption that the mechanisms of a vortex explosion and boundary-layer separation are similar. This idea was developed in [6], where it was found that the vortex breakdown occurred at angles of attack of a wing or a body of revolution reaching 5—10 ^{o} and shock-wave intensity which is close to that of a normal shock wave with a subsonic velocity behind the slip surface. Origination of unsteady oscillating modes in the region of interaction of the streamwise vortex and the shock wave was found in [6, 8, 9]. Quantitative experimental data were obtained in [4, 8]. The investigation results confirmed the presence of recirculation zones in the interaction region. The ratio of a circular velocity to the axial one was entered as a parameter determining an interaction mode, the data on the structure of interaction regimes were obtained. In the case of intersection of the streamwise vortex and the shock wave, interactions of two types are possible: 1) interaction of the streamwise vortex with the shock wave perpendicular to the vortex axis; 2) interaction of the streamwise vortex with the shock inclined to the vortex axis. The first type of interaction was experimentally studied in [4, 8, 9], and the second one was considered in [6, 9]. These works show that in the first case the vortex is broken down, and a reverse flow region with flow unsteadiness appears. There are a few papers where the interaction of the streamwise vortex and the shock was numerically simulated. The results of these works are contradictory. Thus, in some of them vortex-shock wave interaction regimes with vortex explosion were observed [2, 7, 10], and in other works no vortex breakdown was obtained [11]. So, it is necessary to make a mathematical model of a complex spatial (often unsteady) phenomenon: interaction of a streamwise vortex with the shock wave.

Interaction of the streamwise vortex with the shock was analyzed numerically using unsteady 3D equations of Euler and Navier-Stokes. The scheme of a type of the Godunov method is used for solution of the system of equations. The method described in [13 - 14] is applied to determine numerical fluxes at the boundaries of the computational cell. In the form described, the scheme has the second order accuracy in terms of spatial variables. Some computations were performed with the fourth order accuracy in terms of spatial variables. Laminar and turbulent flows were computed. Interaction of the streamwise vortex with both incident and normal shock waves was studied. It was constructed a mathematical model of the vortex to study theoretically the interaction of the streamwise vortex with the shock [2]. In the initial section the vortex was isolated from the external cocurrent flow. The simplest vortex models are constructed within the framework of an axisymmetrical flow.

Such a model allows one to state a boundary condition at the entrance into the calculation domain at solution of three dimensional problem of interaction of the vortex with external disturbance (for example, with a shock wave). The structure of a streamwise vortex formed behind a diamond-shaped body exposed to a supersonic flow was experimentally studied in [8, 9].

The investigation results showed that the structure of the streamwise vortex being formed is close to the structure of the Burgers vortex. In the present work, the Burgers vortex model was supplemented by experimental dependences of the streamwise axial velocity in the vortex center on the vortex radius and flow velocity at infinity [2, 4]. In the case of the incident shock, the influence of basic constitutive parameters (relative streamwise velocity Φ and the vortex circulation Γ) on the flow structure was revealed using a parametric investigation. According to this, three different types of interaction are obtained: weak (Fig. 1), moderate (Fig. 2), and strong (Fig. 3). Weak interaction is characterized by a small distortion of a shock-wave front, minimal variation in a vortex structure and supersonic velocity in the entire flow region. Therefore, the flow structure is not changed in time. This is in agreement with the experimental data presented in [4] which result in that at weak interaction the flow structure remains invariant in time. The calculations show that at the moderate interaction (Fig. 2) the local three-dimensional region of the subsonic flow appears behind the shock-wave front and is propagating downstream under some positive angle to the incoming vortex axis. There are not observed any reverse flows in the subsonic flow region, where a longitudinal velocity component is always positive. A mean deflection angle of the axis of the subsonic region exceeds a wedge angle forming a shock wave by 3 - 4 ^{o}. In the case of strong in-

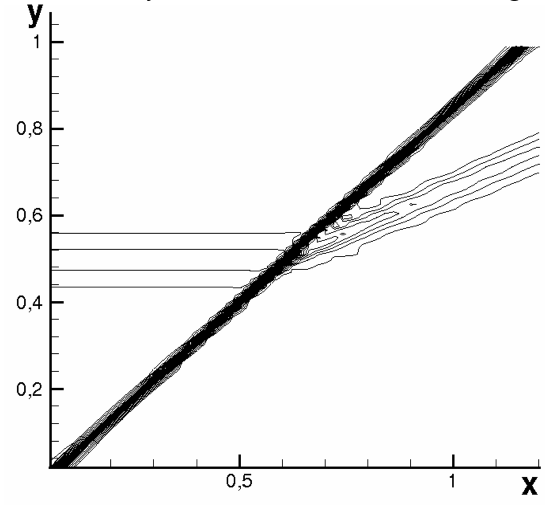

Fig. 1

teraction (Fig.3), a three-dimensional subsonic reverse flow region arises ahead of the main shock-wave front. In its structure, this region is similar to separated flow region

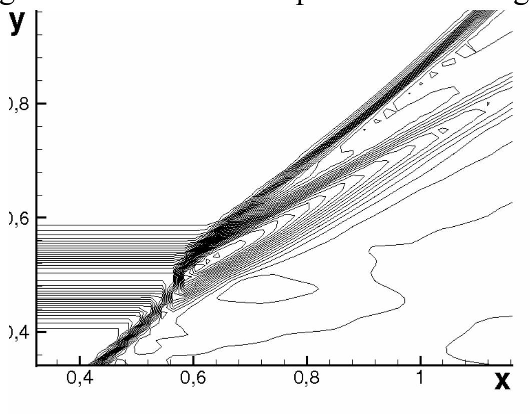

Fig. 2

arisen, for example, in the viscous supersonic flow around a compression corner.

The arising the reverse flow region leads to a significant expansion of vortex cross-section. In all studied regimes of strong interaction, a subsonic reverse flow region was observed, which was separated from the supersonic flow region by a slip surface over entire perimeter. In the region between the slip surface and the shock wave formed ahead of the recirculation region the flow is supersonic, and it is a subsonic one under the slip surface. The size of the reverse flow region depends on the shock-wave strength, i.e., on the wedge angle generating the shock wave. All results presented in Figs 1 – 3 are performed at Mach number of the free-stream of M=3. A relative axial velocity Φ was equal to 1 for the weak interaction, Φ=0.8 ($\Phi = u_x / u_0$) for the moderate interaction regime, and Φ=0.6 for the strong interaction regime.

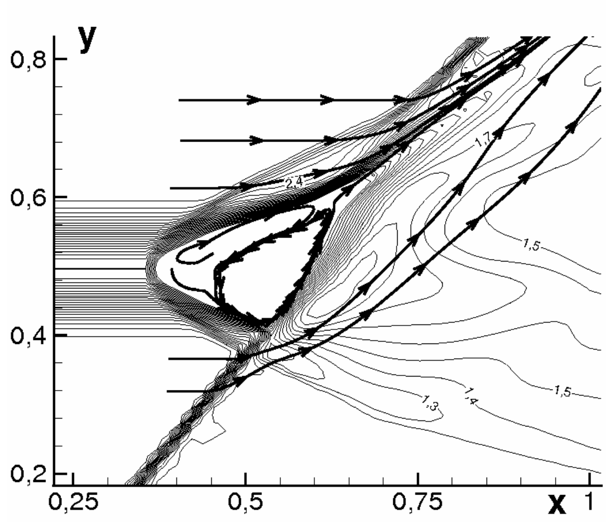

Fig. 3

In the case of a normal shock wave, the flow structure is determined within the range of Mach numbers M=1.6 - 6 of the freestream, that sharply differs from the flow structure at interaction with an inclined shock. It is revealed two interaction modes significantly differed from each other – weak (Fig. 4) and strong (Figs. 5, 6, and 7). In the case of weak interaction (Fig.4), the vortex passes the shock wave, and its shape almost remains the same both upstream and downstream. This result agrees with the experiments performed in [4]. Numerical results show that the vortex structure behind the shock remains unchanged. Rate of azimuth velocity is almost unchanged. Slight vortex expansion is observed behind a normal shock wave. An axial velocity component decreases to a core axis of the vortex. In the case of a weak interaction mode, the reverse flow region is not observed behind the normal shock front. So, the vortex explosion dos not occur. At decreasing in the axial velocity of

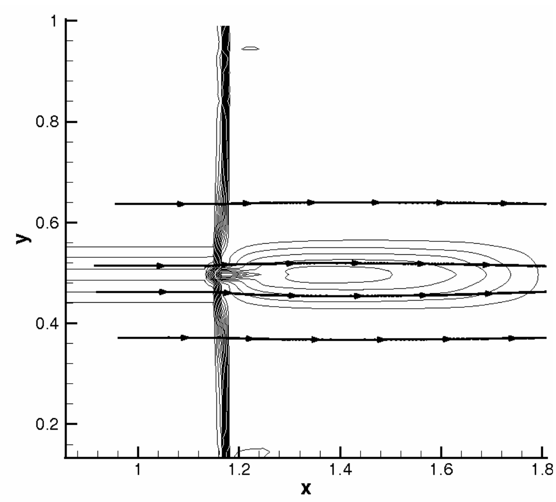

Fig. 4

the incoming vortex for fixed

Mach number, a new phenomenon "vortex explosion" occurs in the flow (Fig.5). A conical shock wave with a blunted top on a vortex axis is arisen in the flow structure. This shock wave is so called boundary separating an un-

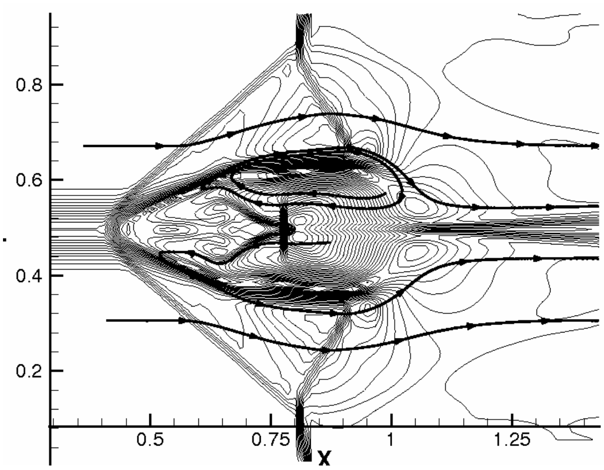

Fig. 5

disturbed flow region from a disturbed one. Inside the cone originating at interaction of the vortex with shock the flow is oscillating and rearranged with time. Since at the vortex circulation the Mach number increases, and the velocity at the vortex axis decreases, for the case of a strong interaction the cone sizes (length) are monotonously increased too.

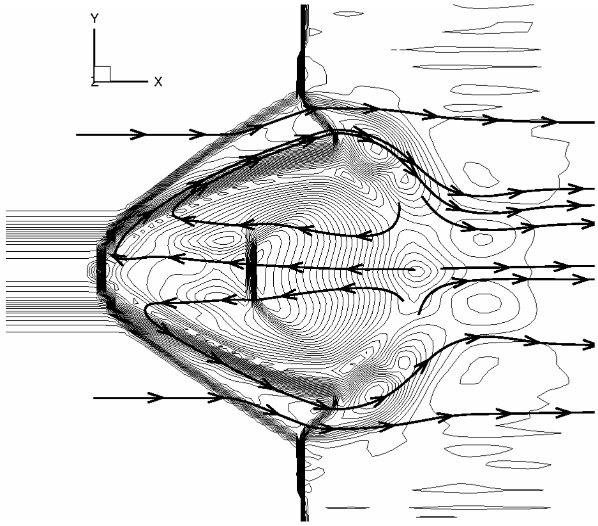

Fig. 6

Herewith, a cone corner is monotonously de creasing. With increase in time the cone height growths, and the angle at the cone apex remains

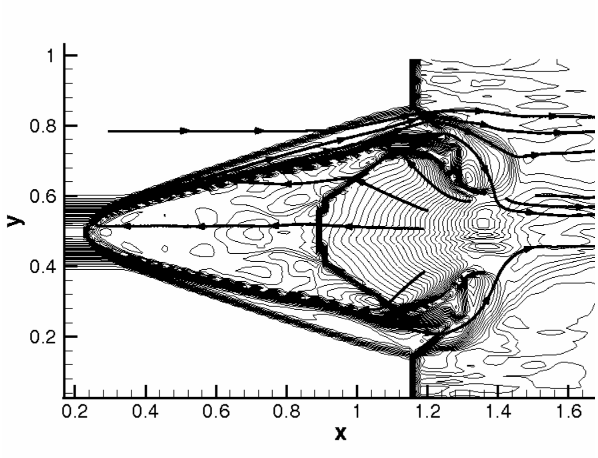

Fig. 7

unchanged. Downstream the shock (i.e., from the apex of a conical shock) a stagnation point is formed on the axis of symmetry. Behind this point the flow sharply changes its direction forming a recirculation region. The stagnation point at a cone top is determined by a negligibly small component of the axial velocity.

The strong interaction modes of the streamwise vortex with both normal shock and inclined shock have similar features. A conical shock wave, a discontinuity surface of parameters and significant region of the recirculation flow are also observed in the flow. At the same parameters of the free stream, a recirculation zone length before the normal shock wave is significantly larger than at strong interaction of the vortex with an inclined wave. The reason is that the subsonic flow behind the normal shock affects the recirculation region arising ahead the normal shock wave. In the case of a shock wave, only a supersonic flow is observed behind it which can not affect the flow structure upstream. The flow structure at interaction of the vortex with the normal shock consists of two different regions: a central subsonic region including a perturbed vortex core and supersonic region located between the perturbed vortex core and the formed conical shock wave. Figs. 5, 6, 7 show a vortex explosion entailed by interaction of the streamwise vortex with a normal shock at Mach numbers M=1.6, 3.0, 5.0 for a certain period of time. Mach number contour lines agree with a complex structure of the shock waves. A conical shock with the top of the incoming vortex axis is like to that observed in a supersonic flow around a blunt body. With distance away from the axis, the angle of slope of this shock decreases till it crosses a normal shock wave. Due to the computations performed using the Euler equations one found two clockwise rotating vortex rings in a cone-type area. A plane of their rotation is perpendicular to the free-stream velocity vector.

The computation using the Navier-Stokes equations show that there are three clockwise vortex rings. The flow was analyzed in time that shows that those vortex ring motion is of a periodic character. Analysis of the flow pattern shows that two vortices are separated by a normal shock wave generated by a upstream supersonic jet. This shock wave is perpendicular to the axis of symmetry. Current lines in the recirculation zone demonstrate that there is a structure like a convergent-divergent nozzle. This nozzle has no solid borders and its walls consist of current lines. Gas flow motion in this nozzle is rather difficult. The jet moves from some section that is a peculiar surface of flow spreading toward the free-stream undisturbed vortex (i.e., moves to the left). To the left of the spreading region in the convergent-divergent nozzle the condition for transition through the sound velocity is realized. Thus, the structure obtained is similar to the flow realized in the Laval nozzle. Therewith, the supersonic jet moving to the left reaches rather high velocities. For M = 5 (Γ=0.4, Φ=0.6, Figure 7) the Mach number on the axis of symmetry ahead the shock equals to 2.5 - 3. When it shifts downstream from the spreading surface, the flow moves to the right with a subsonic velocity. The flow structure can change in time. So, low supersonic velocities can be observed in the jet moving to the right. The Mach number decreases sharply on the shock wave, whereas pressure, density and temperature are increased. Downstream the shock (i.e., from the top of the cone shock) at the axis of symmetry a stagnation point of the flow is formed. Behind that point the flow becomes reverse forming a recirculation zone. The flow stagnation point is formed on the flow axis behind the top of the cone shock. To the left of this point the axial velocity component is positive, and to the right it is negative. In the case of interaction of the vortex with a normal shock wave two stagnation points are formed on the axis of symmetry. The first point is located on the vortex top behind a cone shock wave, the other in the throat of the convergent-divergent nozzle. Both points are the nodes of the flow spreading. In the second stagnation point the left flow has a negative axial component, and the right flow has a positive component of the axial velocity. In the Mach number distribution the second shock can be found. The flow behind the bow shock wave on the axis of symmetry is subsonic. The flow is also subsonic behind the normal shock found on the way of the supersonic jet formed in the convergent-divergent nozzle. This flow is directed toward the undisturbed flow. At interaction of the cone shock wave with the initial shock a lambda shock is generated. It consists of the following components: a cone shock wave, inclined shock wave, and a normal shock wave. All shock waves intercross in one point. The line of a contact break implementing the described structure of the lambda-shock comes from that point. The flow passing through the cone shock wave and inclined shock wave remains supersonic. A contact surface separating a supersonic flow from a subsonic one exists between the cone shock surface and recirculation zone. Below the contact surface the flow is subsonic, above its - supersonic.

Comparison of the results obtained using the Euler and Navier-Stokes equations show that the basic geometrical parameters of the region structure are nearly coincide in the case of the vortex explosion. Therewith, the gas dynamic flow pattern in the region of interaction is different. The numerical data obtained is proved out by the available experimental results.

Numerical simulation of the flow structure in the case of interaction of a streamwise vortex with a shock wave generated by a wedge was performed in the range of free-stream Mach numbers M=2 - 4. The wedge angle was varied within the range 15 - 35^o. It was assumed that the streamwise direction of the vortex axis coincides with the direction of the *x* axis. The increasing wedge angle lad to increase in shock-wave strength. The computations performed within the framework of the project showed that the regime of strong interaction of the vortex with shock wave is the most probable near the critical wedge angle, i.e., a detached shock wave is generated near the wedge. Earlier due to investigation of interaction of a vortex with a free inclined shock wave, it was shown that a shock-wave inclination influences significantly on the interaction regime. It was shown that increasing in the wedge angle by 2^o led to significant change in a type of interaction. There the transition

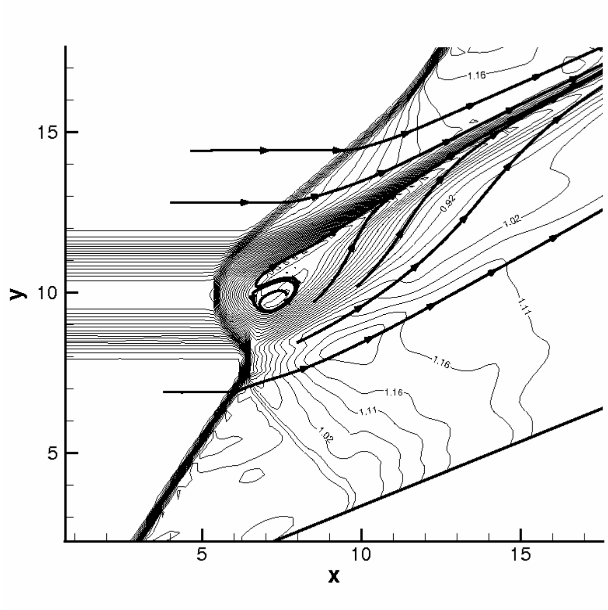

Fig. 8

from the weak interaction to a strong one is performed. The calculations were made for revealing a type of interaction of the streamwise vortex with the wedge surface. In the case of fixed parameters of the free-stream and the parameters peculiar for a streamwise vortex, a monotonous transition from the weak interaction regime to a strong interaction regime is observed. Thus, for M=2 at the wedge angle of 15^o the weak interaction regime occurs. This regime is characterized by absence of a subsonic zone ahead the shock. The pressure variation on the wedge surface was insignificant. A subsequent increasing in the wedge angle for the fixed remaining parameters resulted in arising a moderate interaction regime. In the regime of a moderate interaction a local locked zone of the subsonic flow appears in the place of a vortex impact. It is not observed recirculation zones in that zone. The flow has a stationary character. A low pressure region arises on the wedge plane. The pressure in this region is

10 percent less than the pressure at the wedge in the case of vortex lack. The low pressure region is below of the axial line of the vortex. Subsequent increasing in the wedge angle led to a continuous transition from the moderate interaction to the strong interaction. First, a very little zone of the recirculation flow appears behind the shock front. Then, with increasing wedge angle this zone is increased too. Let us consider the computation results for the strong interaction regime for M=2 and at the wedge angle of 22^o (fig. 8). Critical angle is equal to 23^o. The computation results show that in this case a shock-wave shape is sharply changed. There a local rather extended zone of a subsonic recirculation flow is formed. This fact is proved with visualization of current lines. The effect of vortex splitting into two vortices oppositely directed is observed in the plane perpendicular to the free stream ahead of the shock wave front. As for the moderate interaction regime, on the wedge plane a low pressure region is arisen. In the case of vortex absence, the pressure in that region is 20 percent less than the pressure on the wedge. The low pressure region is under the vortex axial line. Comparison of the linear dimensions of the vortex explosion ahead the shock for a free inclined shock wave show that the vortex explosion zone for a free shock is 1.5 longer than at the wedge surface. The linear dimensions of the cross-section in the interaction region differ insignificantly.

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