Mathematical Modeling of Aircraft Unsteady Aerodynamics at High Angles of Attack in The Problems of Flight Dynamics

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A modeling of aerodynamic loads during aircraft dynamic motion at high incidence flight conditions has been an arena for numerous research efforts during last two decades [1,2,3,4,5]. Different successful techniques and methods taking into account internal vortical and separated flow dynamics have been proposed and verified using various wind tunnel dynamic experiments and flight tests data. Nevertheless, the aerodynamic models at high incidence flight in all industry applications are still based on the traditional aerodynamic derivatives concept. Despite it is well known that this approach is not capable to take into account apparent frequency and amplitude dependencies available in experimental unsteady aerodynamic derivatives at high angles of attack. The considerable dynamic hysteresis effects are neglected also.

Future agile aircraft configurations specially designed for operation at high angles of attack require for dynamics analysis and control law design more adequate aerodynamic modeling methods. The improved unsteady aerodynamic modeling technique at high incidence flight will also facilitate more accurate departure prediction for airliner configurations. It is very important also for airliners upset recovery simulation which is necessary for flight safety improvement.

Usually the research efforts are mainly focused on aerodynamic modeling techniques approximating the wind tunnel experimental aerodynamic responses. Very little attention are given to assessment of accuracy of traditional representation of aerodynamic coefficients, formally extended to high incidence conditions, in terms of aircraft dynamics.

The proposed report presents the results of detailed experimental analysis at high incidence of unsteady aerodynamic effects for slender delta wing and common airliner model. Using these experimental data the state space mathematical models of aerodynamics were developed. Their dynamic components approximate the time lag effects originated from internal flow processes of vortex breakdown and flow separation development. As the state space aerodynamic model includes either linear or nonlinear differential equations it can be naturally combined with the system of aircraft motion equations. In the report the comparative analysis of aircraft longitudinal dynamics at high incidence flight considering two different forms of aerodynamics representation is considered. The first one is based on application of classical aerodynamic derivatives concept. The second one is the unsteady state space aerodynamic model. The investigations were executed for hypothetical tailless aircraft and the airliner.

It is shown that the aerodynamic model based on aerodynamic derivative concept produces a high level of uncertainty in the open-loop system eigenvalues due to its strong dependence on frequency of oscillations and therefore can not be directly applied to dynamics simulation and control law design in the time domain. It is shown also that at high incidence aircraft dynamic response could be qualitatively different if different models of aerodynamics are used.

Mathematical model of delta wing unsteady aerodynamics with account of vortex breakdown dynamic effects

The traditional mathematical model of aircraft unsteady aerodynamics at various angles of attack including flow separation regimes can be represented as follows

$$C_m = C_m(\alpha) + C_{mq*}(\alpha,k)\frac{q\bar{c}}{2V} + C_{m\delta}(\alpha)\delta \quad (1)$$

It is assumed that the unsteady and rotary effects can be described using linear terms proportional to the corresponding aerodynamic derivatives. The nonlinear effects are included in the steady terms only. This mathematical model describes very well the experimental data for low angles of attack. In Fig. 1 the delta wing steady and unsteady pitching moment data for various angles of attack are presented. For low angles of attack where the vortical flow about the wing is stable the dynamic effects are not very considerable and linear upon the pitch rate. At high angles of attack where vortex breakdown takes place the dynamic effects are nonlinear. The significant dependency upon the frequency of oscillations is revealed (see Fig. 1). These effects are distinctly displayed in out-of-phase aerodynamic derivatives (Fig. 2).

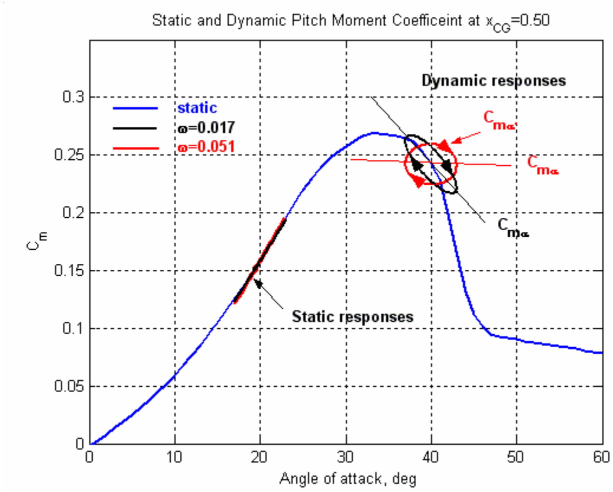

Fig. 1. Static and dynamic dependencies $C_m(\alpha)$ for low and high incidences

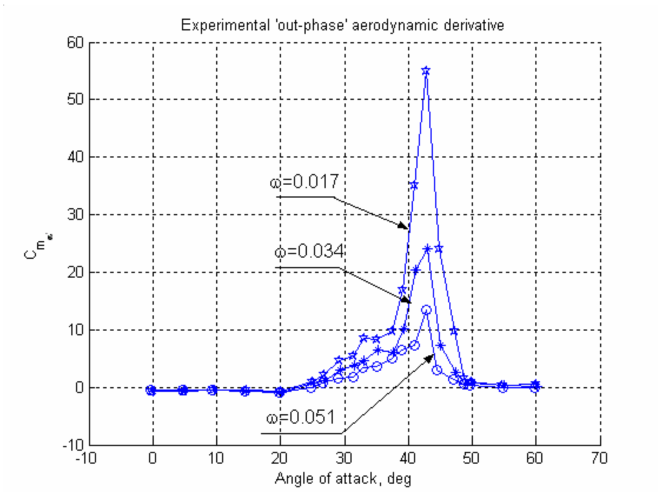

Fig. 2. Aerodynamic derivative $C_{mq} + C_{m\dot{\alpha}}$ obtained in the experiment for various frequencies of forced oscillations

The vortex breakdown is a rather slow process so during the angle of attack variation to simulate its influence on unsteady aerodynamic characteristics the using of additional differential equation is necessary. Then the mathematical model for the unsteady pitching moment could be presented as follows

$$C_m = C_{m_{att}}(\alpha) + C_{mq_{att}}(\alpha)\frac{q\bar{c}}{2V} + C_{m_{dyn}};$$
$$\tau\frac{dC_{m_{dyn}}}{dt} + C_{m_{dyn}} = C_{m_{st}}(\alpha) - C_{m_{att}}(\alpha) \qquad (2)$$

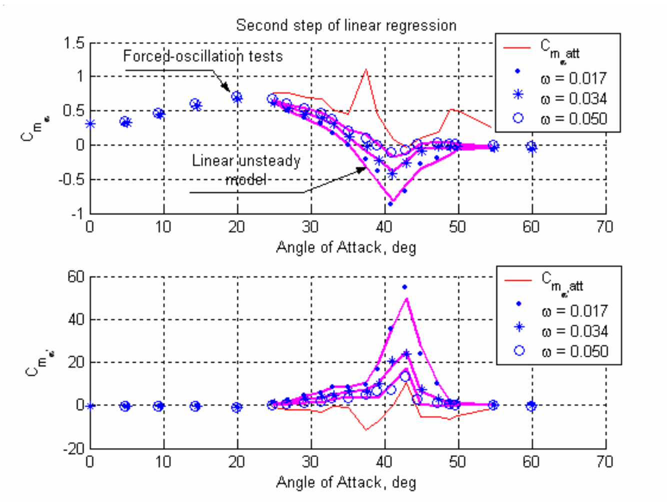

Fig. 3. Mathematical modeling of in-phase and out-of-phase forced oscillations derivatives

By selection f the characteristic time lag constant τ magnitude it is possible to achieve that the mathematical modeling results coincide with the experimental ones for various frequencies of the delta wing model forced oscillations (Fig. 3).

Dynamics of hypothetical delta wing aircraft

To estimate the influence of unsteady aerodynamics representation using various mathematical models on the flight dynamics the simple task of the disturbed longitudinal motion about various trim angles for hypothetical delta wing aircraft was considered. In Fig. 4 the eigen values of corresponding dynamical systems are presented for trim angle of attack $\alpha = 25^0$. Here the vortex flow about the wing is stable and the considerable influence of aerodynamic derivatives upon the frequencies of oscillations does not revealed. Markers of various colors represent the eigen values of open loop dynamical systems for various mathematical models of unsteady aerodynamics: the traditional model using the unsteady and rotary aerodynamic derivatives and nonlinear model with additional differential equations to simulate the dynamic effects.

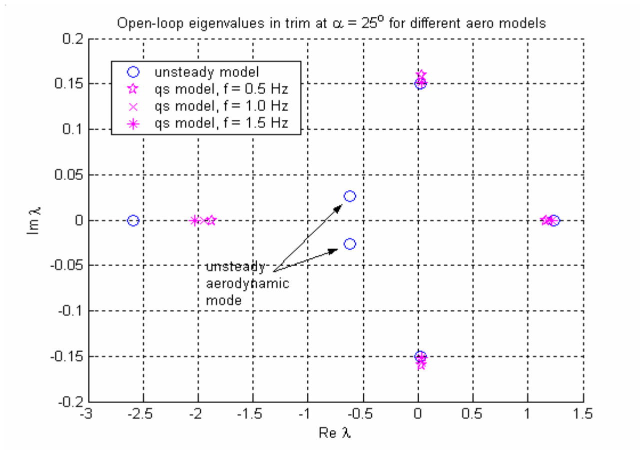

Fig. 4. Open-loop dynamic system eigen values at $\alpha = 25^0$

It is seen that for this angle of attack the eigen values of various dynamical system practically coincide with the exception of eigen values pair corresponding to two additional dynamic equations used to simulate dynamic effects.

The similar comparison results for trim angle $\alpha = 35^0$ are presented in Fig. 5. It is seen that in this case the difference in various dynamical systems eigen values are very considerable. It is the evidence that there is a considerable interaction between the dynamic equations of aircraft longitudinal motion and equations describing the unsteady effects of aerodynamics.

It is evident that the flight control system developed using the traditional mathematical model of unsteady aerodynamics can not control the dynamical system with account of vortex breakdown time lag effects.

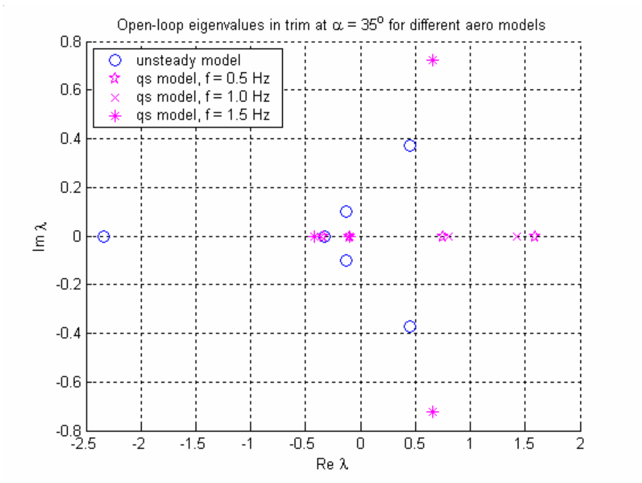

Fig. 5. Open-loop dynamic system eigenvalues at $\alpha = 35^0$

Mathematical model of high aspect ratio wing aircraft unsteady aerodynamics with account of flow separation delay

For a passenger airliner configuration with high aspect ratio wing the experimental investigations of unsteady aerodynamics at high angles of attack were executed also. At this stage the results of longitudinal aerodynamics are considered.

The analysis of steady experimental data for various tested configurations of the aerodynamic model results in the following mathematical model for the normal force and pitching moment coefficients

$$C_N = C_{N\alpha}^B \alpha + C_N^W(\alpha) + \bar{S}_T C_N^T(\alpha + \varphi - \varepsilon)$$
$$C_m = C_m^B(\alpha) + C_{m0}^W + C_m^W(\alpha) x^W(\alpha) - \bar{L}_T \bar{S}_T C_N^T$$

Here "B", "W" and "T" corresponds to the constituents due to fuselage (body), wing and tail. $C_{N\alpha}^B \alpha$ is a linear dependency of fuselage normal force coefficient for the tested region of angle of attack. Corresponding pitching moment dependency is nonlinear $C_m^B(\alpha)$. Nonlinear constituent of the wing normal force coefficient with account of body–wing interference is designated as $C_N^W(\alpha)$. The corresponding wing pitching moment part is $C_m^W(\alpha)x^W(\alpha)$. Where $x^W(\alpha)$ is the coordinate of the wing pressure center relative to CG position in MAC fraction. Term C_{m0}^W was introduced since C_N^W and C_m^W are not necessarily equal to zero simultaneously. Dependency of horizontal tail normal force coefficient upon angle of attack is designated by C_N^T. Horizontal tail inclination angle is φ and ε is a downwash angle at the tail position due to the wing presence.

Relative tail area in wing area fraction is $\bar{S}_T$. Horizontal tail arm in MAC fraction is designated as $\bar{L}_T$.

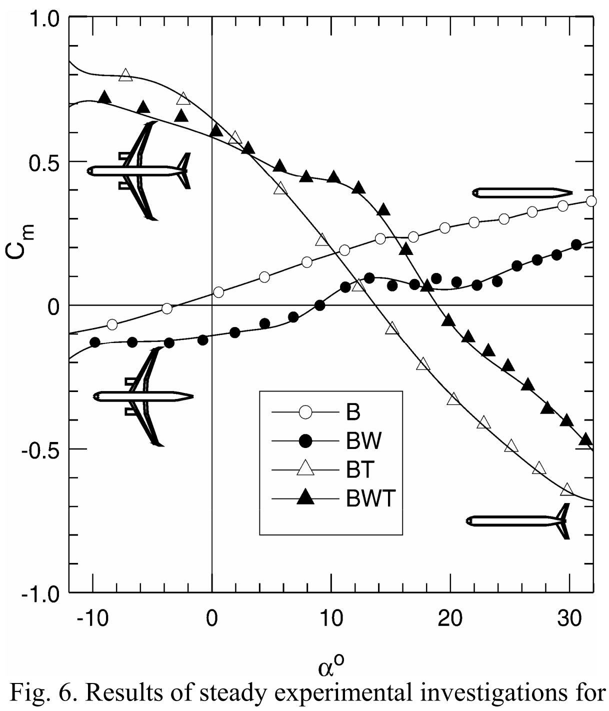

Fig. 6. Results of steady experimental investigations for different model configurations

The experimental results for pitching moment coefficients of various aerodynamic model configurations are presented in Fig. 6 (markers). The corresponding mathematical modeling results using the above expressions are shown by the solid lines. The quantities $C_{N\alpha}^B$ and C_{m0}^W as long as the nonlinear functions $C_N^W(\alpha)$, $C_m^B(\alpha)$, $x^W(\alpha)$, $C_N^T(\alpha)$ and $\varepsilon(\alpha)$ were obtained using the identification technique. Quantities $\overline{S}_T$, $\overline{L}_T$ and φ are the geometrical parameters of the aerodynamic model.

For the model configuration with Body and Wing unsteady normal force coefficient mathematical model could be developed by partitioning on linear (quasi steady) and nonlinear (dynamic) parts

$$C_N^{BW} = C_{N1} + C_{N2}.$$

Linear part is equal to $C_{N1} = C_{N\alpha}\alpha$. To describe the dynamic effects of flow separation the following first order ordinary differential equation is used

$$\tau_1 \frac{d\ C_{N2}}{dt} + C_{N2} = C_{N2}^{st}(\alpha),$$

where τ_1 is a characteristic time lag for wing separation flow development. характерное время запаздывания развития отрыва потока на крыле, $C_{N2}^{st}(\alpha)$ is the steady dependency upon incidence for nonlinear part of Body+Wing configuration normal force coefficient.

The second dynamic equation should be written should be written for wing center of pressure variations

$$\tau_2 \frac{d\ x^W}{dt} + x^W = x_{st}^W(\alpha).$$

This equation is necessary to simulate the dynamic dependencies of pitching moment coefficient.

The third dynamic equation is introduced to simulate the time lag effects of wing downwash at horizontal tail position

$$\tau_3 \frac{d\ \varepsilon}{dt} + \varepsilon = \varepsilon^{st}(\alpha).$$

The characteristic time lags τ_1, τ_2 and τ_3 from these dynamic equations were identified using the results of wind tunnel dynamic tests. The right hand side functions of these dynamic equations were identified also. It was shown [6] that the developed mathematical model enables to describe the complex dependencies of unsteady aerodynamic loads obtained during various amplitudes and frequencies model forced oscillations at wind tunnel.

High aspect ratio wing aircraft longitudinal dynamics at high incidences

The developed mathematical model of unsteady longitudinal aerodynamics for high aspect ratio wing aircraft with account of flow separation delay were used to simulate the flight dynamics at high angles of attack. The simulation of aircraft disturbed motion was executed. The case of vertical gust influence on aircraft trimmed at various incidences was investigated. The results obtained with the use of unsteady aerodynamics traditional mathematical model based on the aerodynamic derivatives concept (linear unsteady model) were compared with the new mathematical model results (nonlinear unsteady model). The goal was to investigate how the account of flow separation development dynamic effects influences on aircraft dynamic responses.

In Fig. 7 the results of comparison are presented for the case of aircraft initially trimmed at angle of attack $\alpha = 7.5^0$. It is seen that in this case the calculated results for both mathematical model practically coincide with each other.

The other picture could be seen after the same gust disturbance on aircraft initially trimmed $\alpha = 10^0$. The results of the numerical calculation for this case are presented in Fig. 8.

In this case if the linear unsteady model of aerodynamics is used the aircraft after the gust disturbance goes to the second trimmed angle at higher angles of attack (longitudinal stall). For given stabilizer deflection required to trim aircraft at $\alpha = 10^0$ the second trim angle exists also for this aircraft due to the nonlinearity of pitching moment coefficient. If the nonlinear unsteady aerodynamics model is used then the aircraft returns to the initial trim angle.

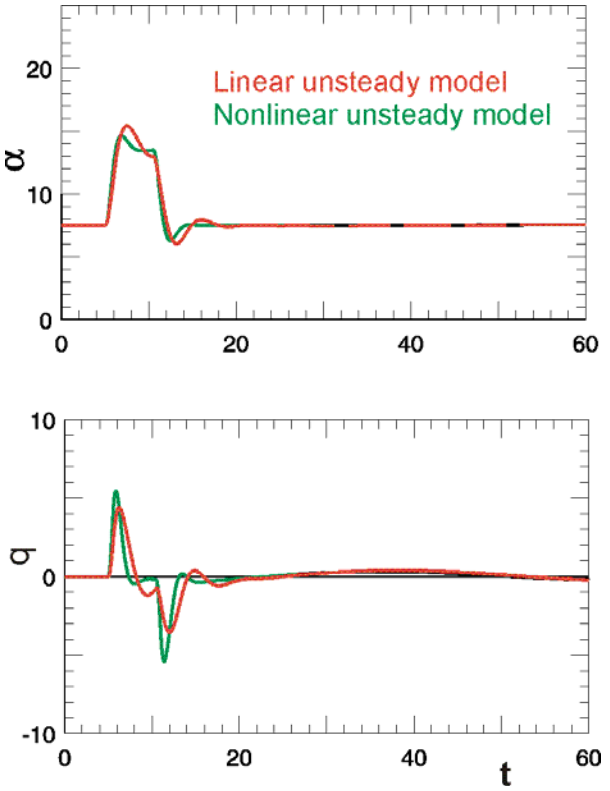

Fig. 7. Gust disturbance response for aircraft trimmed at $\alpha = 7.5^0$

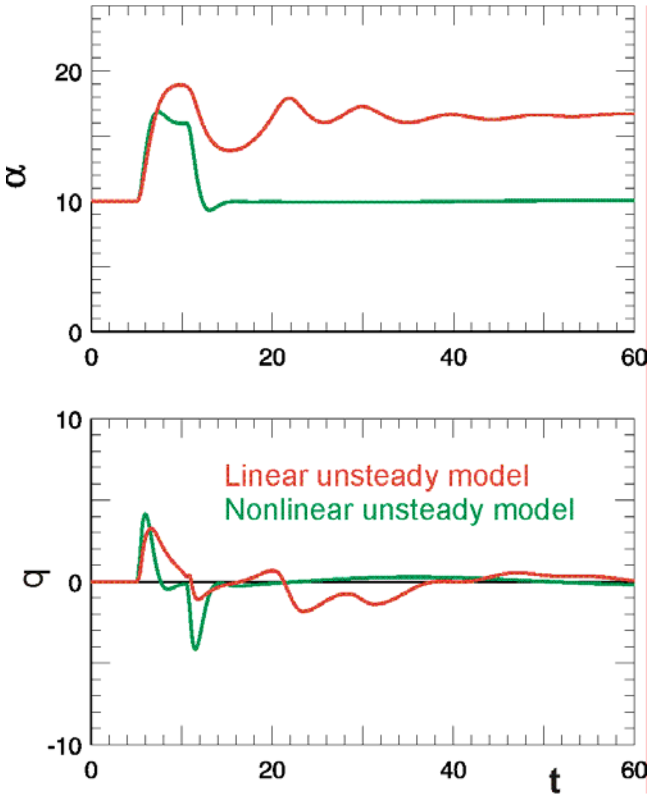

Fig. 8. Gust disturbance response for aircraft trimmed at $\alpha = 10^0$

Hence the stability region of the aircraft motion is wider if the nonlinear unsteady aerodynamic model with account of flow separation time delay (additional differential equations) is used.

Thus, both examples considered (hypothetical delta wing aircraft and large aspect ration wing aircraft) confirm that adequate mathematical modeling of unsteady aerodynamic effects is very important for correct simulation of aircraft flight dynamics at high angles of attack.

The work was supported by the Russian Federation Basic Research Foundation (RFBR Project 03–01–00918) and President grant of scientific schools support No. 2001.2003.1.

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